Resonance Phenomena in Tension Members with Time-Varying Characteristics

Stefan Kaczmarczyk
School Applied Sciences
University College Northampton
Overview

• Introduction: elastic tension members in transport applications
• Dynamic features and modelling
• Non-stationary dynamics and resonance phenomena
• Response prediction and analysis methods
• Applications in vertical transportation: deep mine hoists and building elevators
• Conclusions
Introduction

- Long moving continua: ropes, cables, belts, tethers, among the oldest tools/elements used in engineering;
- Low bending and torsional stiffness;
- Ability to resist large axial loads;
- Used in elevators, hoists, cranes, marine installations and space systems;
- Axially moving, their lengths often vary with time when the system is in operation;
Horizontal transportation systems (1)

Building elevator roping configurations
Vertical transportation systems (2)

Double drum mine hoists
Dynamic features

• Moving slender continua are inherently non-linear;
• The length variation results in slow variation of the natural frequencies rendering the entire system non-stationary;
• The natural frequencies of the installation change with the speed of the transport motion;
• The dynamic forces and response is qualitatively different from the response which would occur if the characteristics were stationary, with transient resonance and vibration interaction phenomena taking place.
Non-stationary dynamics and rope/cable theories

• Classical rope/cable theory: Irvine (1981), Costello (1997);


Modelling procedure

- Assemblage of one-dimensional continuous components
  - Time-variant length
  - Lateral and longitudinal motions
  - Constraints at boundaries

  → Hamilton Principle

  → Non-stationary
  → Non-linear
  → Coupled
  → PDE system

→ Numerical or approximate analytical methods to solve the model
The model of rope with varying length

Dynamic deformation vector:

\[ R(s,t) = R_\Omega(t) + R^i(s) + U(s,t) \]

\[ R_\Omega = [-l,0,0]^T \]

\[ R^i = [s,0,0]^T \]

\[ U = [u(s,t), v(s,t), w(s,t)]^T \]

\[ R(s,t) = [s + u(s,t) - l(t), v(s,t), w(s,t)]^T \]

\[ D(t) = \{s : l(t) < s < L_0\} \]

\[ l(t) = l(0) \pm \int_0^t V(\xi)d\xi \]
System of PDE of motion

\[ \rho(x)U_{,tt} + C [U_{,t}] + L [U] = N [U] + F (x,t,\Omega), \quad x \in D(t), \quad 0 \leq t < \infty, \]

- \( \rho(x) \): mass distribution function
- \( x \): Lagrangian or Eulerian co-ordinate
- \( U(x,t) \): dynamic displacement vector
- \( C, L \): linear operators
- \( N \): non-linear operator
- \( F \): vector of forcing functions with harmonic terms
- \( D(t) = \{x: 0 < x < L(t)\} \)
The rate of variation of parameters

- The small parameter to assess the slow variability of the component length:

\[ \varepsilon = \frac{V}{\omega_0 L_0} \]

- \( \varepsilon \) is directly related to the ratio of the rate of variation of the length of the member (or its axial velocity) and the respective wave velocity:

\[ \omega_0 = \frac{\pi}{2L_0} \quad \varepsilon = \frac{2V}{\pi c} \]

- Facilitates the introduction of the slow time scale \( \tau = \varepsilon t \) to observe the length variation.
The solution methods

• The PDE model can be discretised by expansion in terms of modes of the corresponding linear stationary system;
• The modal expansion leads to the first-order ordinary differential equation (ODE) system with slowly varying parameters;
• An approximate solution can be sought using asymptotic (perturbation) methods or direct numerical integration techniques;
• In some cases, the system of PDEs can be treated directly without discretization and the method of multiple scales can be applied.
The natural frequencies and modes

- Determined from the non-stationary frequency equation for $L = L(\tau)$;
- Lateral:

\[
\left( k - \frac{M}{m} T_0 \beta_n^2 \right) \sin \beta_n L + T_0 \beta_n \cos \beta_n L = 0
\]

\[
\tilde{\omega}_n (\tau) = \overline{c} \beta_n (\tau), \quad \text{where} \quad \overline{c} = \sqrt{\frac{T_0}{m}}
\]

- Longitudinal:

\[
\left( \frac{1}{L_p} - \frac{M_e}{m} \gamma_n^2 \right) \left( \cos \gamma_n L - \frac{M}{m} \gamma_n \sin \gamma_n L \right) - \gamma_n \left( \frac{M}{m} \gamma_n \cos \gamma_n L + \sin \gamma_n L \right) = 0
\]

\[
\omega_n (\tau) = c \gamma_n (\tau), \quad \text{where} \quad c = \sqrt{\frac{EA}{m}}
\]
The effect of transport speed (1)

