Ropes, Cables and Chains: Theory and Applications
Northampton 23rd September 2004

Resonance Phenomena in Tension Members with Time-Varying Characteristics

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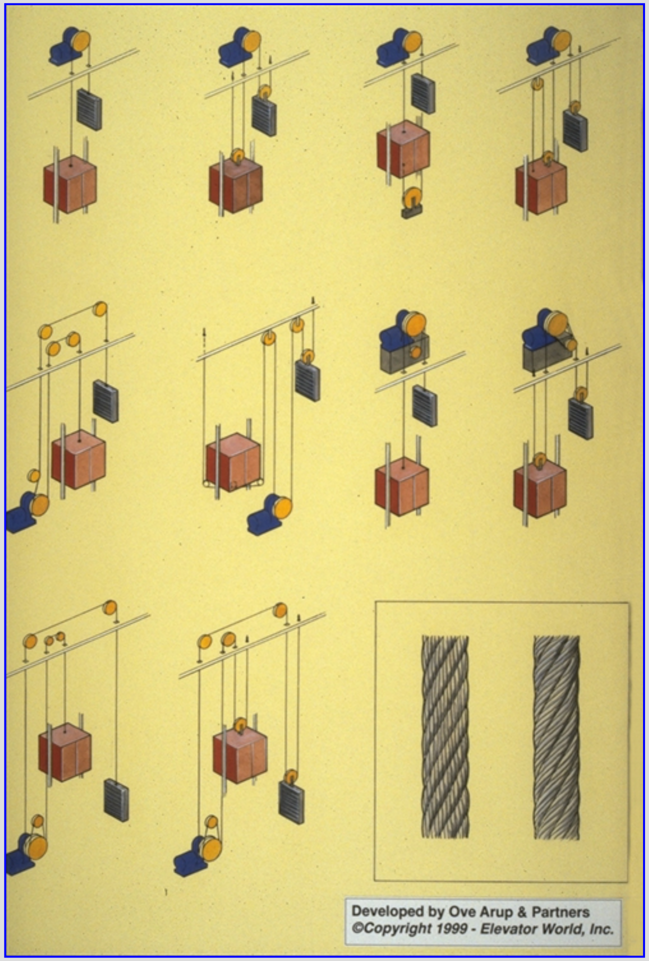
Overview

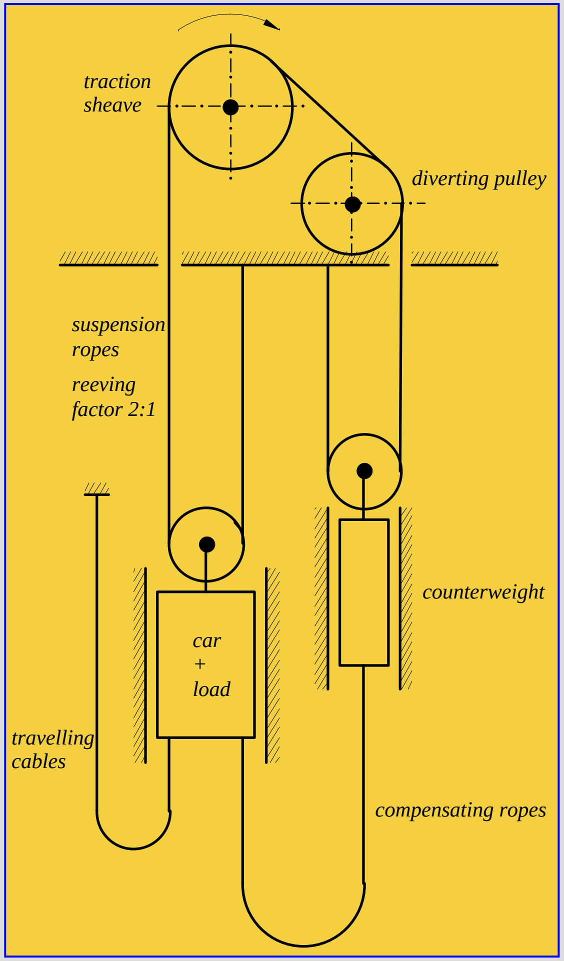
- Introduction: elastic tension members in transport applications
- Dynamic features and modelling
- Non-stationary dynamics and resonance phenomena
- Response prediction and analysis methods
- Applications in vertical transportation: deep mine hoists and building elevators
- Conclusions

Introduction

- Long moving continua: ropes, cables, belts, tethers, among the oldest tools/elements used in engineering;
- Low bending and torsional stiffness;
- Ability to resist large axial loads;
- Used in elevators, hoists, cranes, marine installations and space systems;
- Axially moving, their lengths often vary with time when the system is in operation;

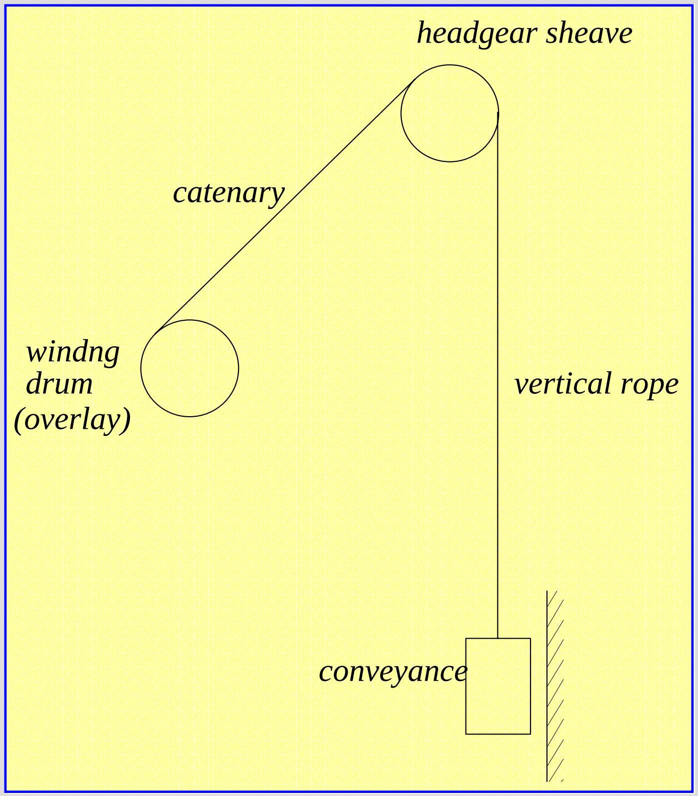
Vertical transportation systems (1)

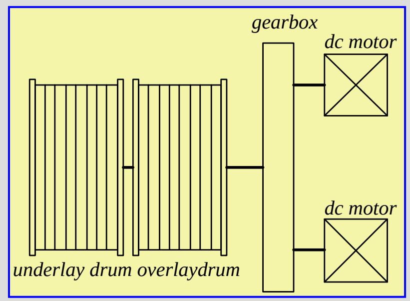




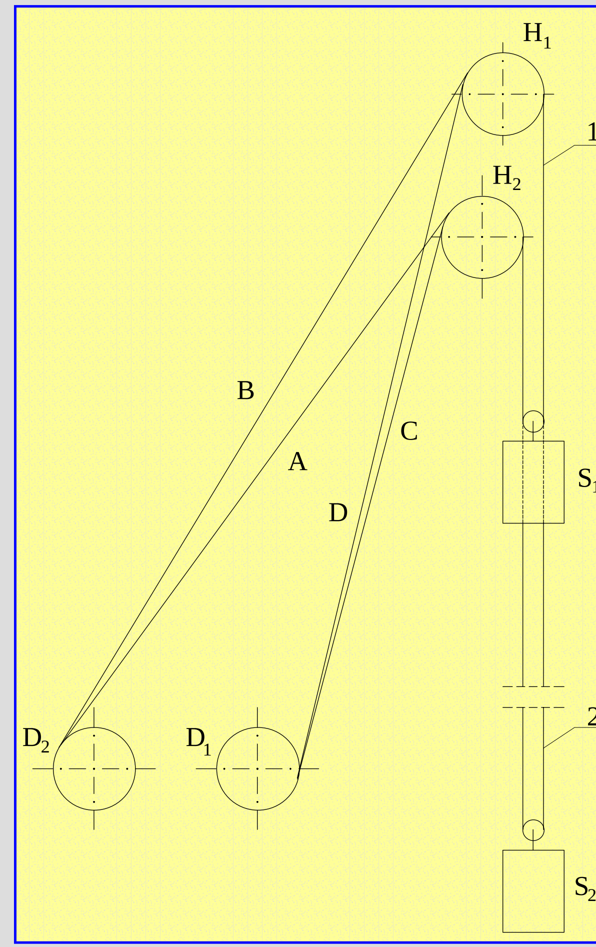
Building elevator roping configurations

Vertical transportation systems (2)





Double drum mine hoists



Dynamic features

- Moving slender continua are inherently non-linear;
- The length variation results in slow variation of the natural frequencies rendering the entire system non-stationary;
- The natural frequencies of the installation change with the speed of the transport motion;
- The dynamic forces and response is qualitatively different from the response which would occur if the characteristics were stationary, with transient resonance and vibration interaction phenomena taking place.

