

## TABLE OF SPRING STIFFNESS Revision B

By Tom Irvine

Email: tomirvine@aol.com

October 5, 1999

---

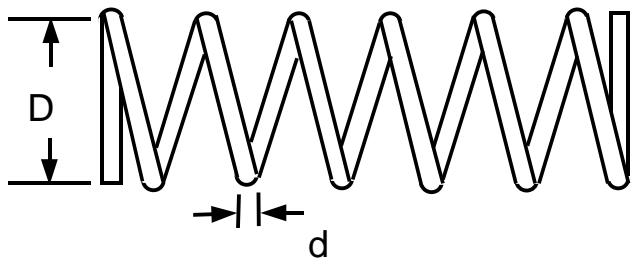
### Common Variables

k = spring stiffness.

G = shear modulus.

E = elastic modulus.

### Circular Wire Helical Spring in Tension or Compression



$$k = \frac{d^4 G}{8D^3 N}$$

D = mean diameter.

d = wire diameter of spring material.

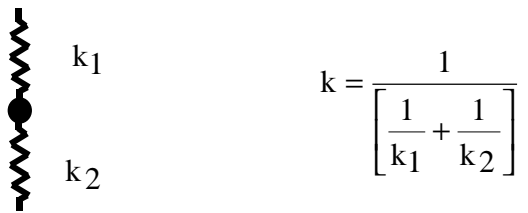
N = number of active turns of wire.

### Circular Wire Helical Spring in Torsion

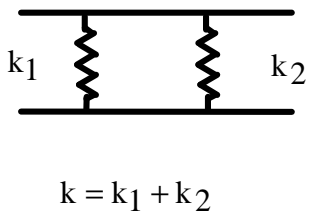
$$k = \frac{d^4 E}{64DN}$$

Dimensions: [length-force/radian]

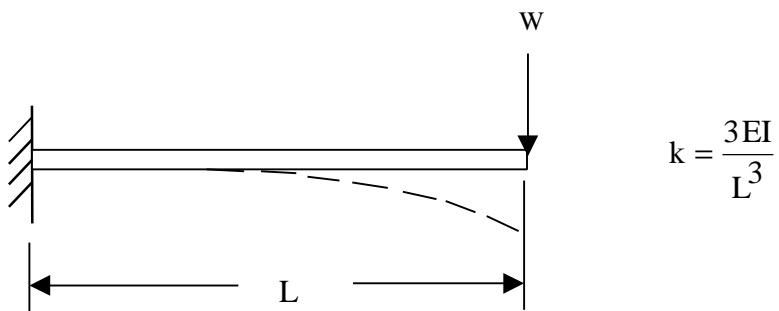
### Springs in Series



### Springs in Parallel

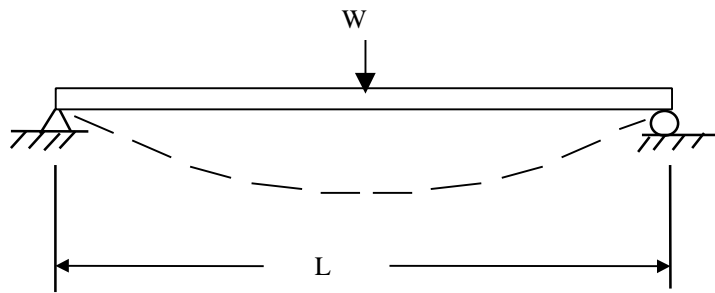


### Bending of Fixed-Free Beam with Concentrated Load



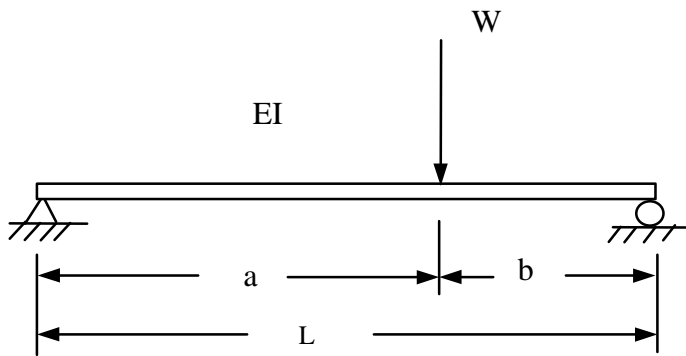
$I$  = the area moment of inertia.