• The natural frequencies decrease as the rope speed $V$ increases:

$$\tilde{\omega}_n = \bar{\omega}_n \left(1 - \nu^2\right)$$

where

$$\nu = \frac{V}{c} = \pi \varepsilon$$

the transport speed parameter
The effect of transport speed (2)

• $\nu_c = 1$ the critical value (the elevator speed equals the lateral wave speed in the stationary rope);

• The frequency of each mode vanishes and the rope experiences divergent instability;

• In suspension ropes tensions are high $\nu \ll \nu_c$, the effect is small;

• In compensating ropes tensions are much lower and the speed parameter may exceed the critical value ($\nu > \nu_c$)
Simulation model

\[ \frac{dy}{dT} = A(T, \tau; \varepsilon)y + N(\tau, y) + F(T, \tau) \]

where:
- \( y \) - modal state vector
- \( A \) - linear coefficient matrix
- \( N \) - coupling vector with quadratic and cubic nonlinear terms
- \( F \) - external excitation vector
- \( \tau \) - slow time (\( \tau = \varepsilon T \))
Non-linear couplings

\[
\mathbf{N}(\tau, y) = \left[ \begin{array}{c} \mathbf{N}^v(\tau, y) \\ \mathbf{N}^w(\tau, y) \\ \mathbf{N}^u(\tau, y) \end{array} \right] = \left[ \begin{array}{c} \mathbf{0} \left[ 1 \times (2N_{lat} + N_{long}) \right] \\ \mathbf{N}^v(\tau, y) \\ \mathbf{N}^w(\tau, y) \\ \mathbf{N}^u(\tau, y) \end{array} \right]^T
\]

\[
\mathbf{N}^v(\tau, y) = [N^v]_{(N_{lat} \times 1)} = -\left( \frac{c}{\varepsilon} \right)^2 \left\{ \hat{\omega}_k^2(\tau) \left[ \frac{1}{L_c} \sum_{n=1}^{N_{long}} z_n + \sum_{n=1}^{N_{lat}} \beta_n^2 (p_n^2 + q_n^2) \right] p_k \right\},
\]

\[
\mathbf{N}^w(\tau, y) = [N^w]_{(N_{lat} \times 1)} = -\left( \frac{c}{\varepsilon} \right)^2 \left\{ \hat{\omega}_k^2(\tau) \left[ \frac{1}{L_c} \sum_{n=1}^{N_{long}} z_n + \sum_{n=1}^{N_{lat}} \beta_n^2 (p_n^2 + q_n^2) \right] q_k \right\},
\]

\[
\mathbf{N}^u(\tau, y) = [N^u]_{(N_{long} \times 1)} = -\frac{EA}{\omega_0^2} \left[ \frac{1}{m_r(\tau)} \sum_{n=1}^{N_{lat}} \beta_n^2 (p_n^2 + q_n^2) \right].
\]
**Transient non-linear interactions**

The natural frequencies are slowly-varying: \( \omega_n = \omega_n(\tau), \ \tau = \varepsilon t \)

<table>
<thead>
<tr>
<th>Non-linearity:</th>
<th>Quadratic</th>
<th>Cubic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resonance:</strong></td>
<td>( \omega_n \approx 2\omega_m ) or ( \omega_n \approx \omega_m \pm \omega_k )</td>
<td>( \omega_n \approx \omega_m, \omega_n \approx 3\omega_m )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \omega_n \approx</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \omega_n \approx</td>
</tr>
<tr>
<td><strong>Internal</strong></td>
<td>( \Omega = \omega_m )</td>
<td>( \Omega =</td>
</tr>
<tr>
<td></td>
<td>( p\Omega = q\omega_m )</td>
<td>( \Omega =</td>
</tr>
<tr>
<td></td>
<td>( \Omega =</td>
<td>\pm \omega_m \pm \omega_k</td>
</tr>
</tbody>
</table>
Vibrations of long moving ropes

- High-rise elevators: ropes over 500 m in length;
- Deep mines: ropes over 2000 m in length;
- Severe vibration problems;
- Rope whirling, miscoiling and/or jumping out of the sheave groove;
- Ride quality compromised;
- Excessive friction wear reducing safe service life;
- Excessive dynamic tension fluctuations leading to high level dynamic stresses.
Double-drum Blair Multi-Rope winder with twin rope compartment drums
Winding house
Headsheave
Vertical rope and skip in the shaft
Hoist Rope Force (1)
Hoist Rope Force (2)