Non-stationary dynamics and rope/cable theories

- Classical rope/cable theory: Irvine (1981), Costello (1997);
- Systems with *slowly* varying parameters, non-stationary oscillations: Mitropolskii (1965), Evan-Iwanowski (1976), Kevorkian (1980), Nayfeh & Asfar (1988), Cveticanin (1991);
- Axially moving continua: Mote Jr. (1966), Perkins & Mote Jr. (1987), Wickert & Mote Jr. (1990), Riedel & Tan (2002).

Modelling procedure

Assemblage of one-dimensional continuous components

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Time-variant length
Lateral and longitudinal motions
Constraints at boundaries

- Non-stationary
- Non-linear
- Coupled

↓

Hamilton Principle — PDE system

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Numerical or approximate analytical methods to solve the model

The model of rope with varying length

Dynamic deformation vector:

$$\mathbf{R}(s,t) = \mathbf{R}_\Omega(t) + \mathbf{R}^i(s) + \mathbf{U}(s,t)$$

$$\mathbf{R}_\Omega = [-l,0,0]^T$$

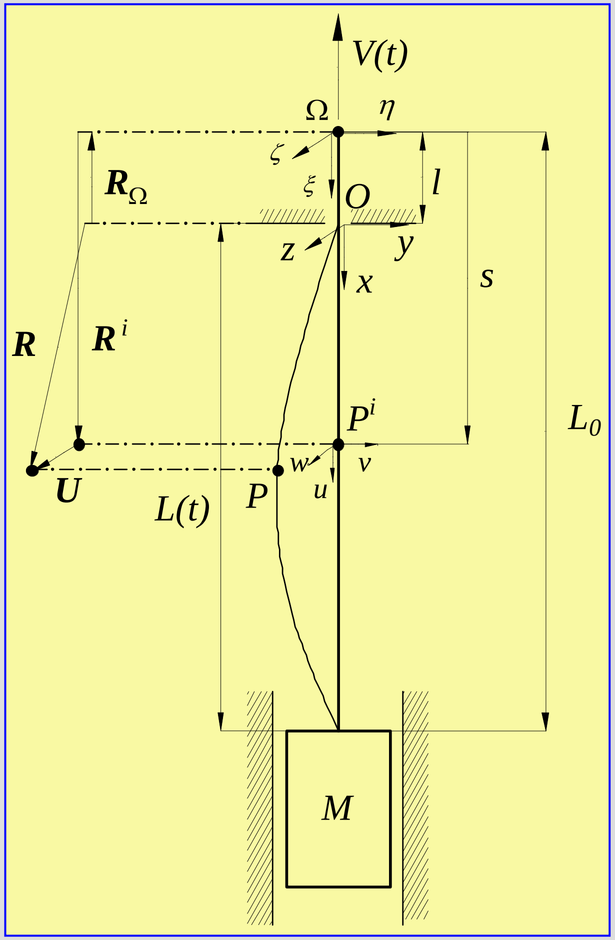
$$\mathbf{R}^i = [s,0,0]^T$$

$$\mathbf{U} = [u(s,t), v(s,t), w(s,t)]^T$$

$$\mathbf{R}(s,t) = [s + u(s,t) - l(t), v(s,t), w(s,t)]^T$$

$$D(t) = \{s : l(t) < s < L_0\}$$

$$l(t) = l(0) \pm \int_0^t V(\xi)\, d\xi$$



System of PDE of motion

$$\rho(x)\mathbf{U}_{,tt} + \mathbf{C}[\mathbf{U}_{,t}] + \mathbf{L}[\mathbf{U}] = \mathbf{N}[\mathbf{U}] + \mathbf{F}(x,t,\Omega), \quad x \in D(t), \; 0 \le t < \infty,$$

$\rho(x)$	mass distribution function
x	Lagrangian or Eulerian co-ordinate
$\mathbf{U}(x,t)$	dynamic displacement vector
$\mathbf{C}$, $\mathbf{L}$	linear operators
$\mathbf{N}$	non-linear operator
$\mathbf{F}$	vector of forcing functions with harmonic terms

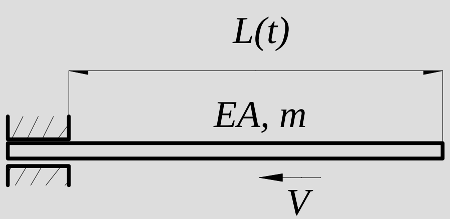
$D(t) = \{x: 0 < x < L(t)\}$

The rate of variation of parameters

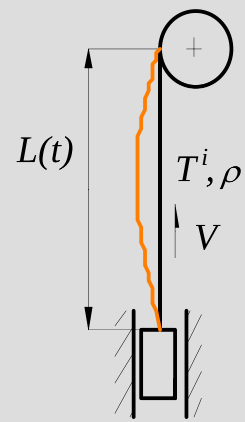
- The small parameter to assess the the slow variability of the component length:

$$\varepsilon = \frac{V}{\omega_0 L_0}$$

- ε is directly related to the ratio of the rate of variation of the length of the member (or its axial velocity) and the respective wave velocity:



$$c = \sqrt{\frac{EA}{m}} \qquad \omega_0 = \frac{\pi}{2L_0} c \qquad \varepsilon = \frac{2}{\pi}\frac{V}{c}$$



$$\bar{c} = \sqrt{\frac{T^i}{\rho}} \qquad \omega_0 = \frac{\pi}{L_0}\bar{c} \qquad \varepsilon = \frac{1}{\pi}\frac{V}{c}$$

- Facilitates the introduction of the slow time scale $\tau = \varepsilon t$ to observe the length variation.

The solution methods

- The PDE model can be discretised by expansion in terms of modes of the corresponding linear stationary system;
- The modal expansion leads to the first-order ordinary differential equation (ODE) system with slowly varying parameters;
- An approximate solution can be sought using asymptotic (perturbation) methods or direct numerical integration techniques;
- In some cases, the system of PDEs can be treated directly without discretization and the method of multiple scales can be applied.

The natural frequencies and modes

- Determined from the non-stationary frequency equation for $L = L(\tau)$;
- Lateral:

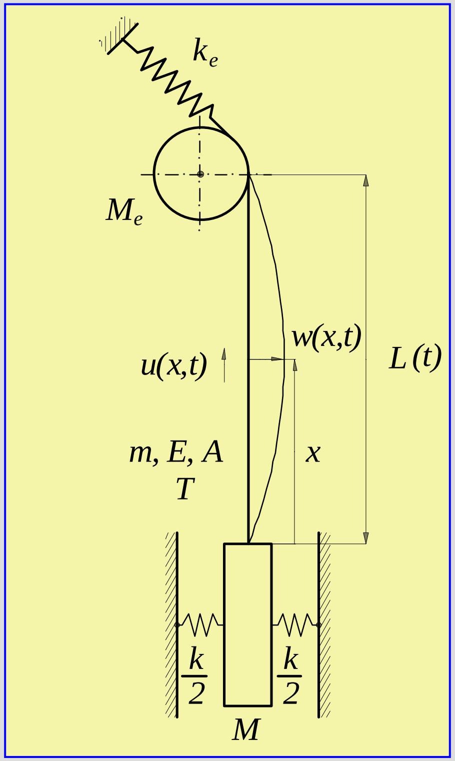
$$\left(k - \frac{M}{m}T_0\beta_n^2\right)\sin\beta_n L + T_0\beta_n\cos\beta_n L = 0$$

$$\hat{\omega}_n(\tau) = \bar{c}\beta_n(\tau), \text{ where } \bar{c} = \sqrt{\frac{T_0}{m}}$$

- Longitudinal:

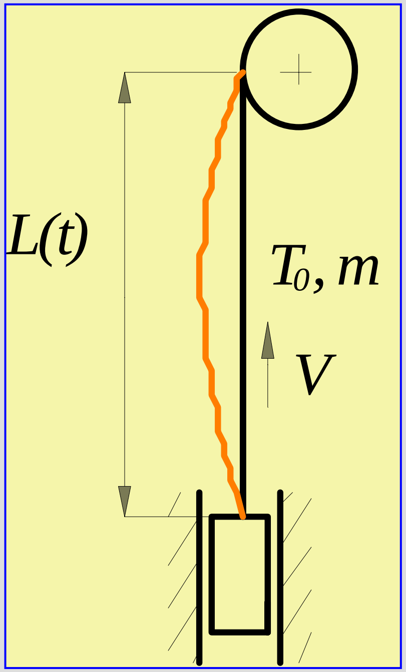
$$\left(\frac{1}{L_p} - \frac{M_e}{m}\gamma_n^2\right)\left(\cos\gamma_n L - \frac{M}{m}\gamma_n\sin\gamma_n L\right) - \gamma_n\left(\frac{M}{m}\gamma_n\cos\gamma_n L + \sin\gamma_n L\right) = 0$$

$$\omega_n(\tau) = c\gamma_n(\tau), \text{ where } c = \sqrt{\frac{EA}{m}}$$



The effect of transport speed (1)

- The natural frequencies decrease as the rope speed V increases:



$$\tilde{\omega}_n = \bar{\omega}_n \left(1 - \nu^2\right)$$

where

$\nu = \dfrac{V}{\bar{c}} = \pi\varepsilon$ the transport speed parameter

The effect of transport speed (2)

- $\nu_c = 1$ the critical value (the elevator speed equals the lateral wave speed in the stationary rope);
- The frequency of each mode vanishes and the rope experiences divergent instability;
- In suspension ropes tensions are high $\nu << \nu_c$, the effect is small;
- In compensating ropes tensions are much lower and the speed parameter may exceed the critical value ($\nu > \nu_c$)

