RELATIVE DISPLACEMENT & ABSOLUTE ACCELERATION Rev A

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Equation of Motion for a SDOF System Subjected to Base Excitation

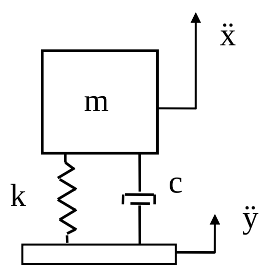

Figure 1.

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m	is the mass
c	is the viscous damping coefficient
k	is the stiffness
fn	is the natural frequency
Q	is the amplification or quality factor
ξ	is the viscous damping ratio
x	is the absolute displacement of the mass
y	is the base input displacement
z	is the relative displacement
A	is the peak absolute response acceleration

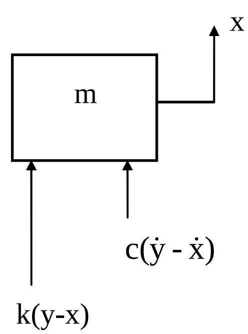

Figure 2.

The equation of motion for a single-degree-of-freedom system subjected to base excitation is

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \tag{1}$$

Define the relative displacement z.

$$z = x - y \tag{2}$$

$$x = z + y \tag{3}$$

Substitute equation (3) into (1).

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{y} \tag{4}$$

$$m(\ddot{z} + \ddot{y}) + c(\dot{z} + \dot{y}) + k(z + y) = c\dot{y} + ky \tag{5}$$

$$m\ddot{z} + m\ddot{y} + c\dot{z} + c\dot{y} + kz + ky = c\dot{y} + ky \tag{6}$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{7}$$

$$\ddot{z} + (c/m)\dot{z} + (k/m)z = -\ddot{y} \tag{8}$$

By convention,

$$(c/m) = 2\xi\omega_n \tag{9}$$

$$(k/m) = \omega_n{}^2 \tag{10}$$

Substitute equations (9) & (10) into (8).

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n{}^2 z = -\ddot{y} \tag{11}$$

An Approximation for Relative Displacement

Now assume that the damping term is approximately zero.

$$\ddot{z} + \omega_n{}^2 z \approx -\ddot{y} \tag{12}$$

Recall

$$\ddot{z} = \ddot{x} - \ddot{y} \tag{13}$$

$$\ddot{x} - \ddot{y} + \omega_n{}^2 z \approx -\ddot{y} \tag{14}$$

$$\ddot{x} + \omega_n{}^2 z \approx 0 \tag{15}$$

The absolute acceleration is thus approximately equal to the relative displacement multiplied by $\omega_n{}^2$. Note that the polarity sign is irrelevant.

$$\ddot{x} \approx -\omega_n{}^2 z \tag{16}$$

Note that

$$\omega_n = 2\pi f_n \tag{17}$$

Let A represent the peak response acceleration.

$$A = |\ \ddot{x}\ | \tag{18}$$

The relative displacement is thus

$$Z \approx \frac{A}{4\pi^2 fn^2} \tag{19}$$

Now assume that Z is in inches and A is in units of G.

$$Z \approx 9.78 \frac{A}{fn^2} \tag{20}$$

Equation (19) is useful for random vibration and for shock response spectrum.

A more accurate equation for sine vibration is given in Appendix A.

Example: Apply Shock Load & Calculate Deflection

A sample SRS is shown in Figure 3.

