The M57A1 motor is a solid-fuel motor originally developed as the third stage for the Minuteman II missile program. The Minuteman II is designated as LGM-30F Mk II. The M57A1 motor has since been used on a variety of suborbital vehicles, such as target vehicles.

This motor has a distinct sinusoidal pressure oscillation which forms in its cavity. The oscillation frequency sweeps downward from 530 Hz to 450 Hz over a 16-second duration. Waterfall FFT plots taken from flight accelerometer data during the M57A1 burn are given in Figures 3 and 4. The magnitude in each plot is the unscaled acceleration.
Figure 2a.
Figure 2b. M57A1
Waterfall FFT  M57A1 Motor Oscillation  Suborbital Vehicle “A”
Flight Accelerometer Data  Motor Adapter Bulkhead Longitudinal Axis

Figure 3.
Waterfall FFT  M57A1 Motor Oscillation  Suborbital Vehicle “B”
Flight Accelerometer Data  Motor Adapter Bulkhead Longitudinal Axis

Figure 4.
The GRMS time histories for the two vehicles are shown in Figure 5.

The amplitude depends on both the severity of the pressure oscillation and the bulkheads' modal characteristics.
SR-19 Motor

Figure 6. HERA Missile

The SR-19 motor is a solid-fuel motor originally developed as the second stage for the Minuteman II missile program. The SR-19 motor has since been used on a variety of suborbital vehicles, such as target vehicles. The HERA missile in Figure 6 had an SR-19 first stage and an M57A1 second stage.

The SR-19 motor has a distinct sinusoidal pressure oscillation which forms in its cavity. The oscillation frequency sweeps downward from 700 Hz to 550 Hz over a 5-second duration, as shown in the Waterfall FFT plot in Figure 7. This data is from a static fire test where the motor was mounted to a frame, with the motor horizontal to the ground.

The corresponding GRMS plot is shown in Figure 8. The amplitude is high because the accelerometer was mounted to the forward motor dome.

Note that the SR-19 motor also has a 2X harmonic from 1400 to 1100 Hz, although this is not shown in the plots. This harmonic attenuates rapidly with distance.

As an aside, the SR-19 also has a nozzle liner ejection event which occurs on about 50% of flights.
Figure 7.
Figure 8.
SR-19 Motor Sweep Rate

The frequency of an acoustic pressure wave \( f \) is

\[
f = \frac{c}{\lambda}
\]

(1)

where

\( c \) is the speed of sound
\( \lambda \) is the wavelength

The frequency sweep rate is

\[
\frac{df}{dt} = -\frac{c}{\lambda^2} \frac{d\lambda}{dt}
\]

(2)

\[
\frac{df}{dt} = -\frac{f}{\lambda} \frac{d\lambda}{dt}
\]

(3)

The SR-19 motor cavity has a complex geometry. The motor has an initial arc-shaped cavity along the inside of the forward motor dome.

The arc length increases at an average rate of 1.64 in/sec. Obviously both the frequency and wavelength vary with time. The average wavelength is about 68 inches. The average frequency is 625 Hz.

\[
\frac{df}{dt} = -\left[\frac{625 \text{ Hz}}{68 \text{ inch}}\right] \left[1.64 \text{ in} / \text{sec}\right]
\]

(4)

The SR-19 frequency sweep rate is thus

\[
\frac{df}{dt} \approx -15 \text{ Hz}
\]

(5)

As an aside, note that the speed of sound in the exhaust gas is about 3500 ft/sec.
Additional Solid-Fuel Motors with Pressure Oscillations

Castor-4B Motor

The Castor-4B motor generates a sinusoidal pressure oscillation. A conservative sine sweep from 60 to 80 Hz with a sweep rate of 7 Hz/min is recommended as a maximum predicted environment.

Figure A-1. TCMP2 Vehicle

The TCMP-2 vehicle had a Castor-4B motor.
PK SR-118, Stage 1 Motor

The PK SR-118 motor has a sinusoidal pressure oscillation. The resulting accelerometer data has been recorded on the three Taurus programs that have used the PK motor as Stage zero. The missions were T1, T3 and T5.

A conservative 45 to 80 Hz sine sweep with a linear rate of 12 Hz/min Hz is recommended as a maximum predicted environment.

PK Stage 3 Motor

Data from three static fire tests is available for the Stage 3 motor.