Simulation model

$$\frac{d\mathbf{y}}{dT} = \mathbf{A}(T, \tau; \varepsilon)\mathbf{y} + \mathbf{N}(\tau, \mathbf{y}) + \mathbf{F}(T, \tau)$$

where:

y- modal state vector

A- linear coefficient matrix

N - coupling vector with quadratic and cubic nonlinear terms

F- external excitation vector

τ- slow time ($\tau = \varepsilon T$)

Non-linear couplings

$$\mathbf{N}(\tau,\mathbf{y}) = \left[\mathbf{0}_{[1\times(2N_{lat}+N_{long})]}, \mathbf{N}^{v^T}(\tau,\mathbf{y}), \mathbf{N}^{w^T}(\tau,\mathbf{y}), \mathbf{N}^{u^T}(\tau,\mathbf{y})\right]^T$$

$$\mathbf{N}^v(\tau,\mathbf{y}) = [N_k^v]_{(N_{lat}\times 1)} = -\left(\frac{c}{\bar{c}}\right)^2 \left\{\hat{\omega}_k^2(\tau)\left[\frac{1}{L_c}\sum_{n=1}^{N_{long}} z_n + \sum_{n=1}^{N_{lat}} \beta_n^2(p_n^2+q_n^2)\right] p_k\right\},$$

$$\mathbf{N}^w(\tau,\mathbf{y}) = [N_k^w]_{(N_{lat}\times 1)} = -\left(\frac{c}{\bar{c}}\right)^2 \left\{\hat{\omega}_k^2(\tau)\left[\frac{1}{L_c}\sum_{n=1}^{N_{long}} z_n + \sum_{n=1}^{N_{lat}} \beta_n^2(p_n^2+q_n^2)\right] q_k\right\},$$

$$\mathbf{N}^u(\tau,\mathbf{y}) = [N_r^u]_{(N_{long}\times 1)} = -\frac{EA}{\omega_0^2}\left[\frac{1}{m_r^v(\tau)}\sum_{n=1}^{N_{lat}} \beta_n^2(p_n^2+q_n^2)\right].$$

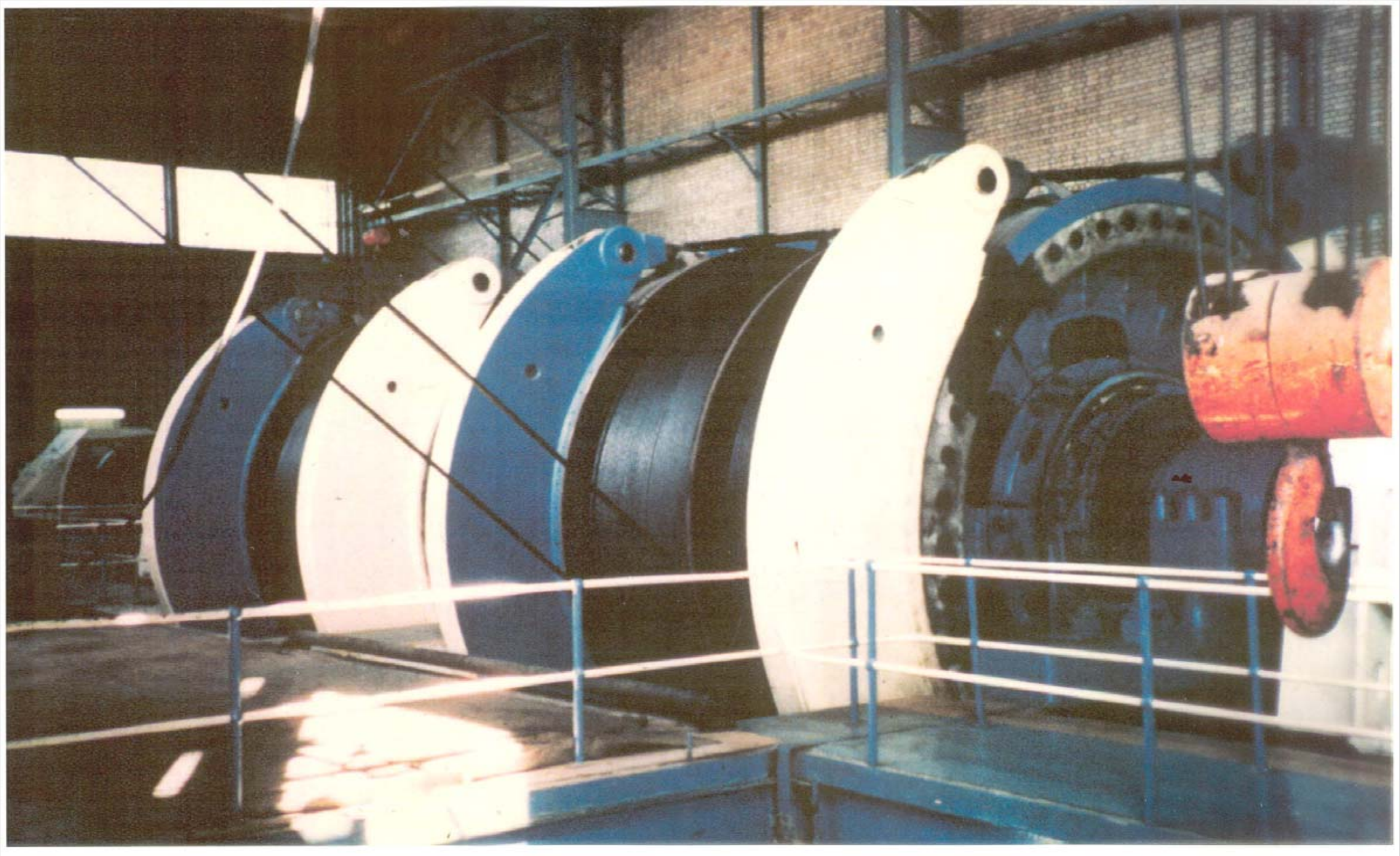
Transient non-linear interactions

The natural frequencies are slowly-varying: $\omega_n = \omega_n(\tau),\ \tau = \varepsilon t$

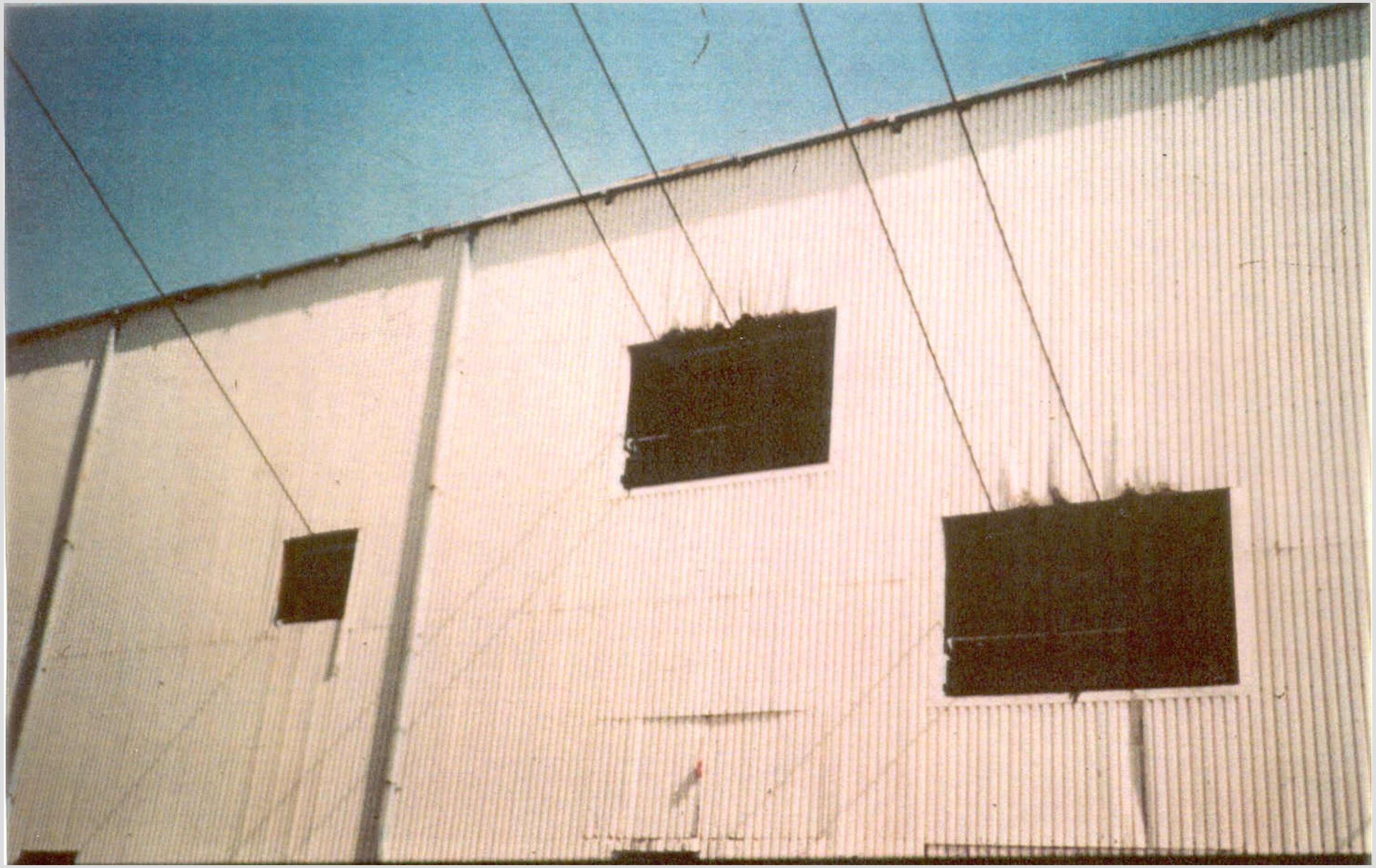
Non-linearity: / Resonance:	Quadratic	Cubic
Internal	$\omega_n \approx 2\omega_m$ or $\omega_n \approx \omega_m \pm \omega_k$	$\omega_n \approx \omega_m, \omega_n \approx 3\omega_m$ $\omega_n \approx \lvert\pm 2\omega_m \pm \omega_k\rvert$ $\omega_n \approx \lvert\pm\omega_m \pm \omega_k \pm \omega_l\rvert$
External/ Parametric	$\Omega = \omega_m$ $p\Omega = q\omega_m$ $\Omega = \lvert\pm\omega_m \pm \omega_k\rvert$	$\Omega = \lvert\pm\omega_m \pm \omega_k \pm \omega_l\rvert$ $\Omega = \lvert\pm 2\omega_m \pm \omega_k\rvert$ $2\Omega = \lvert\pm\omega_m \pm \omega_k\rvert$