Sheave Force kN

Skip Force kN

Shaft Depth m
Sources of excitation

- Inertial load caused by the acceleration/deceleration profile (results in transient longitudinal response);
- Sheave/pulley irregularities and rope – sheave/pulley interactions;
- Rope storage (coiling) mechanism (drum drive systems);
- Guide rail irregularities/ deformations and joint steps;
- Roller guide irregularities;
- Aerodynamic (air flow) effects;
- Building sway;
- Rotating unbalance/ car (conveyance) unbalance;
- Internal (autoparametric) excitations.
The catenary cable - vertical rope model
Rope coiling pattern
Crossover zones of the Lebus drum

The diagram shows the crossover zones of the Lebus drum with labeled dimensions:

- $2R_d$
- $d/2$
- $R_d \beta$

The diagram illustrates the layout of the crossover zones with labeled dimensions to describe the structure and function of the drum.
Excitation functions

\[ v_l = \begin{cases} \frac{1}{2}v_0^v[1 - \cos(2vt)], & 0 \leq t \leq t_\beta \\ 0, & t_\beta \leq t \leq \tau \end{cases} \]

\[ w_l = \begin{cases} \frac{1}{2}w_0[w_0[1 - \cos(\omega t)], & 0 \leq t \leq t_\beta \\ \frac{\pi}{2}, & t_\beta \leq t \leq \tau \end{cases} \]

\[ u_l = \begin{cases} \frac{1}{2}u_0[u_0[1 - \cos(\omega t)], & 0 \leq t \leq t_\beta \\ u_0(\tau - t)/(\tau - t_\beta), & t_\beta \leq t \leq \tau \end{cases} \]

\[ t_\beta = \beta/\omega_d \]

\[ \tau = 2\pi/\Omega \]

\[ \Omega = 2\omega_d \]

\[ \nu = \pi/t_\beta \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time interval s</td>
<td>156</td>
</tr>
<tr>
<td>Nominal speed m/s</td>
<td>15</td>
</tr>
<tr>
<td>Total payload kg</td>
<td>17584</td>
</tr>
<tr>
<td>Sheave inertia kgm²</td>
<td>15200</td>
</tr>
<tr>
<td>Drum radius m</td>
<td>2.14</td>
</tr>
<tr>
<td>Crossover arc rad</td>
<td>0.2</td>
</tr>
<tr>
<td>Cable diameter mm</td>
<td>48</td>
</tr>
<tr>
<td>Cable density kg/m</td>
<td>8.4</td>
</tr>
<tr>
<td>Cable effective area m²</td>
<td>$1.028 \times 10^{-3}$</td>
</tr>
<tr>
<td>Cable Young’s modulus N/m²</td>
<td>$1.1 \times 10^{11}$</td>
</tr>
<tr>
<td>Catenary length m</td>
<td>74.95</td>
</tr>
<tr>
<td>Maximum shaft depth m</td>
<td>2100</td>
</tr>
</tbody>
</table>
Simulation results

• Relationship between the crossover excitation frequency, the lateral and longitudinal natural frequencies: $\Omega_k, \bar{\omega}_m, \omega_n$, respectively

• Displacement response

• Cable/rope tensions
Frequency map: $V = 15$ m/s

$$
\varepsilon = \frac{V}{\omega_{\min} L_{\text{max}}} \ll 1
$$

$$
\omega_{\min} = \omega_1 \big|_{L_{\text{max}}} = 1.5 \text{ rad} / \text{s}
$$

$$
L_{\text{max}} = 2100 \text{ m}
$$

$$
\varepsilon = 0.005
$$
Resonance conditions

- \( (\bar{\omega}_k)_{in-plane} = (\bar{\omega}_k)_{out-of-plane} \) 1:1 autoparametric resonance

