STRUCTURAL DYNAMICS TESTING USING AN IMPULSE FORCE

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Introduction

A structure can be characterized in terms of its dynamic properties, such as its natural frequencies, damping ratios, mode shapes. Modal tests are performed to measure these parameters.

A modal test is carried out by exciting the structure with a measured force. The force can be imparted by an electromagnetic shaker or by an impulse hammer. The response of the structure is measured by accelerometers.

The shaker method yields a steady-state response. The impulse hammer yields a transient response.

This report focuses on the impulse hammer method. Specific attention is given to the response window, which is used to prevent an error source called "leakage."

Example

Input and Response Time Histories

The following example is generated from analytical functions. A linear, single-degree-of-freedom system is considered as the test item.

An impulse force is applied to a structure with a modal hammer. The force time history is a half-sine pulse with a 500 Newton amplitude and a duration of 0.002 seconds, as shown in Figure 1.

The displacement response time history is shown in Figure 2. Assume that the response transducer was mounted adjacent to the hammer strike location.[1]

Note that the input force and the response displacement are plotted over different time intervals in Figures 1 and 2, respectively.

[1] Accelerometers are typically used to measure the response. The displacement can be obtained by double-integrating the acceleration. Some charge amplifiers have analog integration circuits for this purpose. Note that highpass filtering is necessary, however, to remove any baseline shifts prior to integration. Again, displacement is the response parameter considered in this report. Nevertheless, the methods can be readily adapted to an acceleration response.

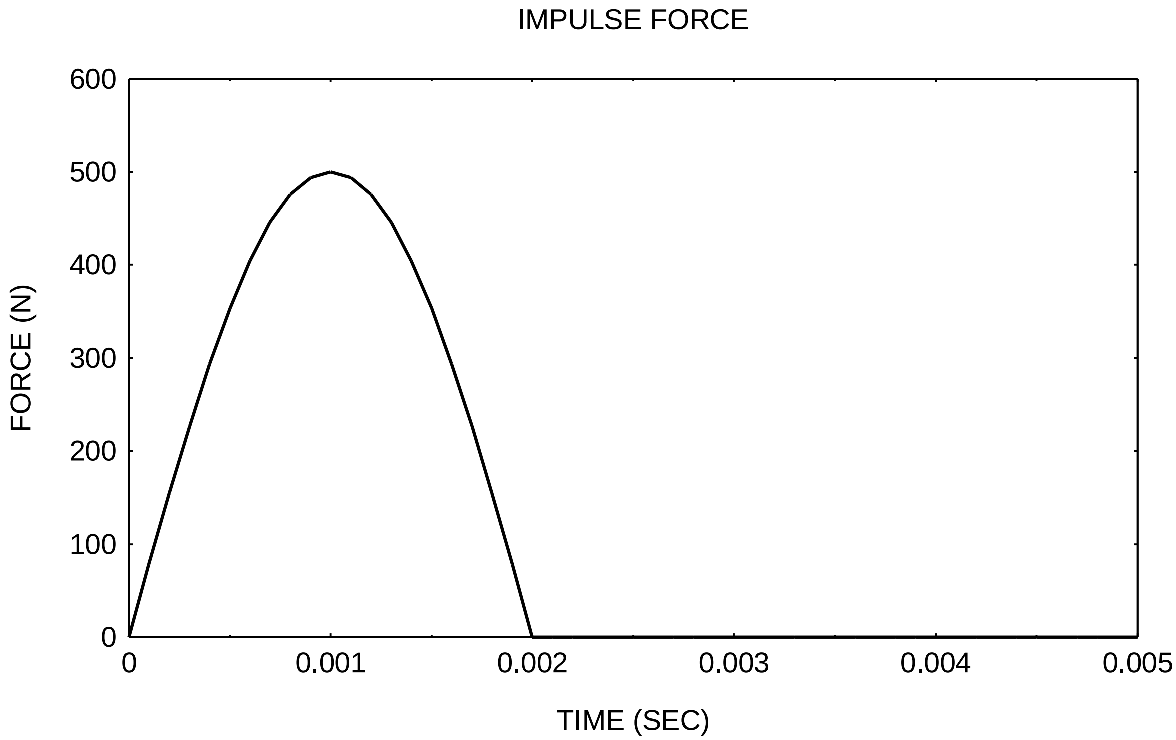

Figure 1.

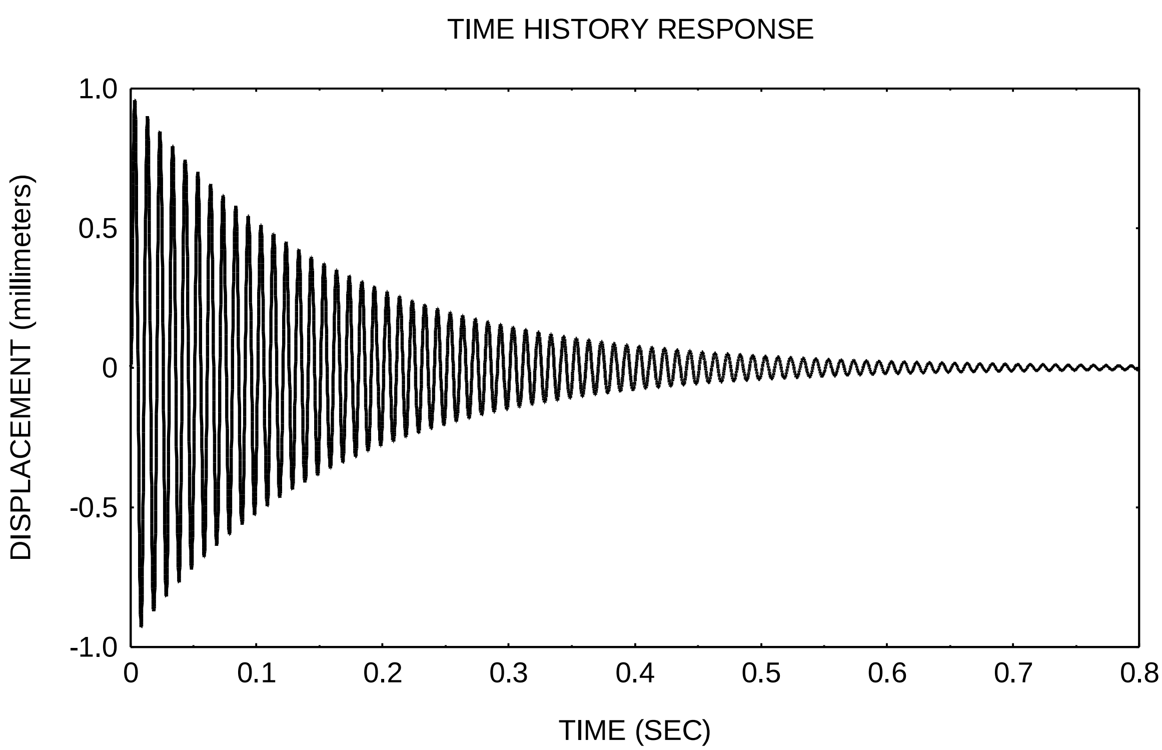

Figure 2.

Time Domain Analysis

The natural frequency of the system can be calculated essentially by inspection. There are twenty positive peaks over the zero to 0.2 second duration, as shown in Figure 3. This corresponds to a natural frequency of 100 Hz.