Vibrations of long moving ropes

- High-rise elevators: ropes over 500 m in length;
- Deep mines: ropes over 2000 m in length;
- Severe vibration problems;
- Rope whirling, miscoiling and/or jumping out of the sheave groove;
- Ride quality compromised;
- Excessive friction wear reducing safe service life;
- Excessive dynamic tension fluctuations leading to high level dynamic stresses.



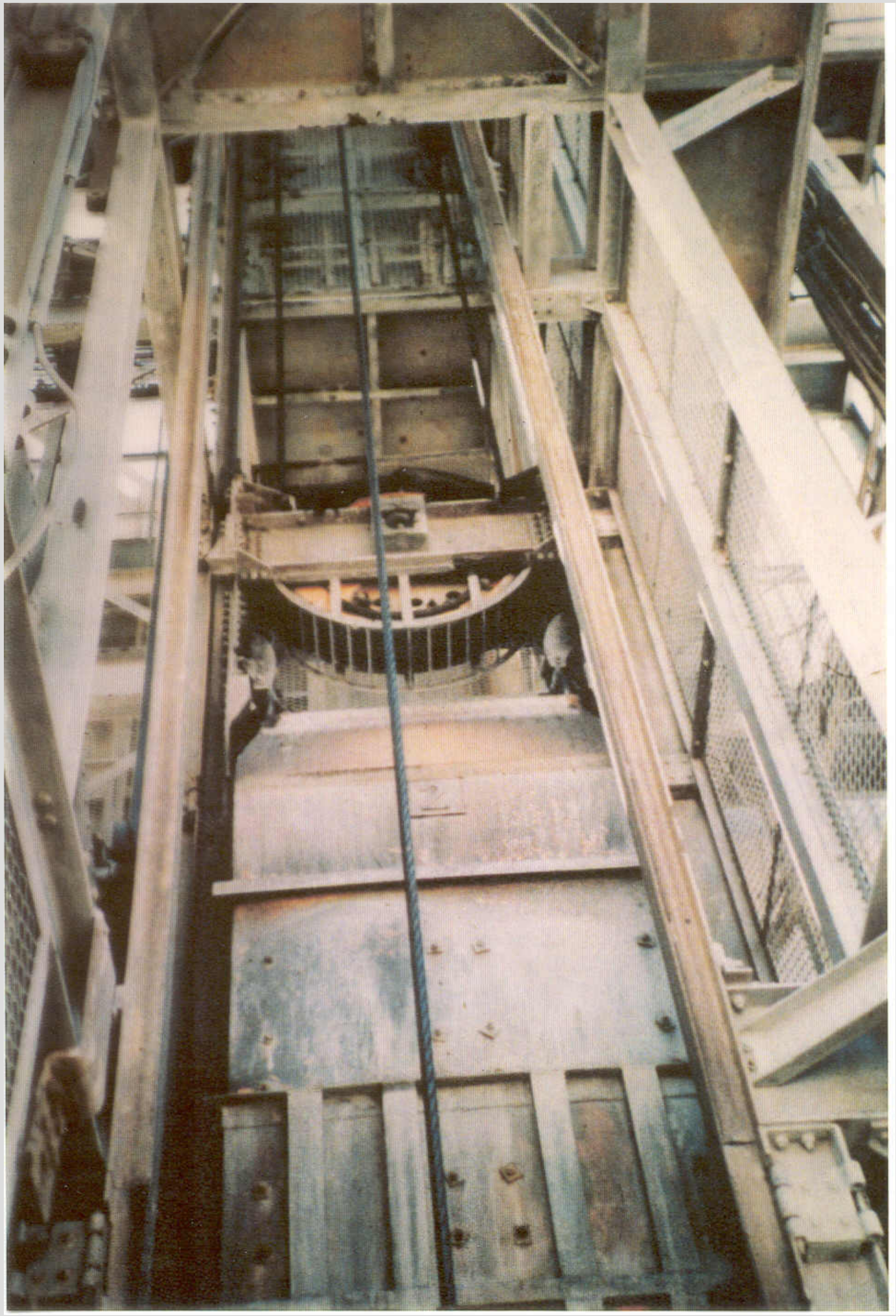
Double-drum Blair Multi-Rope winder with twin rope compartment drums