- 750m: \( \omega_2 \approx 2\bar{\omega}_1 \) 2:1 autoparametric resonance

\[ \Omega_1 \approx \omega_2 \approx \bar{\omega}_2 \quad \text{and} \quad \Omega_2 \approx 2\bar{\omega}_2 \] primary external resonances

\[ \Omega_2 \approx \omega_2 + \bar{\omega}_2 \] summed combination resonance
Displacement response: \( V = 15 \text{ m/s} \)
Catenary cable 1\textsuperscript{st} quarter point motion
Rope tensions: $V = 15 \text{ m/s}$
Frequency map: $V = 19.5$ m/s
Displacement response: $V = 19.5 \text{ m/s}$
Rope tensions: $V = 19.5$ m/s
Elevator suspension rope model
# The system parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car mass</td>
<td>$P$</td>
<td>2000</td>
<td>kg</td>
</tr>
<tr>
<td>Mass of rated load</td>
<td>$Q$</td>
<td>1250</td>
<td>kg</td>
</tr>
<tr>
<td>Balance</td>
<td>$B$</td>
<td>40</td>
<td>%</td>
</tr>
<tr>
<td>Equivalent mass of diverter pulley</td>
<td>$M_e$</td>
<td>80</td>
<td>kg</td>
</tr>
<tr>
<td>Rope length between traction sheave and pulley</td>
<td>$L_p$</td>
<td>1.04</td>
<td>m</td>
</tr>
<tr>
<td>Travel height</td>
<td>$H$</td>
<td>60</td>
<td>m</td>
</tr>
<tr>
<td>Well height</td>
<td>$W$</td>
<td>70</td>
<td>M</td>
</tr>
<tr>
<td>Car height</td>
<td>$H$</td>
<td>3.2</td>
<td>m</td>
</tr>
<tr>
<td>Hoist rope Young’s modulus</td>
<td>$E$</td>
<td>60.0</td>
<td>kN/mm²</td>
</tr>
<tr>
<td>Hoist rope diameter</td>
<td>$d$</td>
<td>19</td>
<td>mm</td>
</tr>
<tr>
<td>Hoist rope mass per unit length</td>
<td>$m$</td>
<td>1.3</td>
<td>kg/m</td>
</tr>
<tr>
<td>Compensating rope mass per unit length</td>
<td>$m_c$</td>
<td>1.6</td>
<td>kg/m</td>
</tr>
</tbody>
</table>
Longitudinal resonance frequencies

- car side
- cw side
Lateral resonance frequencies (1)

--- car side

---- cw side
Lateral resonance frequencies (2)

\[ \omega_M = \sqrt{\frac{k}{M}} \]

\[ L(t) \]

\[ M \]

\[ \omega_1 \]

\[ \omega_2 \]

\[ \omega_3 \]

\[ \omega_M \]

Suspension rope length \( L \) m

Lateral Frequency Hz

0 1 2 3 4 5 6 7 8 9 10 20 30 40 50 60

Lateral Frequency Hz

0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5

\[ \frac{L}{2} \]
External, the lateral and the 1\textsuperscript{st} longitudinal natural frequencies (car side)

\begin{align*}
\omega_1 & \\
\omega_4 & \\
\Omega & 
\end{align*}

\begin{axis}[
    title={},
    xlabel={Suspension rope length $L$ [m]},
    ylabel={Frequency [Hz]},
    xmin=0, xmax=70,
    ymin=0, ymax=30,
    xtick={0,10,20,30,40,50,60,70},
    ytick={0,5,10,15,20,25,30},
    legend entries={Lateral, Vertical, External},
]
\addplot[style={black, line width=1pt}]{...};
\addplot[style={red, line width=1pt}]{...};
\addplot[style={blue, dashed, line width=1pt}]{...};
\end{axis}
Resonance: building sway effect

- Building often excited near its own natural frequency;
- Periodic sway at low frequencies results;
- Elevator ropes strongly affected;
- Resonance conditions occur during elevator travel;
- *Whirling* motions may result.
Passage through lateral resonance

\[ w_0 = A_0 \sin \Omega t \]

\[ x = L/2 \]

Transient resonances

\[ \omega_1, \omega_2, \Omega \]

Response \( w(L/2,t) \) m

Frequency Hz
Multiple transient resonances

\[ w_0 = A_0 \sin \Omega t \]

\[ T, m \quad g \quad V \]

\[ x = \frac{L}{2} \]
Passage through resonance at various speeds

- $V = 2.5 \text{ m/s}$
- $V = 10 \text{ m/s}$
Conclusions

- Despite recent developments in new technologies elastic suspension components in vertical transport systems are susceptible to oscillations;
- The non-linear and non-stationary nature of long moving ropes in transport installations is often responsible for adverse dynamic behaviour of the entire system;
- Large dynamic responses occur due to the transient resonance phenomena;
- The prediction of resonance conditions is of primary importance in the design of vertical transport installations;
- Subsequently, a suitable control strategy can be sought to minimize the resonance effects;
- The resonance effects can be reduced/shifted through the speed and/or acceleration changes.
References