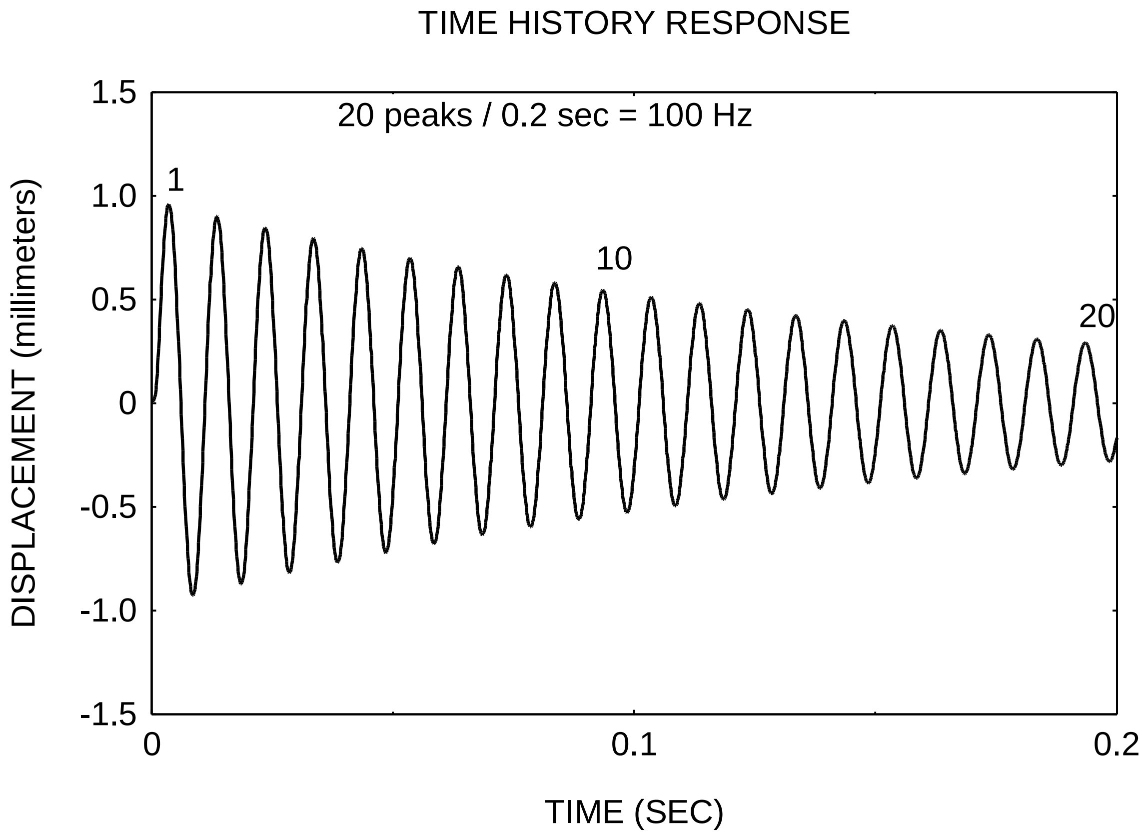

Figure 3.

The “direct count” method is as accurate as any Fourier transform, or any other numerical method, for determining the natural frequency of a well-behaved system.

Some “real world” systems actually exhibit linear, single-degree-of-freedom behavior. Nevertheless, the impulse force would be expected to excite a number of vibration modes for a “general system,” which is a multi-degree-of-freedom system.

Bandpass filtering can be used to view the response of single vibration mode assuming that adjacent vibration modes have a reasonable frequency separation, say at least a one octave separation. Thus, the direct count method can be applied to well-behaved multi-degree-of-freedom systems.

Returning to the time history in Figure 2, the next step is to estimate the damping. There are several methods for doing this. For example, Reference 1 gives the “log decrement” method.

The author’s preferred method, however, is to perform a trial-and-error curve fit of the response envelope, assuming exponential decay. This method is particularly useful if the response has some minor non-linearity.

The envelope results are shown in Figure 4.

The displacement response time history y(t) is assumed to follow the equation

$$y(t) = A \exp(-\xi \omega_n t) \sin(\omega_d t + \phi) \tag{1}$$

where

- A is the amplitude,
- ξ is the damping ratio,
- ω_n is the natural frequency,
- ω_d is the damped natural frequency,
- ϕ is the phase angle.

Note that

$$\omega_d = \omega_n \sqrt{1-\xi^2} \tag{2}$$

More precisely, equation (1) gives the natural response, which is the response after the force input permanently, returns to zero.

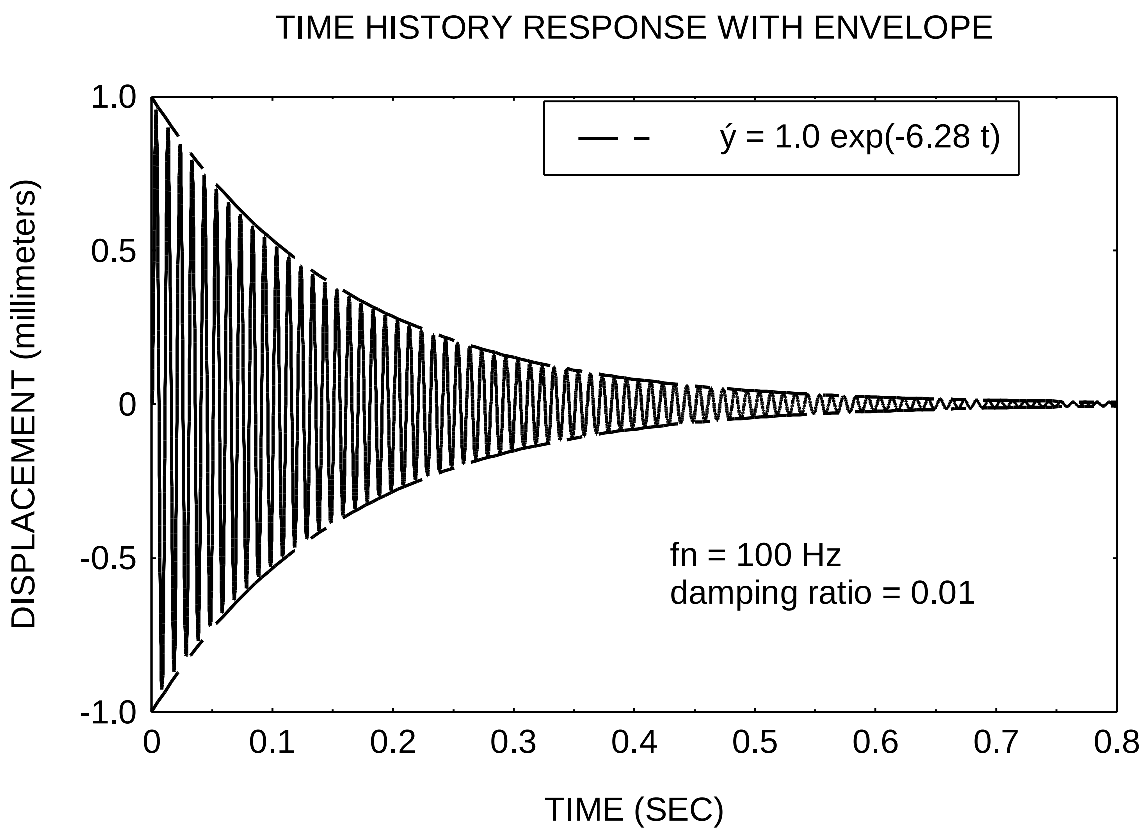

Figure 4.

Now let $\hat{y}(t)$ be the envelope function as follows.

$$\hat{y}(t) = A \exp(-\xi \omega_n t) \tag{3}$$

$$y(t) = \hat{y}(t) \sin(\omega_d t + \phi) \tag{4}$$

Again, the natural frequency is simply determined by counting the number of peaks over an interval. The natural frequency f_n is the number of peaks divided by the time interval. The natural frequency has units of Hz if the time interval is in units of seconds.

The natural frequency can also be expressed as ω_n in units of radians per second.

$$\omega_n = 2\pi f_n \tag{5}$$

Returning to the example in Figure 4, the following envelope equation was determined by trial-and-error inspection.

$$\hat{y} = 1.0 \exp(-6.28\,t) \tag{6}$$

Now apply equation (6) to equation (3).

$$A \exp(-\xi\,\omega_n\,t) = 1.0 \exp(-6.28\,t) \tag{7}$$

The equality requires that

$$\xi\,\omega_n = 6.28 \text{ rad/sec} \tag{8}$$

Divide through by the frequency variable.

$$\xi = \frac{6.28}{\omega_n} \tag{9}$$

Substitute equation (5) into equation (9).

$$\xi = \frac{6.28}{2\pi f_n} \tag{10}$$

Again, the natural frequency is 100 Hz.

$$\xi = \frac{6.28 \text{ rad/sec}}{2\pi(100) \text{ rad/sec}} \tag{11}$$

The damping ratio is thus

$$\xi = 0.01 \tag{12}$$

Time Domain Analysis with a Window Applied

Consider that an exponential window is to be applied to the data. The need for such an envelope in certain cases will be demonstrated later in this tutorial.

Assume that the window has the decay parameter σ such that the response equation is modified as follows.

$$y_e(t) = A \exp(-\sigma\,t) \exp(-\xi\,\omega_n\,t) \sin(\omega_d t + \phi) \tag{13}$$

The subscript e denotes exponential window.