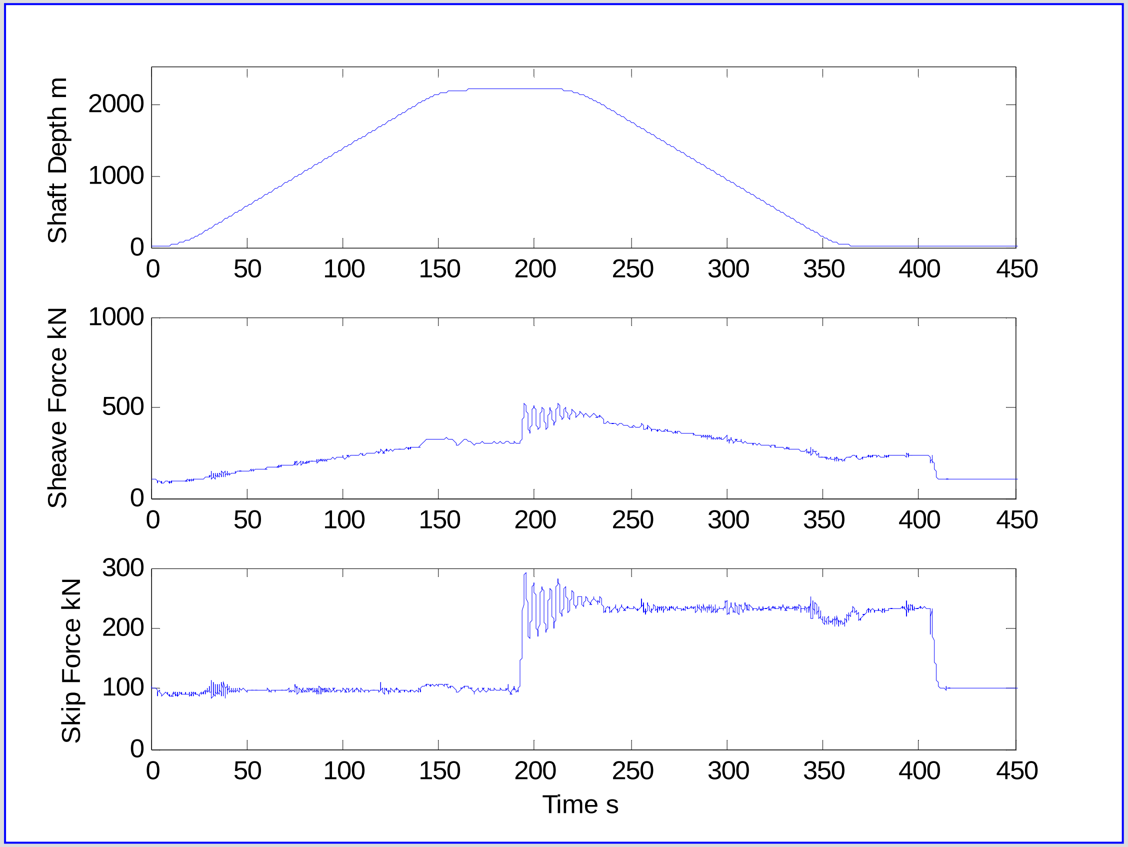
Winding house

Headsheave

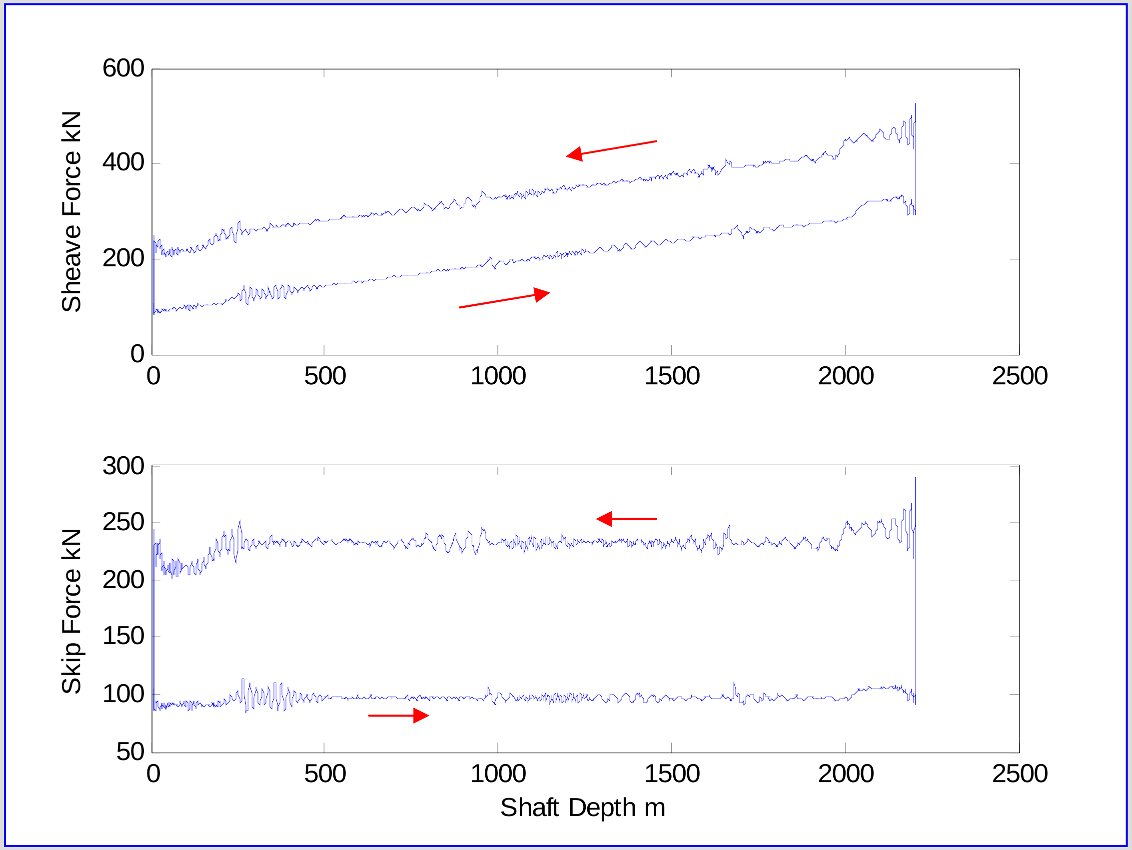
Vertical rope and skip in the shaft



Hoist Rope Force (1)



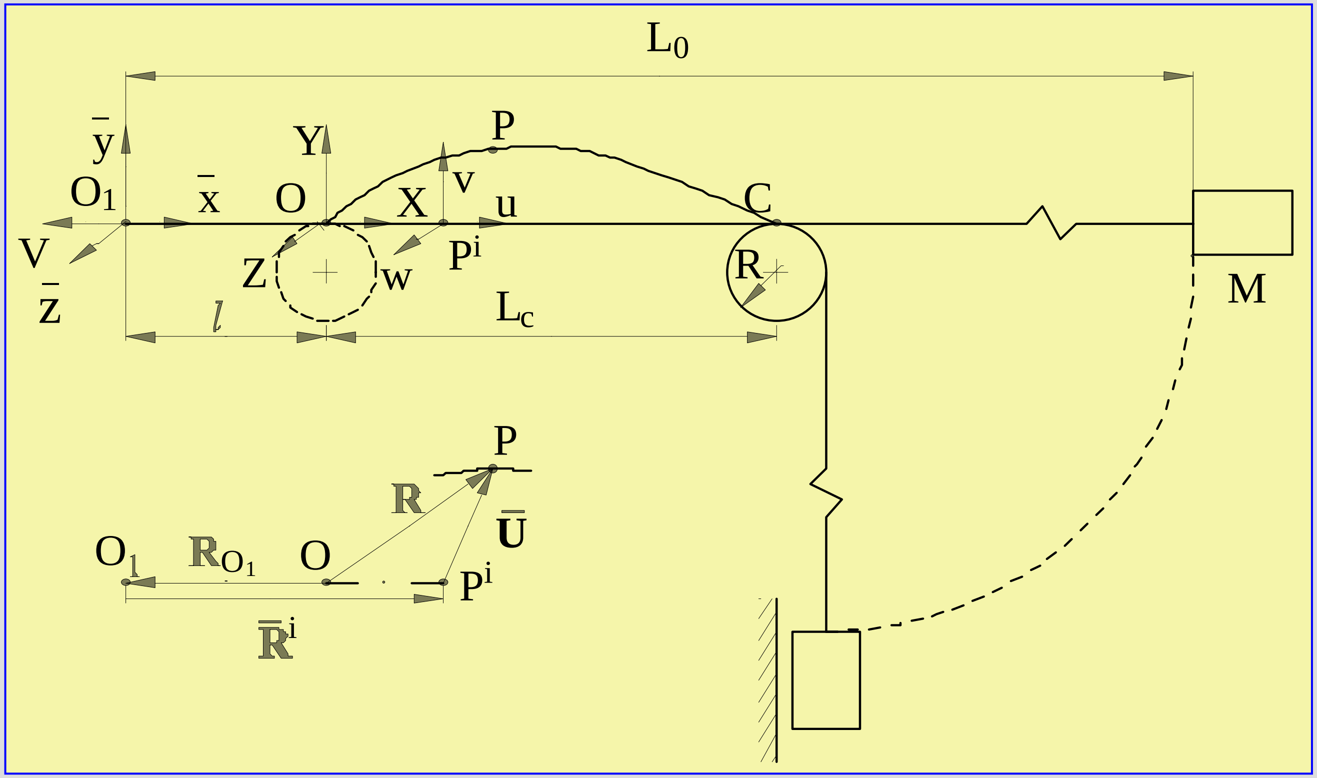
Hoist Rope Force (2)



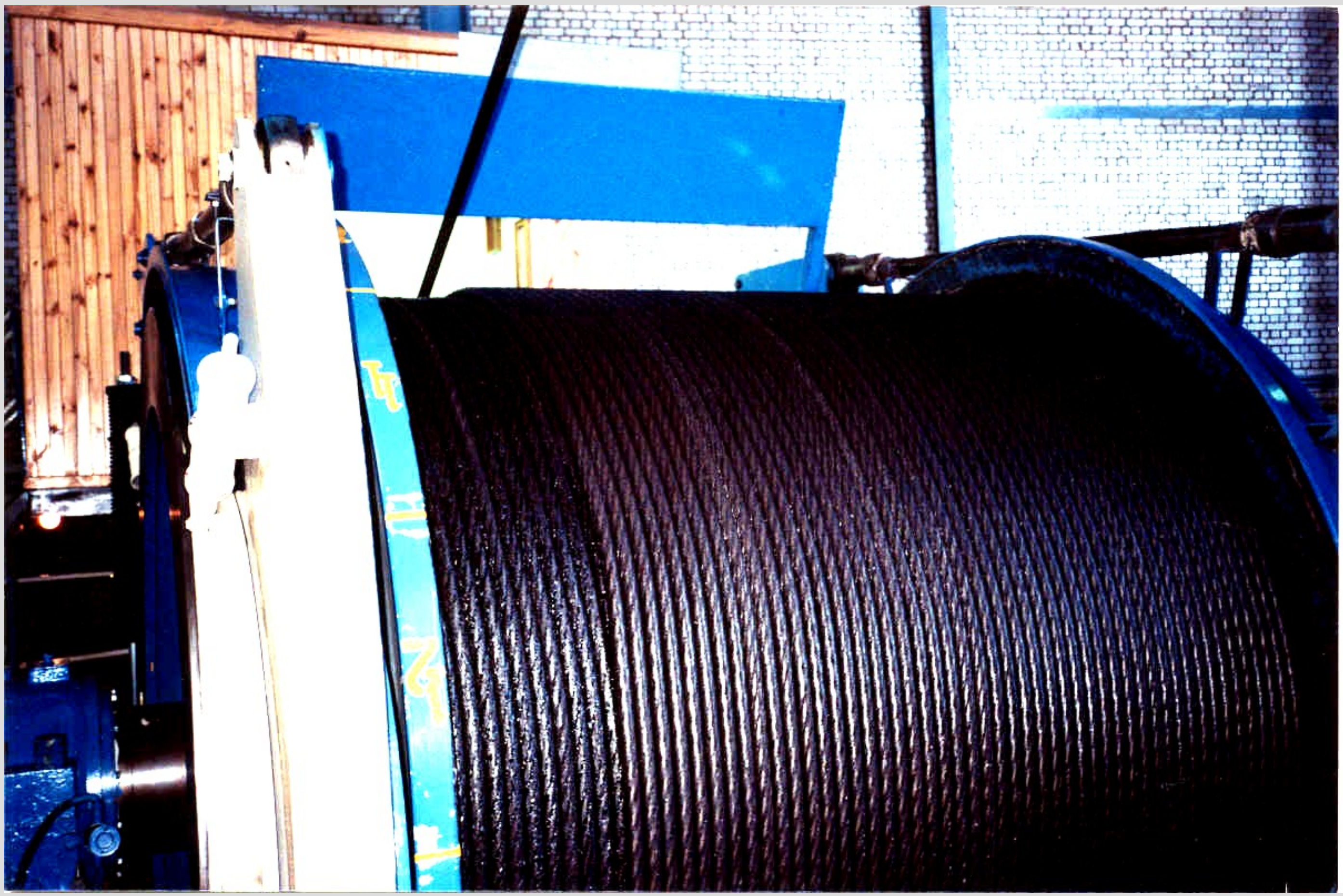
Sources of excitation

- Inertial load caused by the acceleration/deceleration profile (results in transient longitudinal response);
- Sheave/pulley irregularities and rope – sheave/pulley interactions;
- Rope storage (coiling) mechanism (drum drive systems);
- Guide rail irregularities/ deformations and joint steps;
- Roller guide irregularities;
- Aerodynamic (air flow) effects;
- Building sway;
- Rotating unbalance/ car (conveyance) unbalance;
- Internal (autoparametric) excitations.