Bending of Beam Simply-Supported at Both Ends with Concentrated Load at Center Location



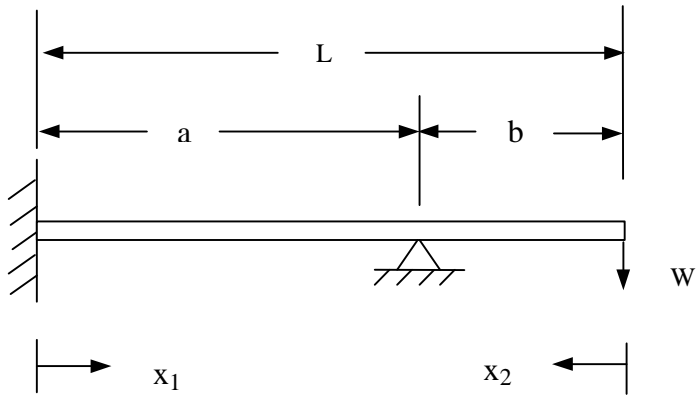
$$k = \frac{48EI}{L^3}$$

Beam with Simply-Supported at Both Ends with Concentrated Load at an Arbitrary Location



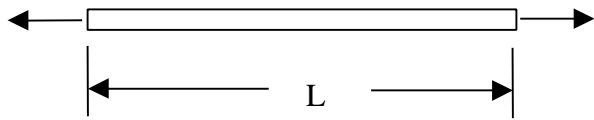
The stiffness at the load point is  $k = \left\{ \frac{3EI}{a^2b^2} \right\}$ .

### Clamped-Pinned-Free Beam



The stiffness at the free end is  $k = \frac{12EI}{b^2 [4b + 3a]}$ .

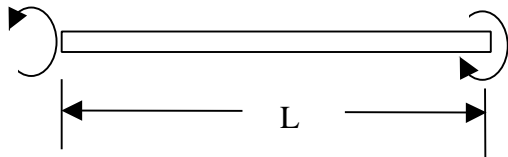
### Rod or Bar Longitudinal Stiffness



$$k = \frac{EA}{L}$$

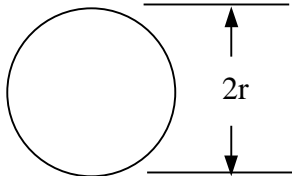
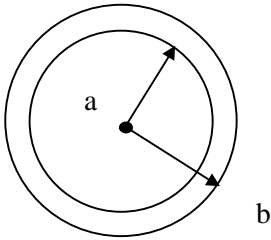
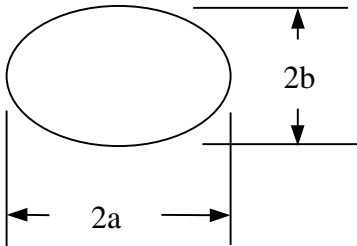
A is the cross-sectional area.

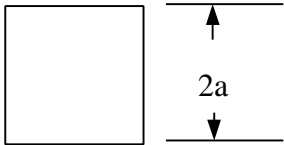
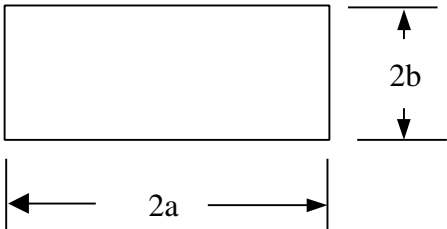
### Rod or Bar Torsional Stiffness



$$k = \frac{GJ}{L}$$

J is the torsion constant of the cross-section. Examples are shown in Table 1.

Table 1. Torsion Constants for Torsional Stiffness	
Cross-section Shape	Torsion Constant J
<p>Solid Circular Section</p> 	$J = \frac{1}{2} \pi r^4$
<p>Hollow Concentric Circular Section</p> 	$J = \frac{1}{2} \pi [b^4 - a^4]$
<p>Solid Elliptical Section</p> 	$J = \frac{\pi a^3 b^3}{a^2 + b^2}$

Cross-section Shape	Torsion Constant J
<p>Solid Square Section</p> 	$J = 2.25 a^4$
<p>Solid Rectangular Section</p> 	$J = a b^3 \left[ \frac{16}{3} - 3.36 \frac{b}{a} \left( 1 - \frac{b^4}{12 a^4} \right) \right], \quad a \geq b$

1. W. Thomson, Theory of Vibration with Applications, 2<sup>nd</sup> edition, Prentice-Hall, Englewood Cliffs, N.J., 1981.
2. W. Young, Roark's Formulas for Stress & Strain, 6<sup>th</sup> edition, McGraw-Hill, New York, 1989.
3. Vibration and Shock Mount Handbook, Product Catalog A-816, Stock Drive Products, New York, 1990.