Equation (13) can be simplified as

$$y_e(t) = A \exp[-(\sigma + \xi \omega_n) t] \sin(\omega_d t + \phi) \tag{14}$$

Now let $\hat{y}_e(t)$ be the envelope function for the exponential window method as follows.

$$\hat{y}_e(t) = A \exp[-(\sigma + \xi \omega_n) t] \tag{15}$$

$$y_e(t) = \hat{y}_e(t) \sin(\omega_d t + \phi) \tag{16}$$

Now assume that the data acquisition system is set at a sample rate of 10,000 samples per second and that it is limited to 2048 samples. The duration is thus 0.2048. Choose

$$\sigma = 33.734 \ \ \text{rad/sec} \tag{17}$$

The formula for choosing this value is explained later in this report. The single-degree-of-freedom system from the previous example would then have the displacement response shown in Figure 5.

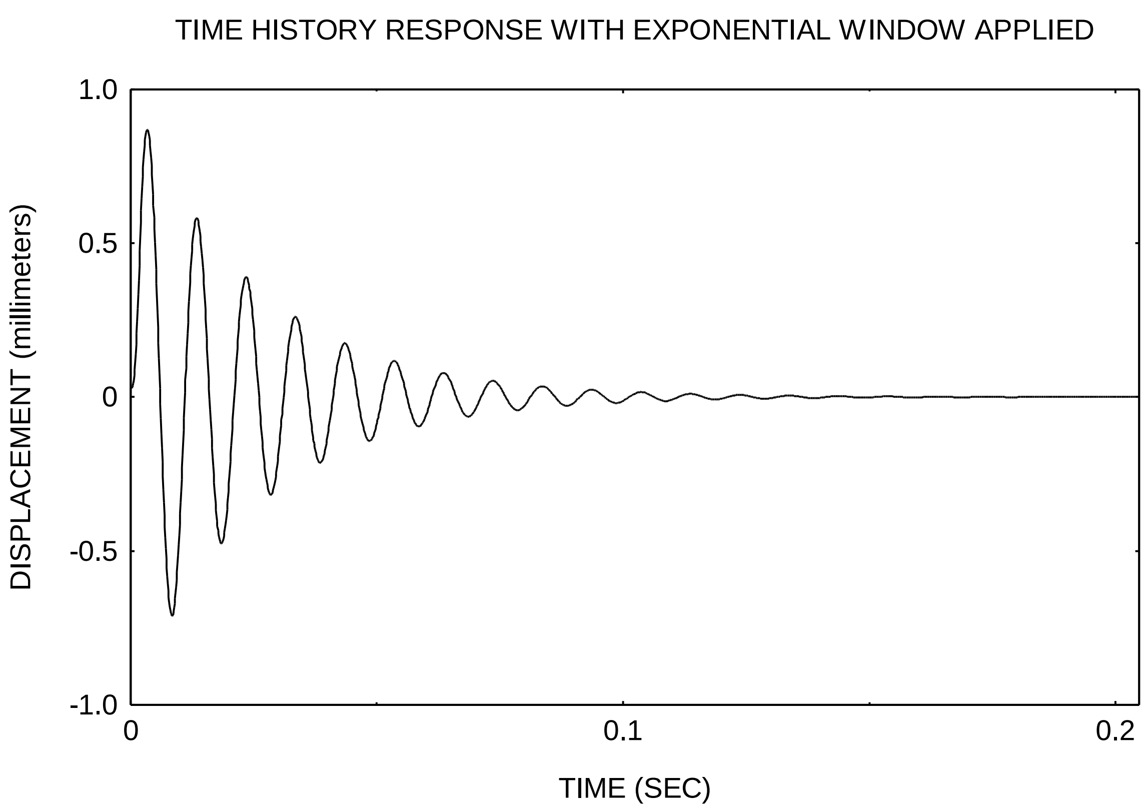

Figure 5.

The natural frequency in Figure 5 remains at 100 Hz. The response with an exponential-decay curve-fit of the envelope is shown in Figure 6.

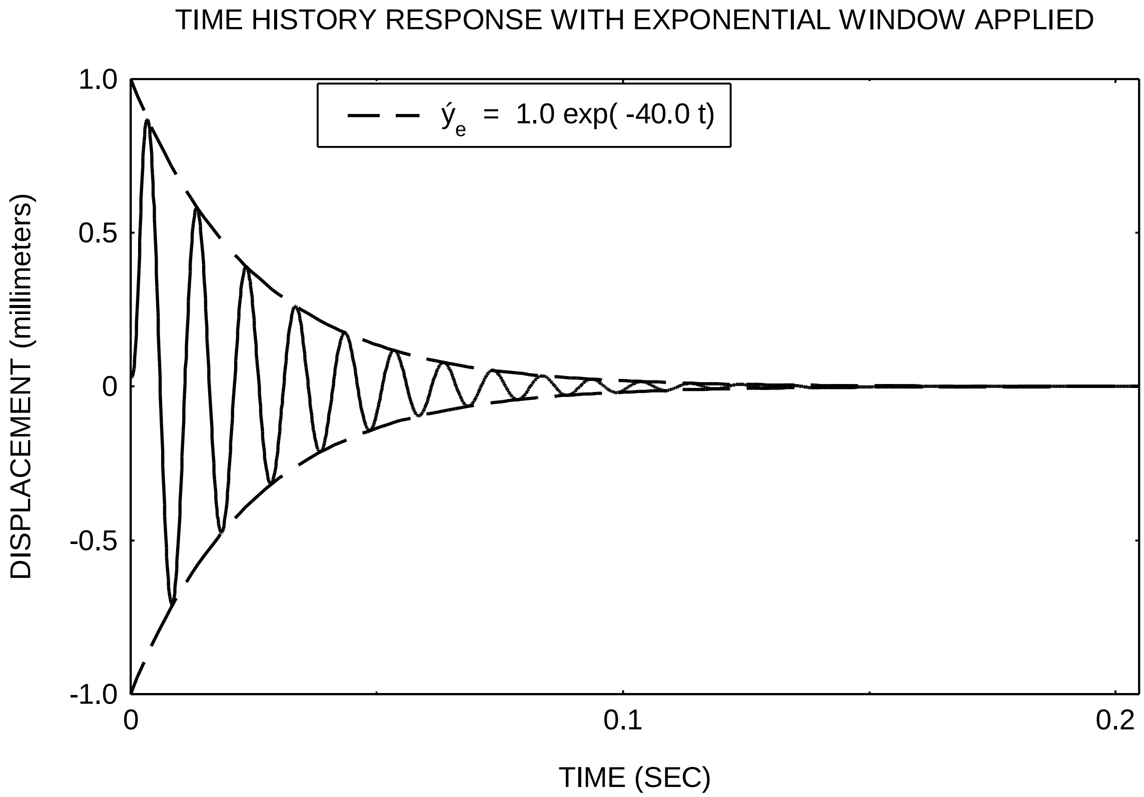

Figure 6.

The envelope equation in Figure 6 is

$$\hat{y}_e = 1.0 \exp(-40.0\,t) \tag{18}$$

Again, this equation was obtained by trial-and-error.

Compare equations (15) and (18).

$$A \exp\left[-\left(\sigma + \xi\,\omega_n\right)t\right] = 1.0 \exp(-40.0\,t) \tag{19}$$

By inspection,

$$\sigma + \xi\omega_n = 40.0 \ \text{rad/sec} \tag{20}$$

$$\xi\omega_n = 40.0 - \sigma \tag{21}$$

$$\xi = \frac{40.0 - \sigma}{\omega_n} \tag{22}$$

$$\xi = \frac{40.0 - \sigma}{2\pi f_n} \tag{23}$$

$$\xi = \frac{(40.0 - 33.734) \ \text{rad/sec}}{2\pi (100) \ \text{rad/sec}} \tag{24}$$

The damping ratio is thus

$$\xi = 0.01 \tag{25}$$

This is the same value that was calculated for the initial case, in which a window was not applied.

The reader should be wondering, "Why bother with an exponential window?"

A real-world system may have several modes which are spaced closely in frequency. A frequency-domain method is thus required to identify the natural frequencies and damping ratios. There are several potential error sources associated with frequency domain processing. One is called leakage. The purpose of the exponential window is to prevent leakage as discussed in the following section.

Frequency Domain Analysis

Fourier Transforms

Another method for evaluating the data is the frequency domain method. The measured time-domain data is converted to frequency-domain data via the Fourier transform. The Fourier transform of the input force is shown in Figure 7. The Fourier transform of the response displacement is shown in Figure 8.