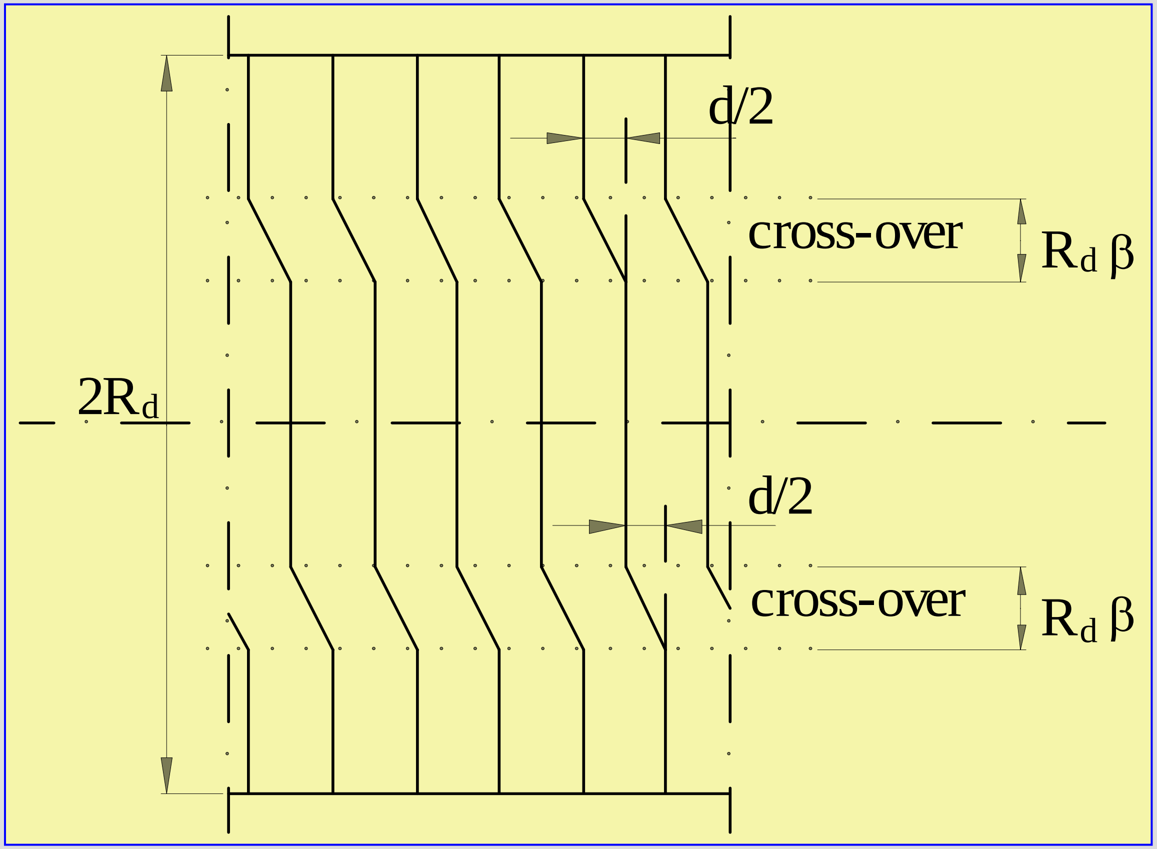
The catenary cable - vertical rope model



Rope coiling pattern



Crossover zones of the Lebus drum



Excitation functions

$$v_l = \begin{cases} \frac{1}{2} v_0^n [1 - \cos(2\nu t)], & 0 \le t \le t_\beta \\ 0, & t_\beta \le t \le \tau \end{cases}$$

$$w_l = \begin{cases} \frac{1}{2} w_0 [1 - \cos(\nu t)], & 0 \le t \le t_\beta \\ \frac{d}{2}, & t_\beta \le t \le \tau \end{cases}$$

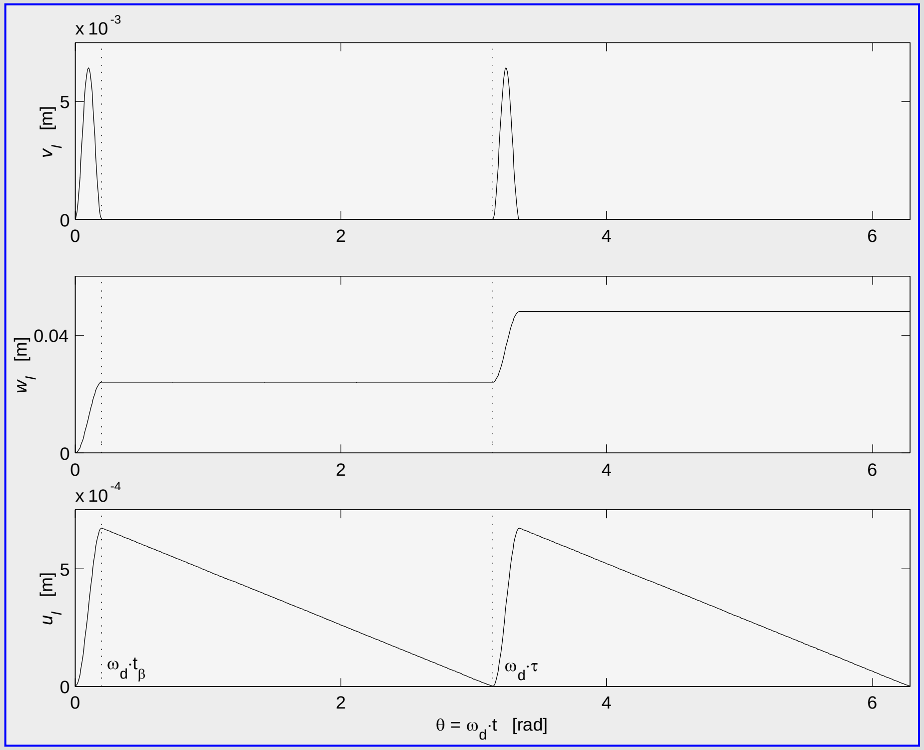
$$u_l = \begin{cases} \frac{1}{2} u_0 [1 - \cos(\nu t)], & 0 \le t \le t_\beta \\ u_0 (\tau - t)/(\tau - t_\beta), & t_\beta \le t \le \tau \end{cases}$$

$$t_\beta = \beta/\omega_d$$

$$\tau = 2\pi/\Omega$$

$$\Omega = 2\omega_d$$

$$\nu = \pi/t_\beta$$



Hoist parameters

Parameter	Value
Time interval s	156
Nominal speed m/s	15
Total payload kg	17584
Sheave inertia kgm^2	15200
Drum radius m	2.14
Crossover arc rad	0.2
Cable diameter mm	48
Cable density kg/m	8.4
Cable effective area m^2	$1.028 \cdot 10^{-3}$
Cable Young's modulus N/m^2	$1.1 \cdot 10^{11}$
Catenary length m	74.95
Maximum shaft depth m	2100

Simulation results

- Relationship between the crossover excitation frequency, the lateral and longitudinal natural frequencies: $\Omega_k, \overline{\omega}_m, \omega_n$, respectively
- Displacement response
- Cable/rope tensions