The first test had a significant 950 Hz oscillation from 30 to 68 seconds. This oscillation did not occur during the other tests, however. The source of this excitation remains unclear.

The second test had a notable 650 Hz oscillation throughout the motor burn.

The third test had an oscillation that began at 700 Hz and swept down to 600 Hz, over the entire motor burn.

The cause of the oscillations in the 600 Hz to 700 Hz domain may be due to the gas generator or TVC system. This explanation needs to be verified, however.

Furthermore the PK Stage 3 has a transient oscillation beginning at 500 Hz and sweeping downward to 420 Hz, over the first twelve seconds after motor ignition. This event appears to be localized to the nozzle adapter flange.

Figure A-2. Minotaur IV Vehicle

The Minotaur IV first stage is a Peacekeeper SR-118 motor. The first flight of this vehicle is scheduled for 2008.
Minuteman III, SR73-AJ/TC-1

The Minuteman III, LGM-30G vehicle third stage motor is an Aerojet/Thiokol SR73-AJ/TC-1 solid-fuel motor.

The SR73 motor has four pressure oscillation frequencies:

- 170 to 210 Hz
- 310 to 380 Hz
- 780 to 860 Hz
- 1620 to 1700 Hz

Figure A-3. Minuteman III, LGM-30G Vehicle
The Taurus first stage is referred to as “Stage zero.” A Castor 120 motor is used as the booster for commercial customers. A Peacekeeper SR-118 motor is used for certain U.S. Government mission.

The vehicle on the left has a Castor 120 as its stage zero.

The Castor 120 motor is essentially a commercial version of the Peacekeeper SR-118 motor. The Castor 120 has a similar sinusoidal oscillation as the SR-118.

Thus, a conservative 45 to 80 Hz sine sweep with a linear rate of 12 Hz/min Hz is recommended as a maximum predicted environment.
Black Brant V Mk 1

The Black Brant has a pressure oscillation frequency at 110 Hz with integer harmonics thereof.

Figure A-5.

The Black Brant is the Third Stage of the vehicle in the above figure.
The Orion Launch Abort System, Abort Motor, has a tail-off frequency at 685 Hz with a harmonic at 1370 Hz, as taken from the PA-1 Test flight data.

The acoustic mode prediction for a closed right cylinder estimates the first radial/tangential acoustic mode to be approximately 690 Hz (based on end of burn radius of 17.7 inch).

This resonance exists in the dynamic pressure data and is considered to be a real phenomenon of the low pressure end of burn gas dynamics of the reverse flow motor.
Space Shuttle Solid Rocket Booster (SRB)

Figure A-7.

<table>
<thead>
<tr>
<th>Booster</th>
<th>Oscillation Frequency (Hz)</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-segment</td>
<td>15</td>
<td>Space Shuttle</td>
</tr>
<tr>
<td>5-segment</td>
<td>12</td>
<td>Ares</td>
</tr>
</tbody>
</table>

Note that the Ares vehicle was cancelled before its first planned flight.

Solid Rocket Motor Upgrade (SRMU)

Figure A-7. Titan IV Launch

The three-segment SRMU had an oscillation frequency of 18 Hz.

APPENDIX B

Excerpts from AIAA Paper

![Graph showing motor fundamental acoustic mode frequency as a function of motor length.](image)

**Fig. 2** Motor fundamental acoustic mode frequency: heavy line corresponds to one-dimensional combustion chamber with closed ends and bars indicate ranges observed in static firing data.

A. Motor Acoustic Modes

It was first shown in 1973 that the internal pressure oscillations from resonant burning are caused by coupling of vortex shedding with the acoustic modes of the motor chamber.\(^{11}\) Because an infinite number of acoustic modes exist, the pressure oscillations may theoretically have many frequency components. However, static firing data for numerous motor designs indicate that the pressure oscillations are dominated by coupling with, at most, the first two acoustic modes.\(^{7,9,10,12,13}\)

When the motor is modeled as a cavity with closed ends, the frequencies of its longitudinal acoustic modes can be approximated by

\[
f_a = \frac{nc}{2L} \quad (1)
\]

in which the speed of sound \(c\) is taken herein as 3500 ft/s. Equation (1), with \(n = 1\), is shown in Fig. 2 for several solid rocket motors.
Reference