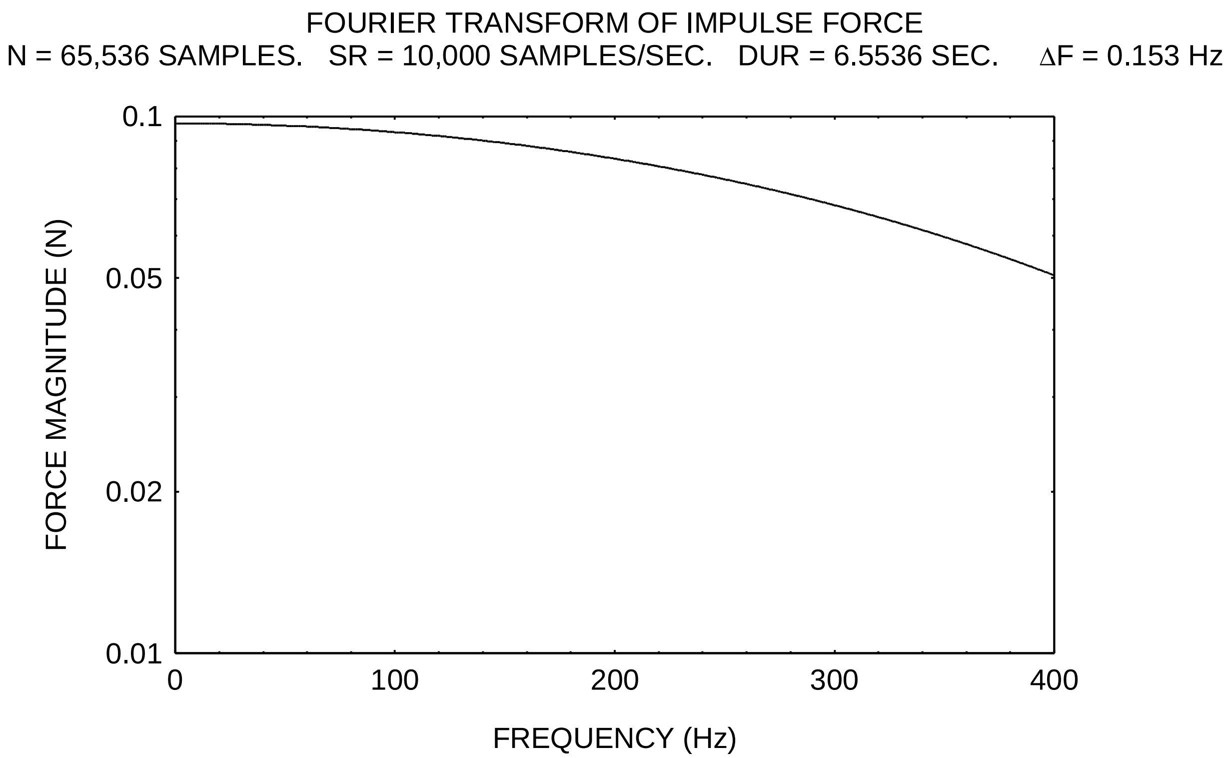

Figure 7.

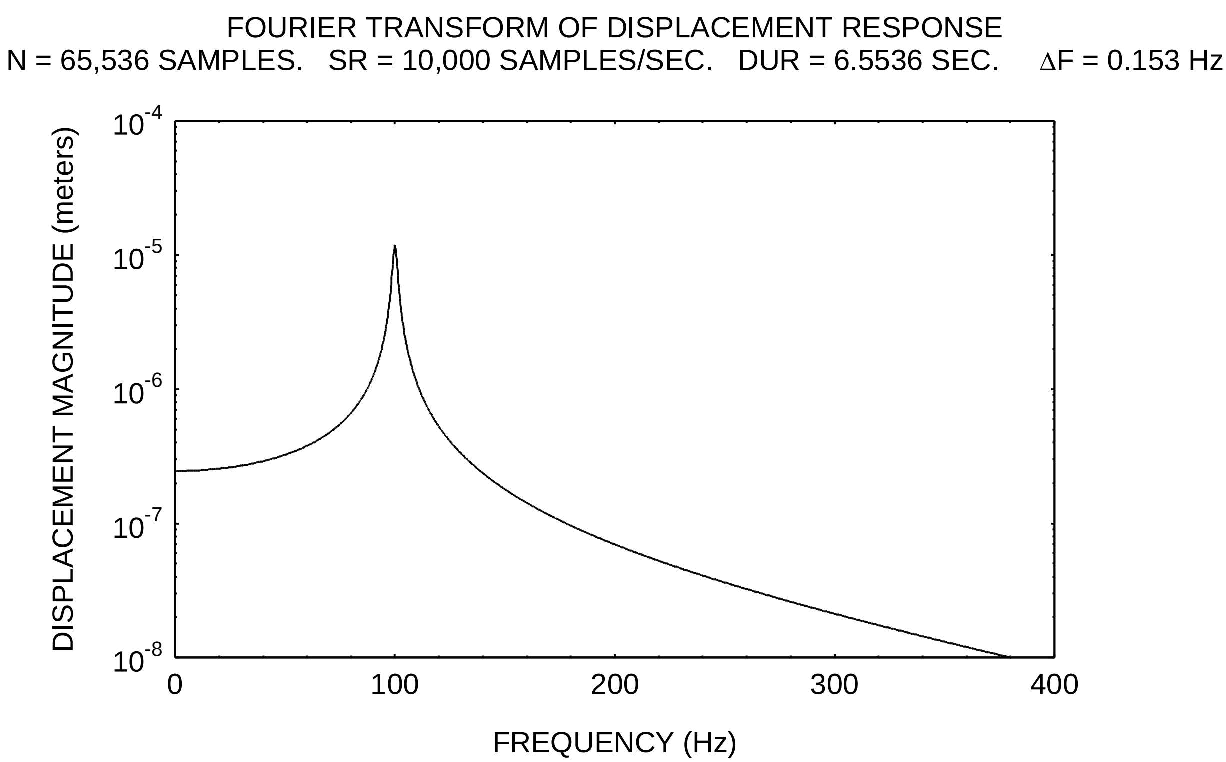

Figure 8.

Compliance or Receptance Function

The compliance function is the ratio of the output displacement to the input force, expressed over the frequency domain. The compliance function is also called the receptance function. A third name is the admittance function. The compliance function is shown in Figure 9.

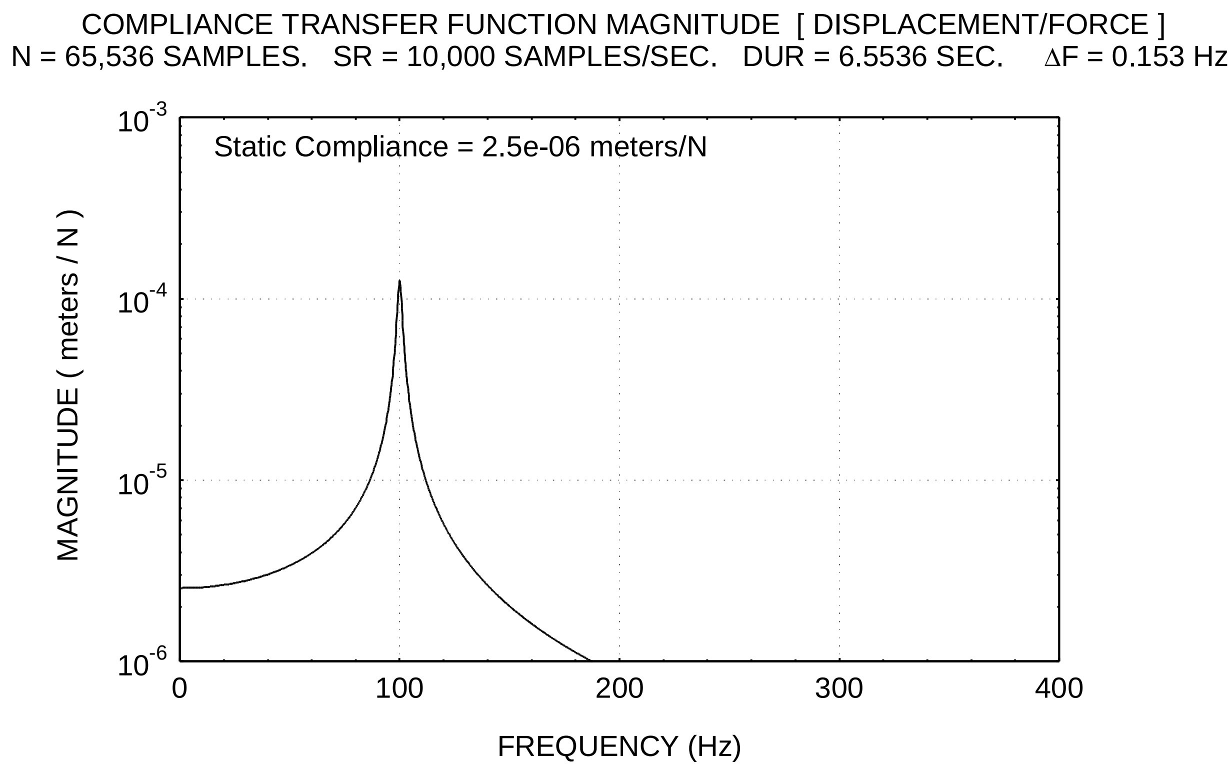

Figure 9.