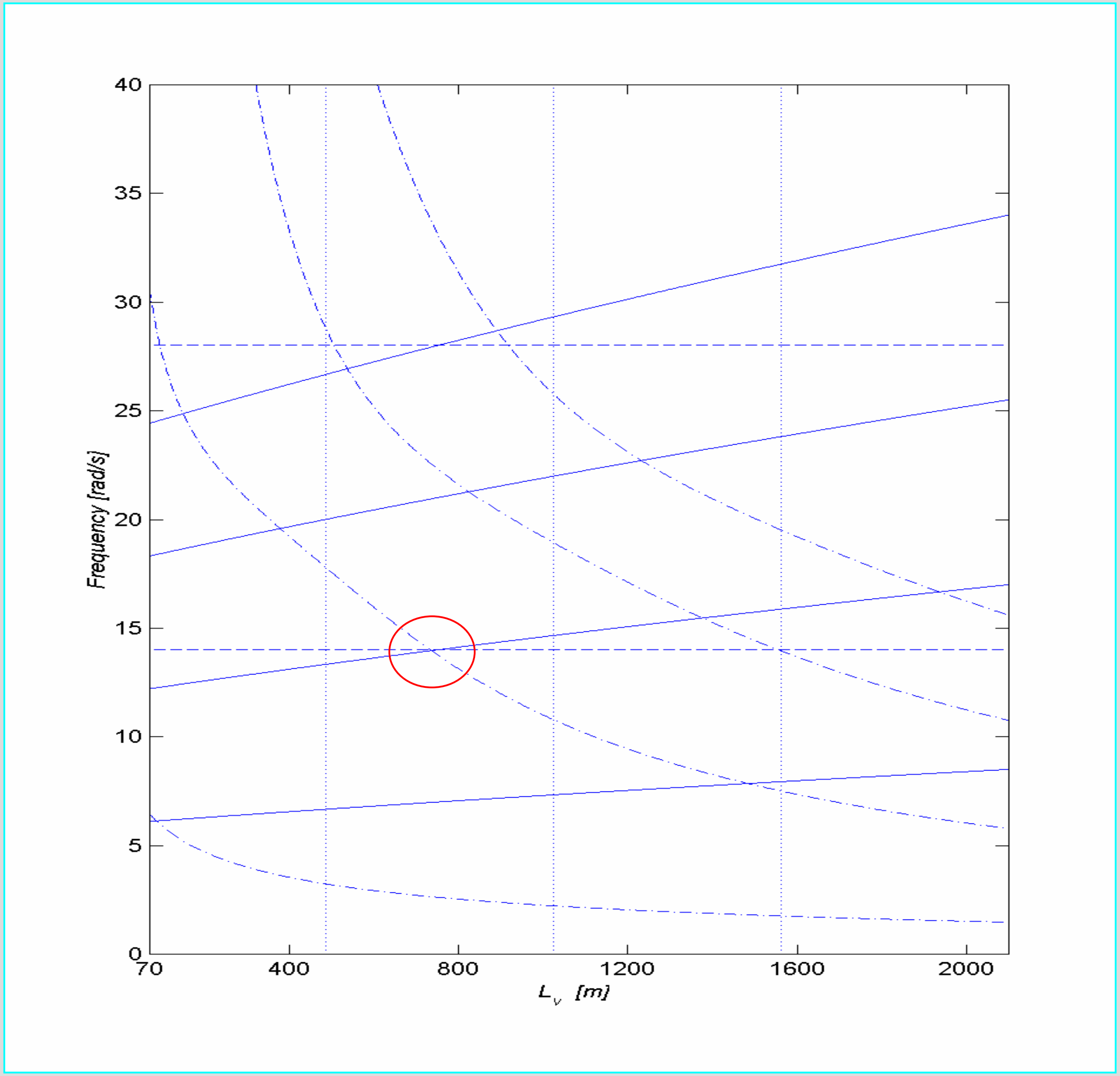
Frequency map: V = 15 m/s

$$\varepsilon = \frac{V}{\omega_{\min} L_{\max}} << 1$$

$$\omega_{\min} = \omega_1 \big|_{L_{\max}} = 1.5\, rad/s$$

$$L_{\max} = 2100\, m$$

$$\varepsilon = 0.005$$



Resonance conditions

- $\left(\bar{\omega}_k\right)_{in-plane} = \left(\bar{\omega}_k\right)_{out-of-plane}$ 1:1 autoparametric resonance
- 750m: $\omega_2 \approx 2\bar{\omega}_1$ 2:1 autoparametric resonance

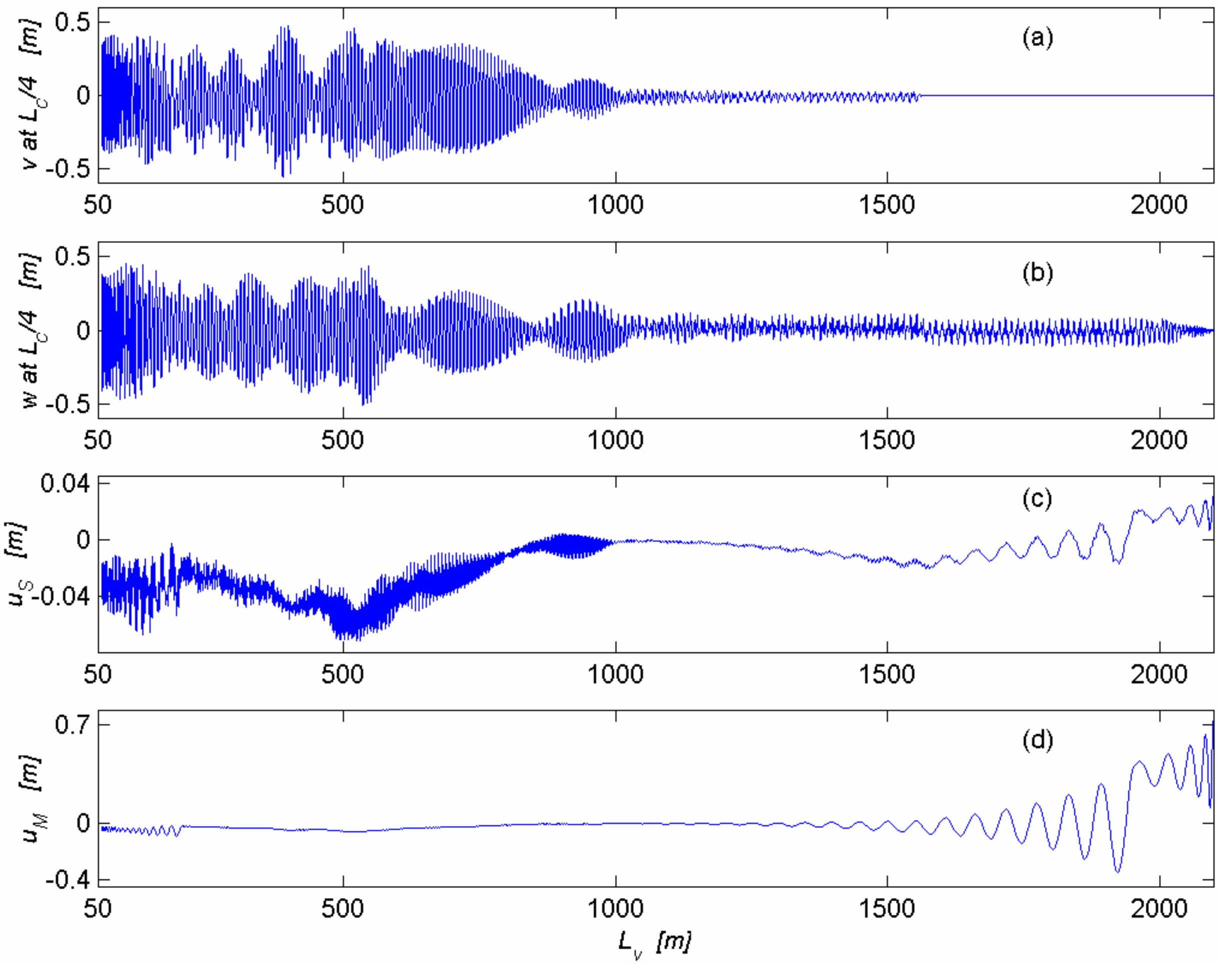
  $\omega_2 \approx \bar{\omega}_2$ 1:1 autoparametric resonance

  $\Omega_1 \approx \omega_2 \approx \bar{\omega}_2$ and $\Omega_2 \approx \bar{\omega}_4$ primary external resonances

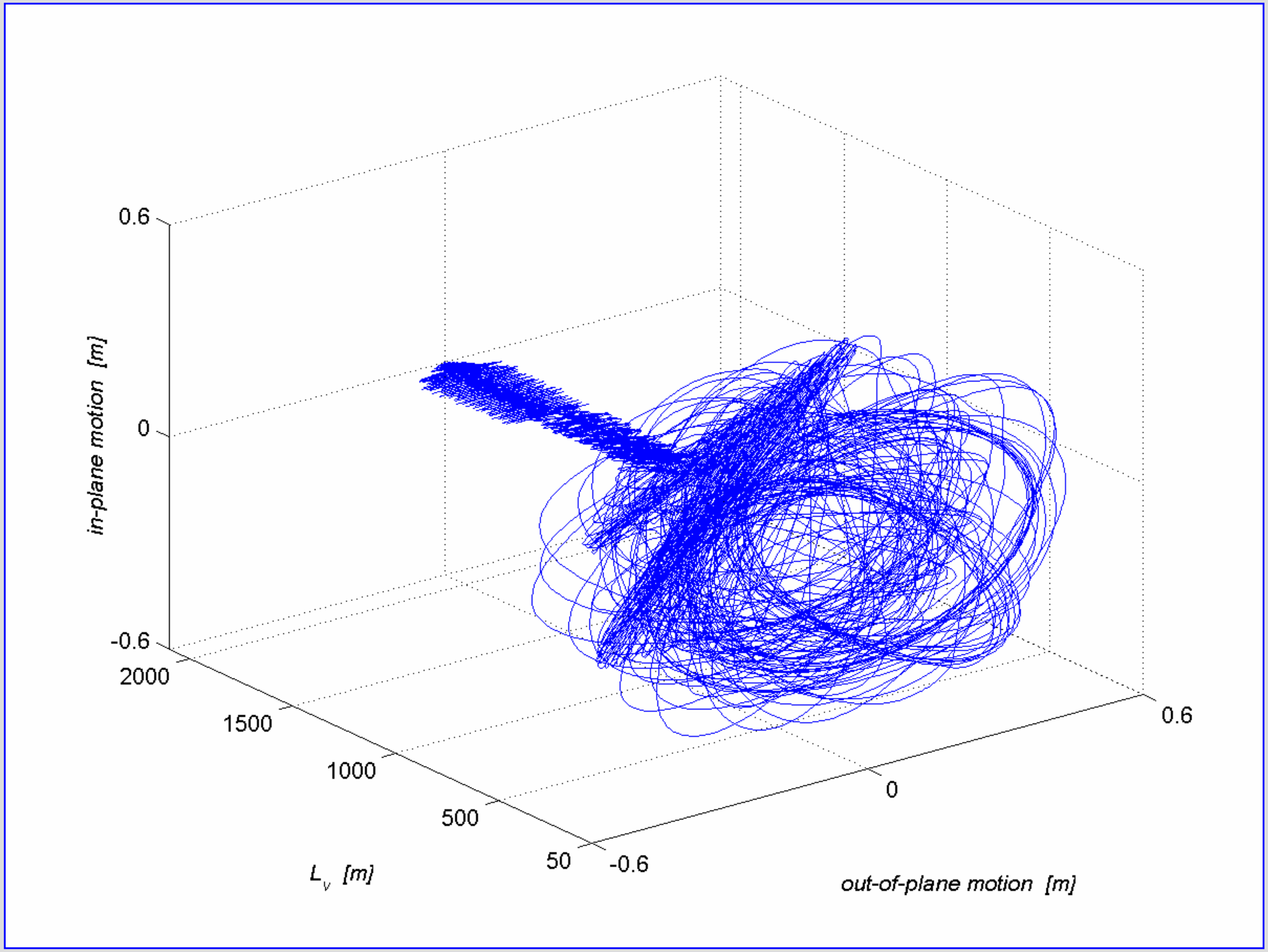
  $\Omega_1 \approx 2\bar{\omega}_1$ and $\Omega_2 \approx 2\bar{\omega}_2$ principal parametric resonances