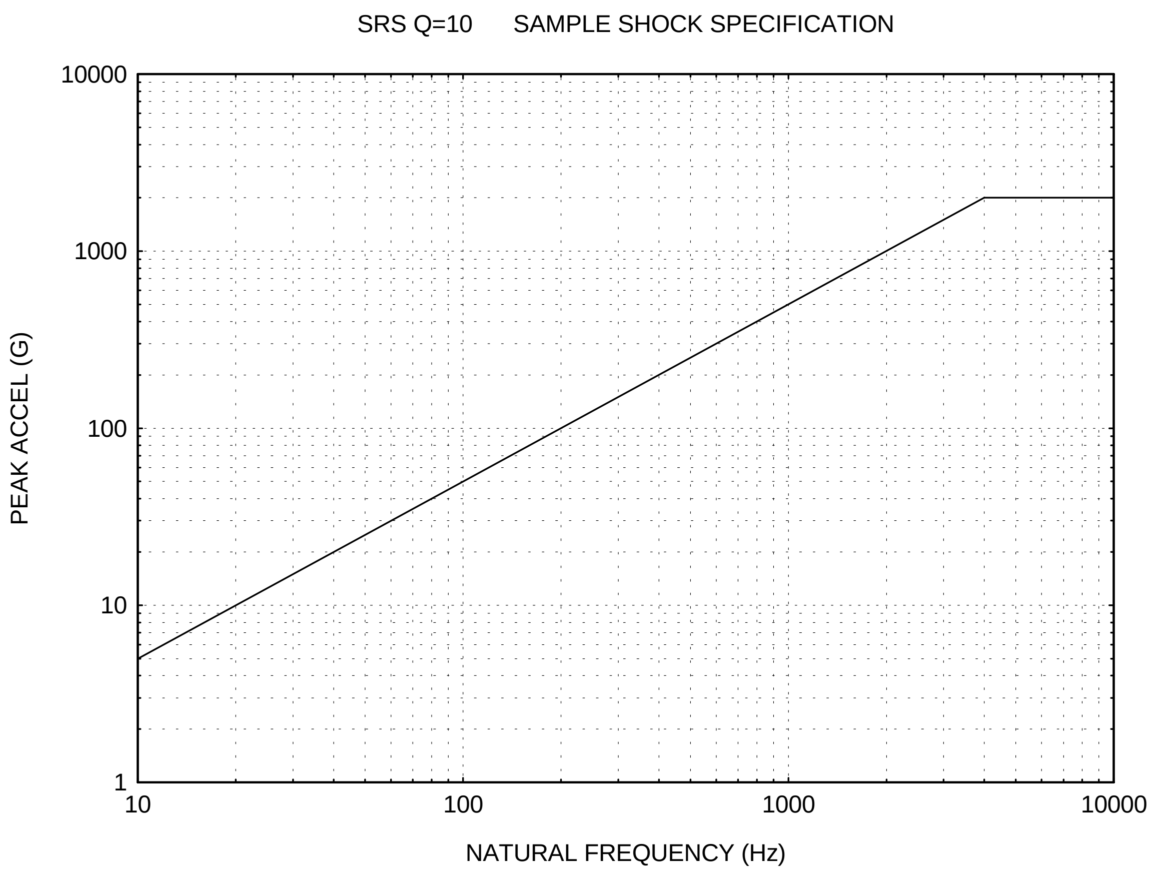

Figure 3.

Assume a certain hardmounted component behaves as a single-degree-of-freedom system with a natural frequency of 200 Hz. The SRS curve shows that the component's peak response is 100 G for this frequency.

Now assume that the same component is mounted via isolators such that its natural frequency decreases to 40 Hz. The SRS curve shows that the component's peak response is 20 G for the isolated configuration. The peak acceleration response is thus reduced by 14 dB.

The relative displacement for isolated component in the example is

$$Z = 9.78 \frac{20 \text{ G}}{(40 \text{ Hz})^2} \tag{21}$$

$$Z = 0.12 \text{ inch} \tag{22}$$

<u>Reference</u>

1. T. Irvine, Steady-State Relative Displacement Response to Base Excitation, Vibrationdata, 2004.

2. T. Irvine, The Steady-State Response of a Single-Degree-of-Freedom System Subjected to a Harmonic Base Excitation, Vibrationdata, 2004.

APPENDIX A

<u>Steady-State Sine Vibration</u>

Let $H(f)$ be the transfer function that is the ratio of the relative displacement to the base input acceleration.

$$\left| H(f) \right| = \frac{1}{4\pi^2 \, f_n^{\,2} \sqrt{\left[1-(f/fn)^2\right]^2 + \dfrac{(f/fn)^2}{Q^2}}} \tag{A-1}$$

A derivation of equation (A-1) is given in Reference 1.

Let $W(f)$ be the transfer function that is the ratio of the absolute response acceleration to the base input acceleration.

$$\left| W(f) \right| = \frac{\sqrt{1+\dfrac{(f/fn)^2}{Q^2}}}{\sqrt{\left[1-(f/fn)^2\right]^2 + \dfrac{(f/fn)^2}{Q^2}}} \tag{A-2}$$

A derivation of equation (A-2) is given in Reference 2.

Divide equation (A-1) by (A-2).

$$\frac{|\,H(f)\,|}{|\,W(f)\,|}=\left\{\frac{\dfrac{1}{4\pi^2\ f_n{}^2\ \sqrt{\left[1-(f/fn)^2\right]^2+\dfrac{(f/fn)^2}{Q^2}}}}{\dfrac{\sqrt{1+\dfrac{(f/fn)^2}{Q^2}}}{\sqrt{\left[1-(f/fn)^2\right]^2+\dfrac{(f/fn)^2}{Q^2}}}}\right\} \tag{A-3}$$

$$\frac{|\,H(f)\,|}{|\,W(f)\,|}=\frac{1}{4\pi^2\ f_n{}^2\sqrt{1+\dfrac{(f/fn)^2}{Q^2}}} \tag{A-4}$$

The relative displacement Z is thus

$$Z=\frac{A}{4\pi^2\ f_n{}^2}\frac{1}{\sqrt{1+\dfrac{(f/fn)^2}{Q^2}}} \tag{A-5}$$

Again, A is the peak response acceleration.

Equation (A-5) converges to (19) if $\dfrac{(f/fn)^2}{Q^2} << 1$.