Note that the compliance value converges to $2.5(10^{-6})$ m/N as the frequency decreases to zero. The static stiffness is simply the reciprocal value, 400,000 N/m. This value can be used as a scale factor to render the compliance function in a normalized form, as shown in Figure 10.

Note that experimental determination of the static stiffness value can be challenging, particularly if there is any low-frequency error in the data.

A close-up view of the peak is shown in Figure 11, along with the half-power points.

NORMALIZED COMPLIANCE TRANSFER FUNCTION MAGNITUDE [STIFFNESS*DISPLACEMENT/FORCE]
N = 65,536 SAMPLES. SR = 10,000 SAMPLES/SEC. DUR = 6.5536 SEC. ΔF = 0.153 Hz

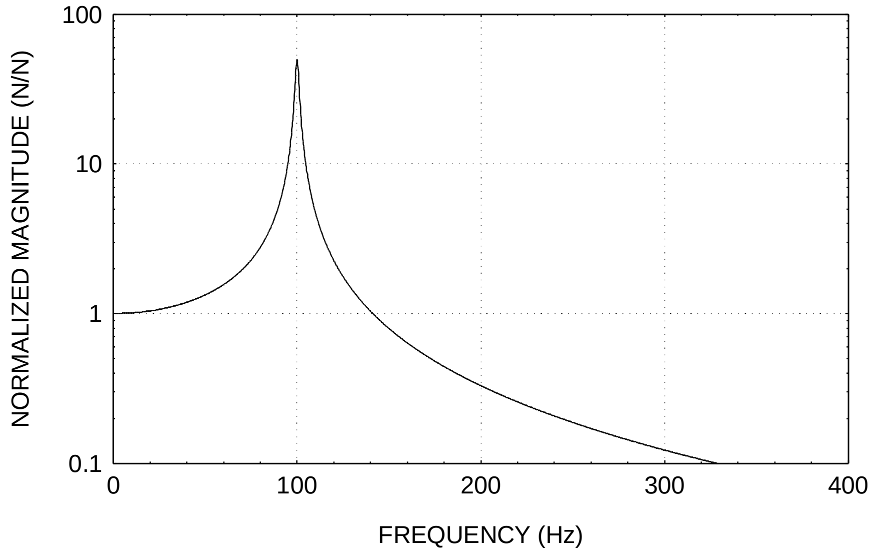

Figure 10.

NORMALIZED COMPLIANCE TRANSFER FUNCTION MAGNITUDE [STIFFNESS*DISPLACEMENT/FORCE]
N = 65,536 SAMPLES. SR = 10,000 SAMPLES/SEC. DUR = 6.5536 SEC. ΔF = 0.153 Hz

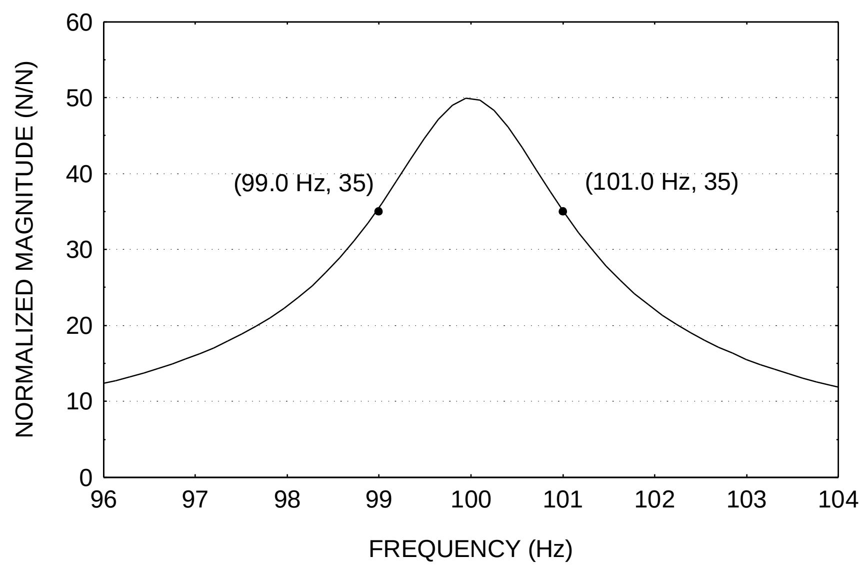

Figure 11.

Peak Value Damping Calculation

The 100 Hz natural frequency is evident in Figures 9 through 11. The peak value is Figure 11 is 50. The damping ratio is determined as follows

$$\frac{1}{2\xi} = 50 \tag{26}$$

$$\xi = 0.01 \tag{27}$$

This is the same value that was obtained in the time-domain analysis.

Half-power Bandwidth Damping Calculation

An alternate method is half-power bandwidth method. The half-power points are the points at –3 dB with respect to the peak value. This is equivalent to $(1/\sqrt{2})$ times the peak value. These points are shown in Figure 11.

Let Δf_h be the frequency separation between the two half-power points. The damping and natural frequency are related to the frequency separation as follows

$$2\xi f_n = 2\pi \Delta f_h \tag{28}$$

This equation is appropriate for cases where $\xi < 0.1$.

The damping is thus

$$\xi = \frac{\Delta f_h}{2 f_n} \tag{29}$$

The frequency separation is 2 Hz in the example, as shown in Figure 11. Again, the natural frequency is 100 Hz. Thus

$$\xi = \frac{2\,\text{Hz}}{2\,(100\text{ Hz})} \tag{30}$$

$$\xi = 0.01 \tag{31}$$

Comparison

The half-power method has several advantages over the peak method. The peak method requires proper amplitude normalization. On the other hand, the half-power method is independent of the amplitude scale factor.

Furthermore, the peak method is only valid for single-degree-of-freedom systems. The half-power method, however, may be applicable to more complex systems, as discussed in Reference 2.

Data Concerns

There are some obvious pitfalls to these approaches. First, the frequency resolution must be sufficiently narrow to characterize the peak. This becomes increasingly difficult as the damping value decreases. This concern is related to the leakage error.

In addition, a more appropriate method of testing would be to measure perhaps ten sets of response and input pairs. A frequency response function is calculated for each pair. The overall frequency response function is the arithmetic average of the individual functions. This approach yields more a more reliable estimate of the true frequency response function.

Leakage Concern

The data in the frequency domain example was considered out to a duration of 6.5536 seconds. The reciprocal is equal to the frequency resolution, which is 0.153 Hz. The frequency separation between the two –3 dB points is 2 Hz. Thus, there are about thirteen points inside the segment between the –3 dB points. This is a sufficient number to characterize the peak.

On the other hand, a data acquisition system might not have enough memory to capture and store a time history signal with 65,536 samples in a data segment.

This limitation is disappearing due to technology advances in computer memory. Nevertheless, consider that a system is limited to a smaller sample set, say 2048 points. This sample size would correspond to a duration of 0.2048 seconds, at a sample rate of 10,000 samples per second. [2]

As a result, the time history would be truncated before the signal decayed to zero, as shown in Figure 12.

[2] Note that the sample rate must be high enough to accurately measure both the force input and the response.

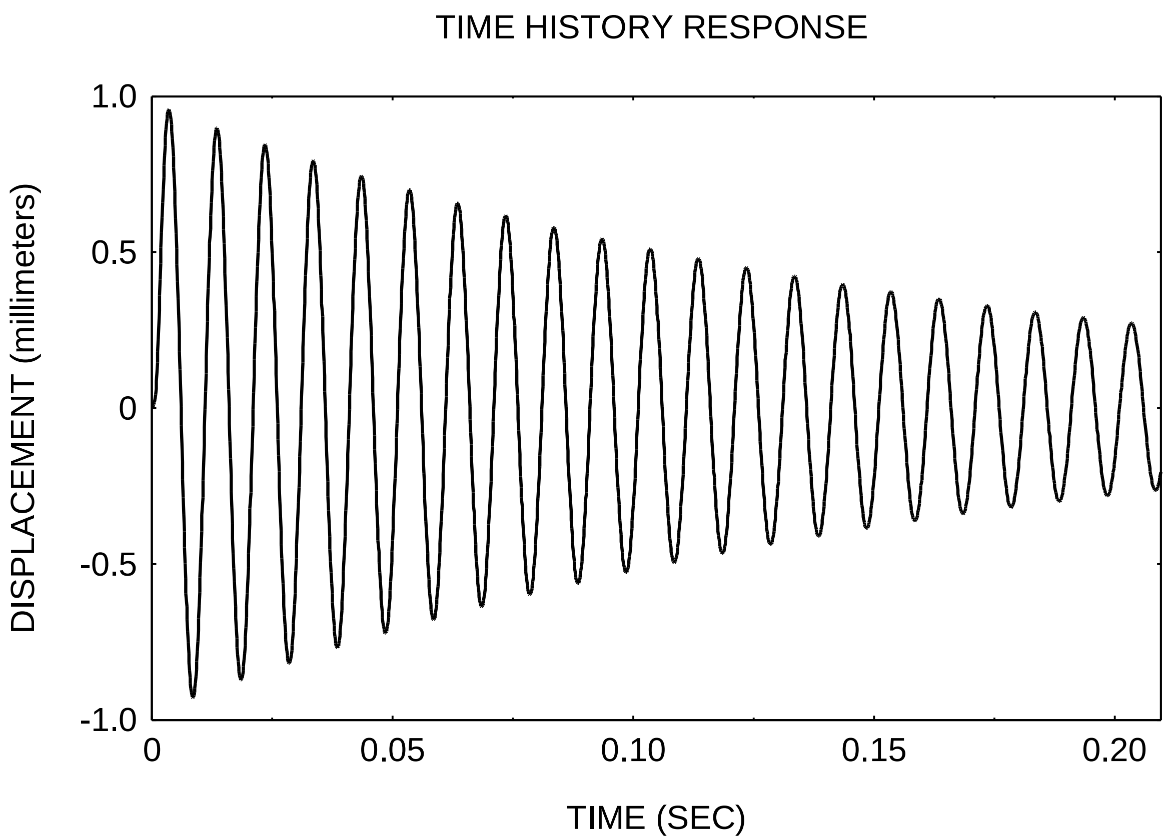

Figure 12.

There are two problems from this truncation. Both are symptoms of leakage.

The first problem is that a spurious ripple effect might appear on the Fourier transforms and frequency response function plots. This is shown in Reference 3.

The second problem is that the frequency resolution would become too coarse. The reciprocal of 0.2048 second sample duration is 4.883 Hz. This reciprocal is also the frequency resolution for the Fourier transform. This resolution would be more than twice as wide as the 2.0 Hz half-power bandwidth. The resulting frequency response function is shown in Figure 13.

The half-power bandwidth method yields a damping value of $\xi = 0.039$ for the 2048 sample curve in Figure 13. This is nearly four times the true damping ratio.

The solution for the leakage error is an exponential window.

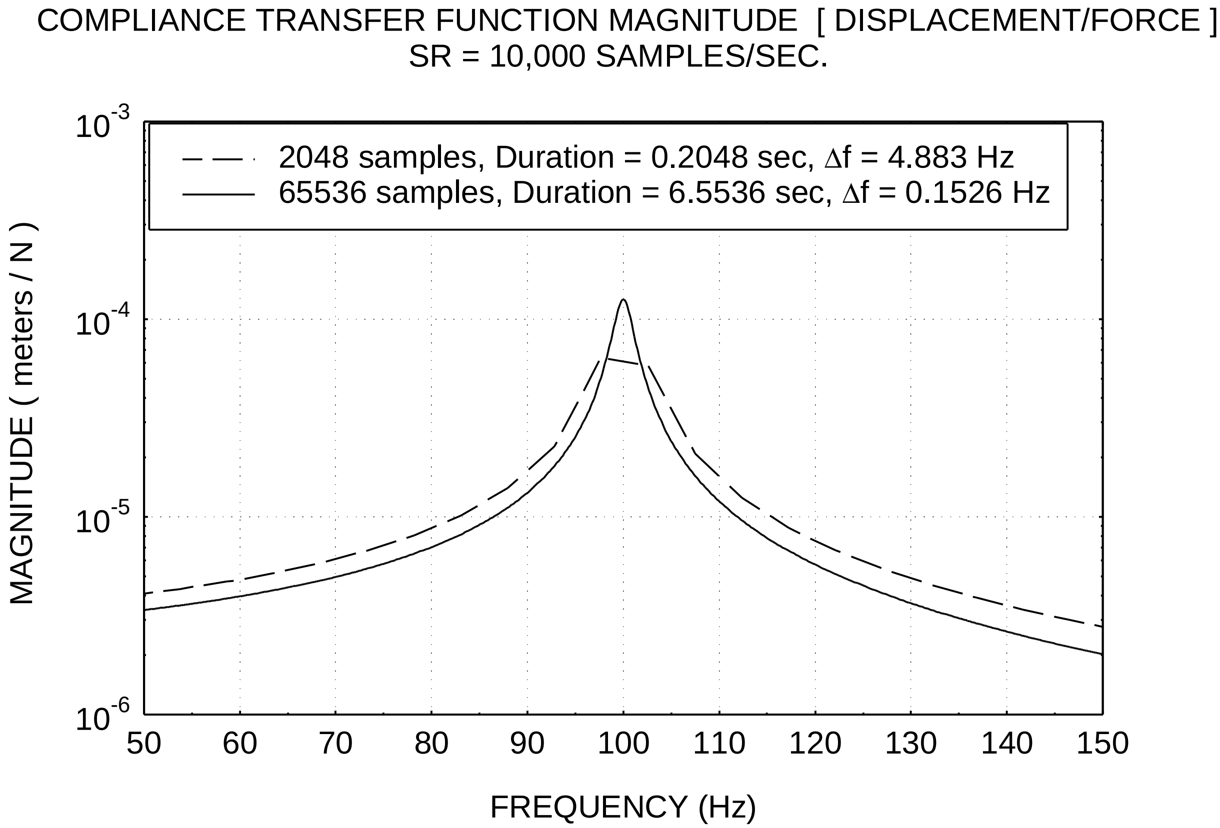

Figure 13.

Exponential Window

Again, the concern is that a transient signal might not decay to a negligible value within a data block. An example would be the response of a lightly damped structure to an impulsive load. A leakage error would then result in the Fourier transform.

An exponential window can be applied to suppress leakage. The following formula is recommended in Reference 4 for a time-domain exponential window:

$$u(t) = \begin{cases} 0\,, & \text{for } t < 0 \\ \exp\left(-6.9078\dfrac{t}{T}\right), & \text{for } 0 \le t \le T \\ 0\,, & \text{for } t > 0 \end{cases} \tag{32}$$

where T is the data block duration.

The equivalent frequency-domain magnitude function is:

$$U(f) = \frac{0.69078\ T}{\sqrt{(0.69078)^2 + (2\pi f\ T)^2}} \tag{33}$$

This window decays from a value of unity at the start of the block to a value of 0.001 (-60 dB) at the end of the block.

This exponential window is desirable because it appears in the resulting Fourier spectrum as additional, well-defined linear damping of the structural response. It thus offers the opportunity to correct estimates of structural damping made from the Fourier spectrum for window effects.

Returning to the example problem, the exponential window in equation (32) was previously applied to the time history as shown in Figures 5 and 6.

The corresponding compliance function is shown in Figure 14. A close-up view of the peak is shown in Figure 15, along with the –3 dB points.

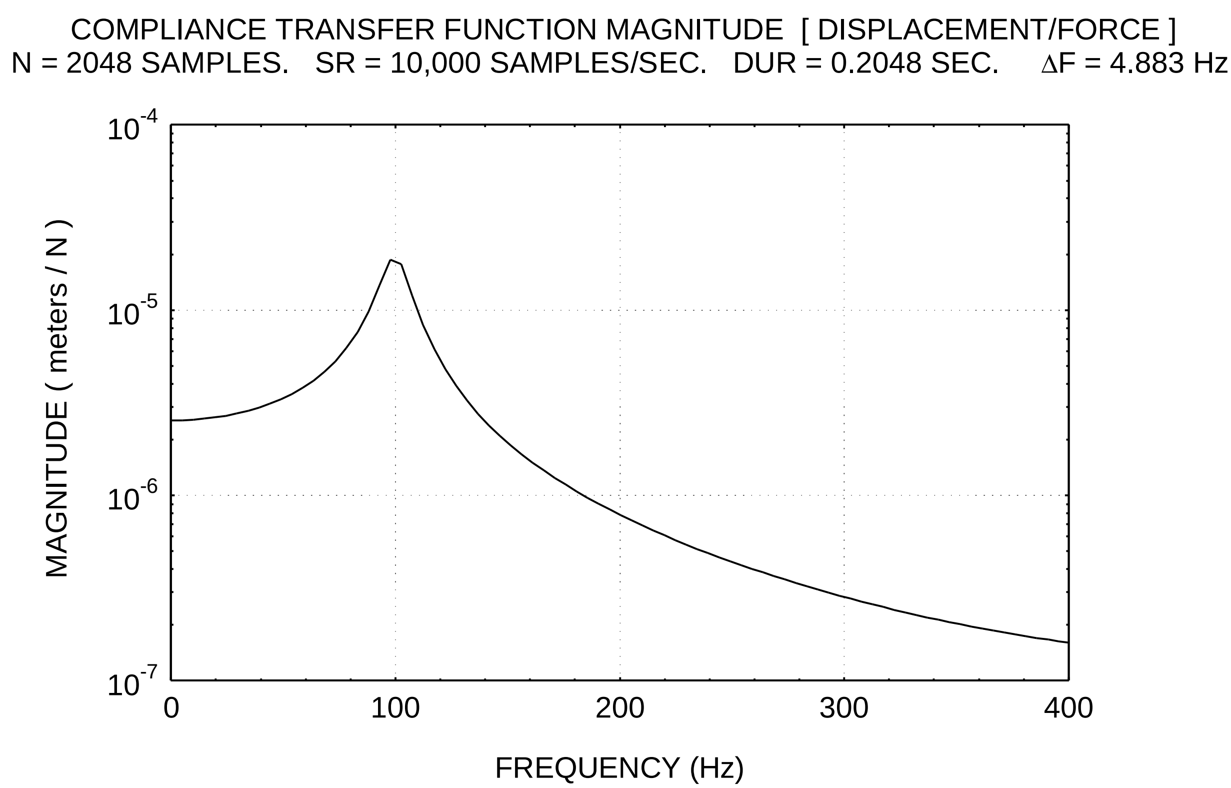

Figure 14.

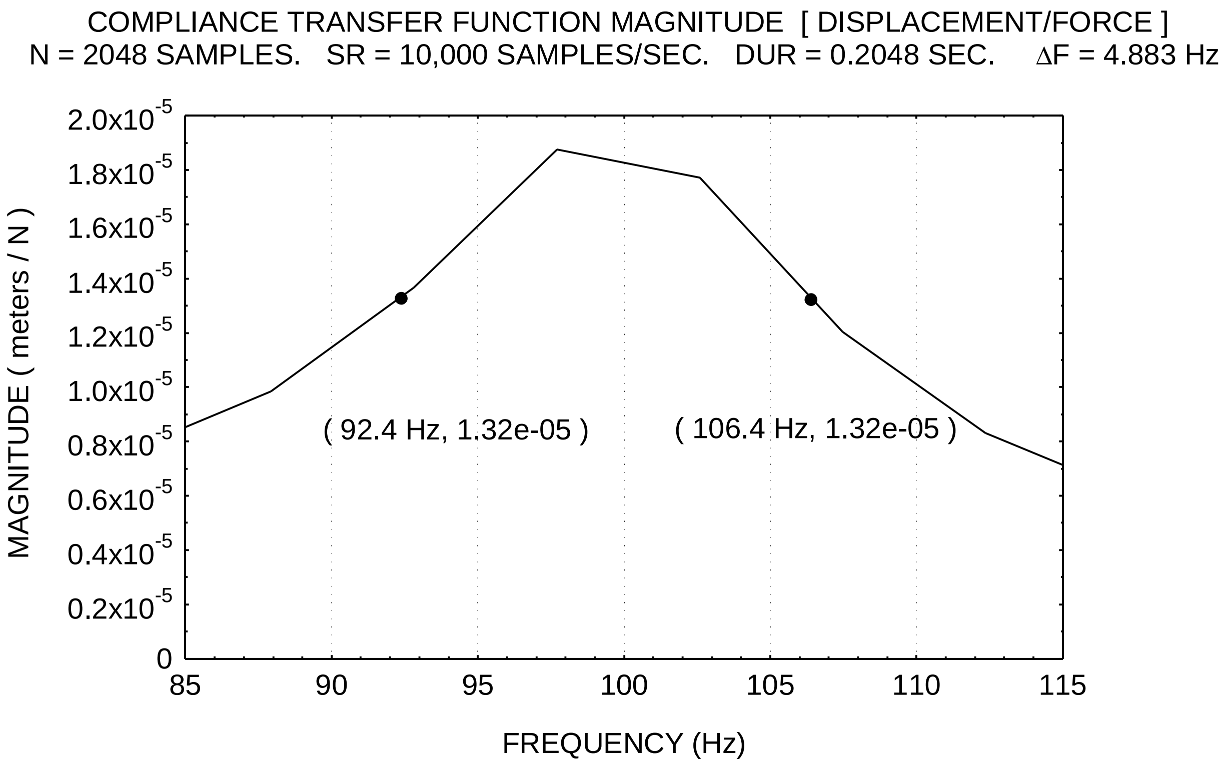

Figure 15.

First, calculate the apparent damping ratio ξ_a.

$$2\,\xi_a\,f_n = 2\pi\,\Delta f_h \tag{34}$$

The apparent damping is thus

$$\xi_a = \frac{\Delta f_h}{2\,f_n} \tag{35}$$

The frequency separation is 14 Hz in the example, as shown in Figure 15. Again, the natural frequency is 100 Hz. Thus

$$\xi_a = \frac{14\,\text{Hz}}{2\,(100\ \text{Hz})} \tag{36}$$

$$\xi_a = 0.070 \tag{37}$$

The true damping ratio is estimated through the following steps.

$$\sigma + \xi\,\omega_n = \xi_a\,\omega_n \tag{38}$$

$$\xi\,\omega_n = \xi_a\,\omega_n - \sigma \tag{39}$$

$$\xi = \frac{\xi_a\,\omega_n - \sigma}{\omega_n} \tag{40}$$

$$\xi = \frac{\xi_a\,\omega_n - \sigma}{2\pi f_n} \tag{41}$$

$$\xi = \frac{2\pi\,\xi_a\,f_n - \sigma}{2\pi f_n} \tag{42}$$

$$\xi = \frac{[2\pi\,(0.070)(100) - 33.734]\ \mathrm{rad/sec}}{2\pi\,(100)\ \mathrm{rad/sec}} \tag{43}$$

The damping ratio is thus estimated as

$$\xi = 0.016 \tag{44}$$

This is 60% higher than the true value $\xi = 0.01$.

This error is partially due to the fact that there were only three points inside the half-power bandwidth in Figure 15. Furthermore, numerical error may result when the apparent damping is much greater than the true damping.

Greater resolution is needed. This requires a longer duration.

Now repeat the example with a window of 0.4096 seconds duration, with 4096 samples. The exponential window decay value is calculated from equation (32).

$$\sigma = 16.865\ \ \mathrm{rad/sec} \tag{45}$$

The time history is shown in Figure 16. The compliance function is shown in Figure 17. A close-up view showing the half-power points is given in Figure 18.

TIME HISTORY RESPONSE WITH EXPONENTIAL WINDOW APPLIED, 4096 SAMPLES

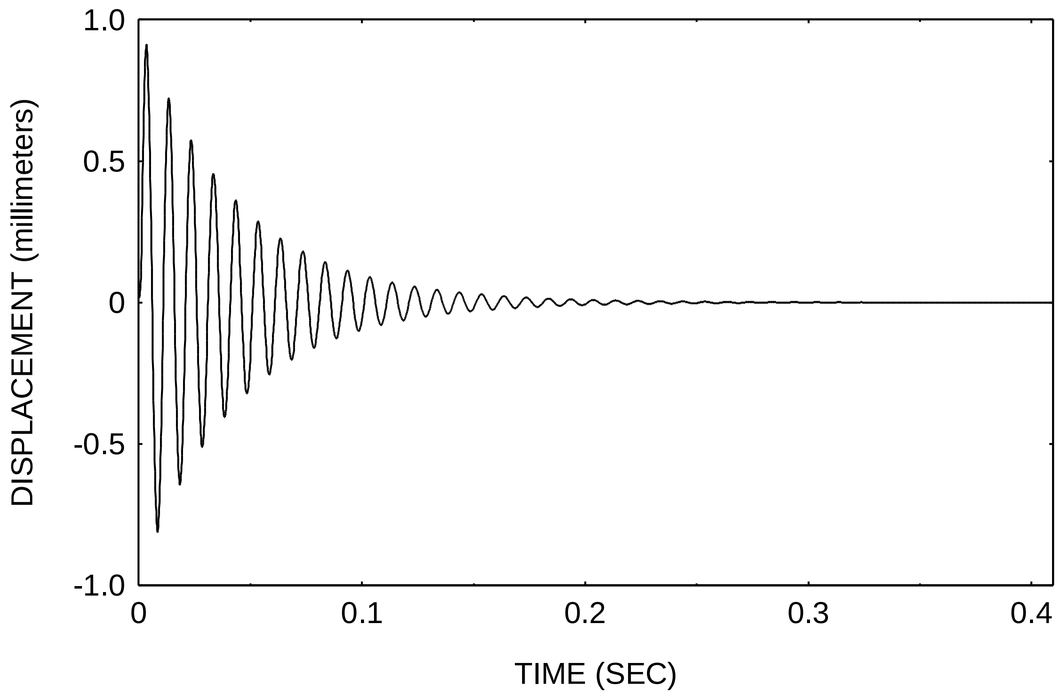

Figure 16.

COMPLIANCE TRANSFER FUNCTION MAGNITUDE [DISPLACEMENT/FORCE]
N = 4096 SAMPLES. SR = 10,000 SAMPLES/SEC. DUR = 0.4096 SEC. ΔF = 2.441 Hz

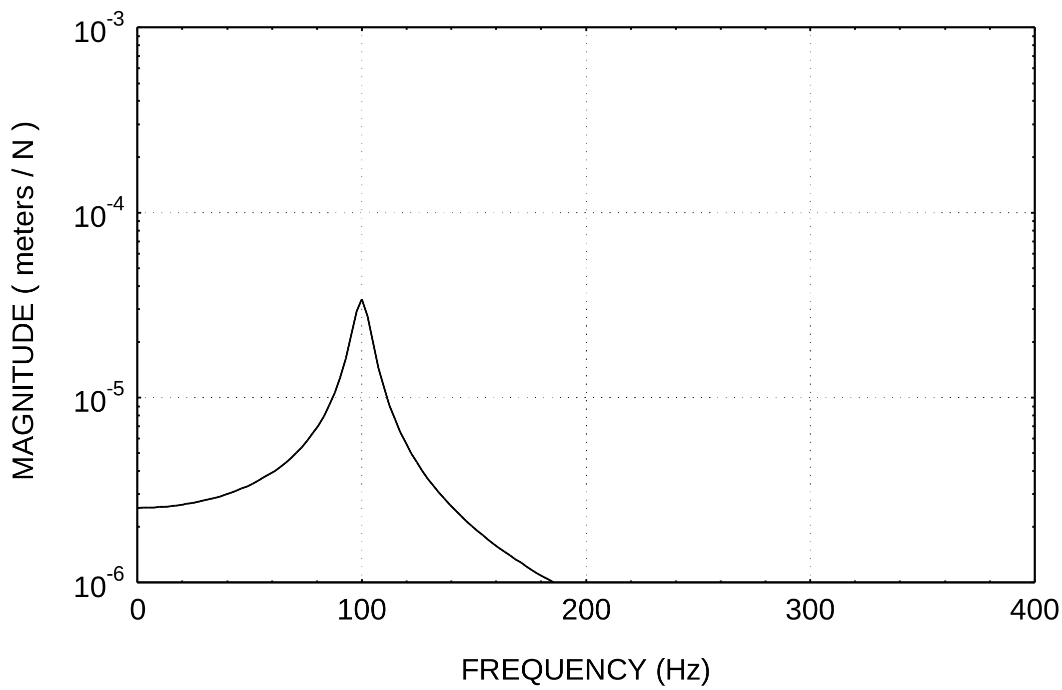

Figure 17.

COMPLIANCE TRANSFER FUNCTION MAGNITUDE [DISPLACEMENT/FORCE]
N = 4096 SAMPLES. SR = 10,000 SAMPLES/SEC. DUR = 0.4096 SEC. ΔF = 2.441 Hz

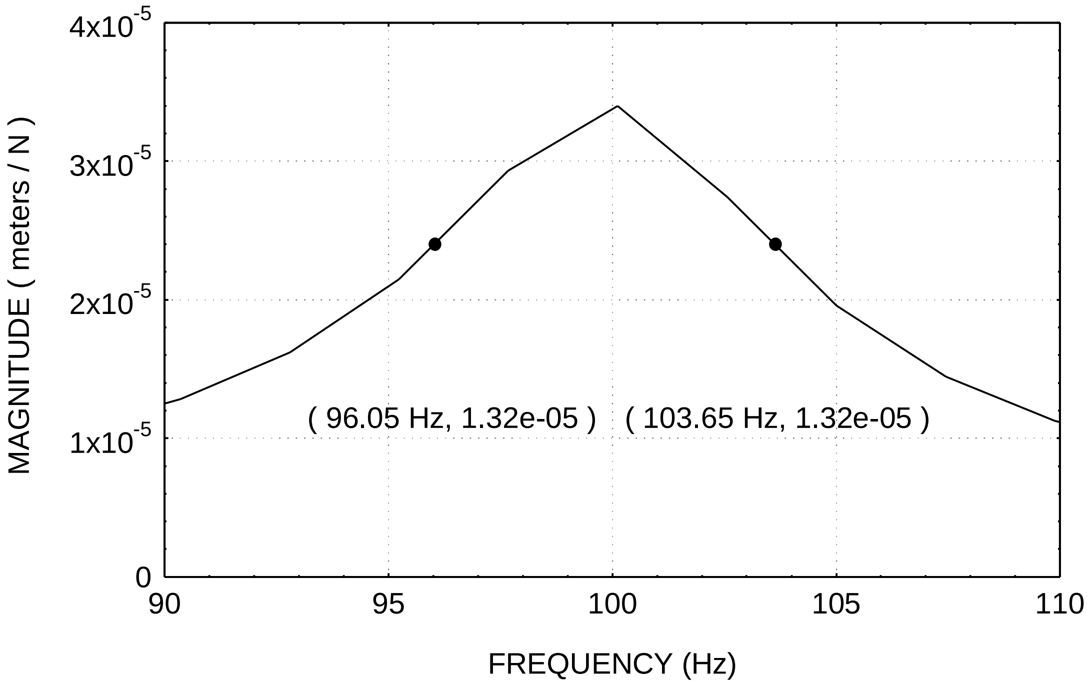

Figure 18.

Again, the apparent damping is

$$\xi_a = \frac{\Delta f_h}{2\, f_n} \tag{46}$$

The frequency separation is 7.6 Hz in the example shown in Figure 18. Again, the natural frequency is 100 Hz. Thus

$$\xi_a = \frac{7.6\ \text{Hz}}{2\ (100\ \text{Hz})} \tag{47}$$

$$\xi_a = 0.038 \tag{48}$$

The true damping ratio is estimated as

$$\xi = \frac{2\pi\,\xi_a\, f_n - \sigma}{2\pi\, f_n} \tag{49}$$

Substitute equations (45) and (48) into (49).

$$\xi = \frac{[2\pi (0.038)(100) - 16.865] \text{ rad/sec}}{2\pi (100) \text{ rad/sec}} \tag{50}$$

$$\xi = 0.011 \tag{51}$$

This is only 10% higher than the true value $\xi = 0.01$.

The exponential window was thus a successful tool in identifying the damping ratio given a limited time window. The ideal approach, however, would be to use a sufficiently long sampling duration to obviate the need for the exponential window.

Again, the half-power bandwidth method can be used without normalizing or scaling the frequency response function.

Advanced Methods

Reference 5 discusses some of the methods covered in this tutorial, as well as advanced techniques. An example is the complex exponential method which is appropriate for multi-degree-of-freedom methods.

References

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