  $\Omega_2 \approx \omega_2 + \bar{\omega}_2$ summed combination resonance

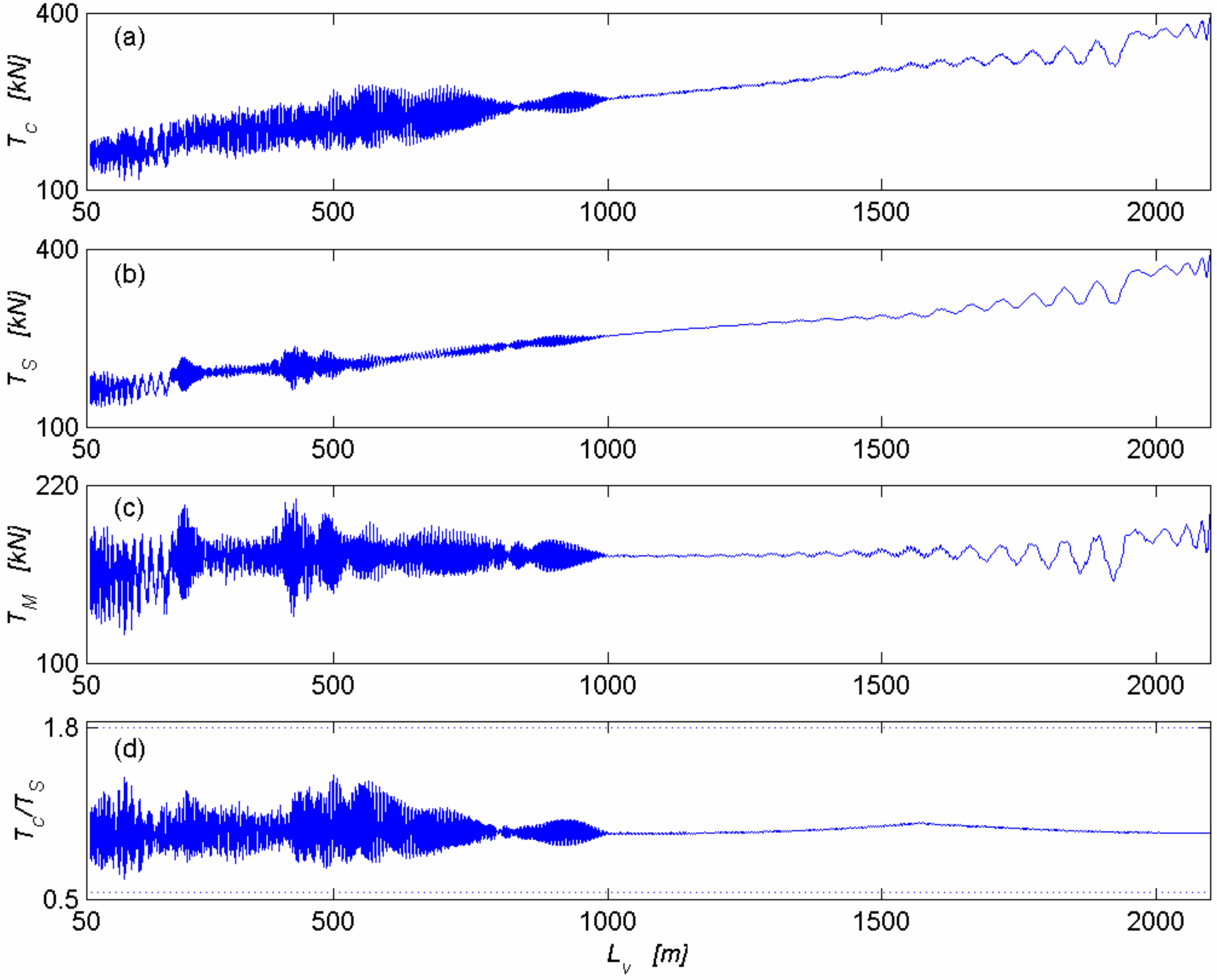
Displacement response: V = 15 m/s



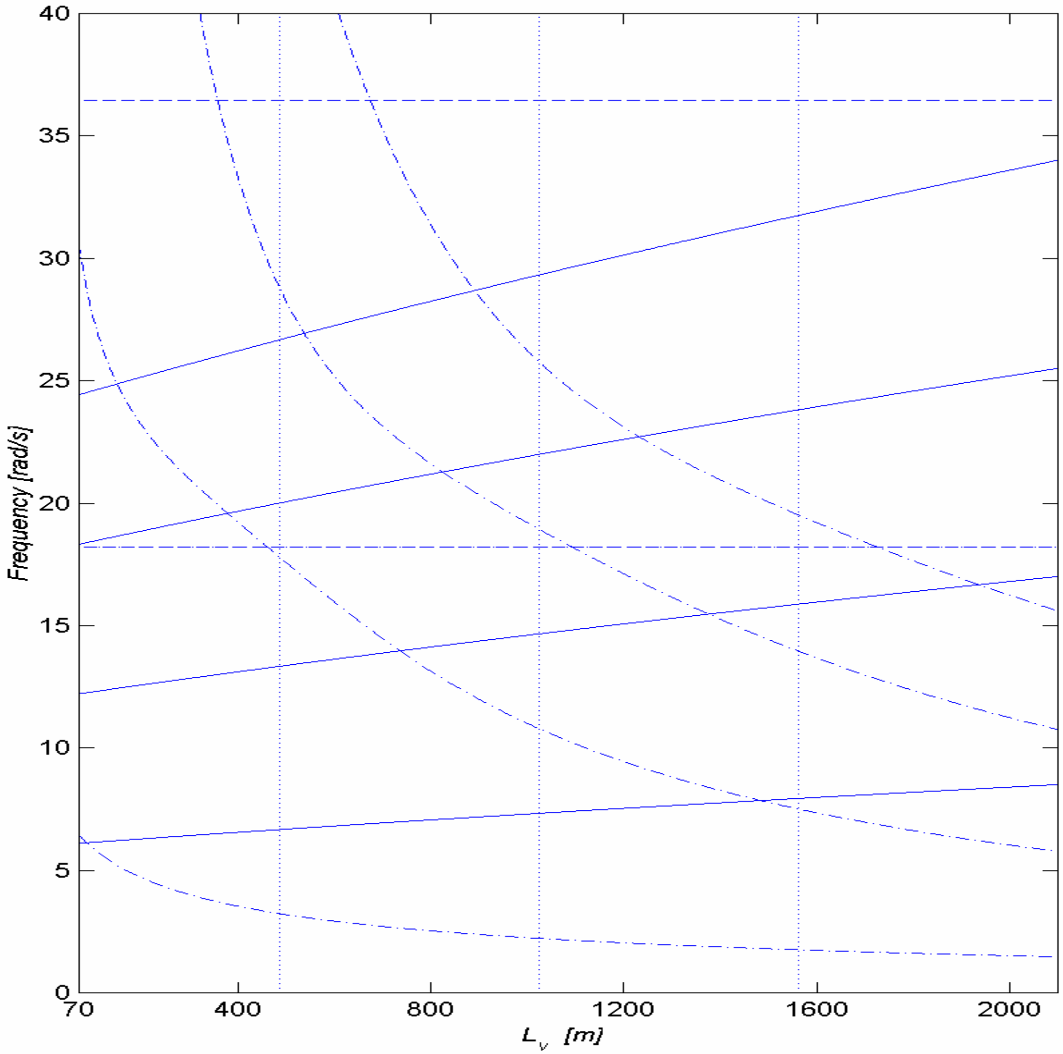
Catenary cable 1st quarter point motion



Rope tensions: V = 15 m/s

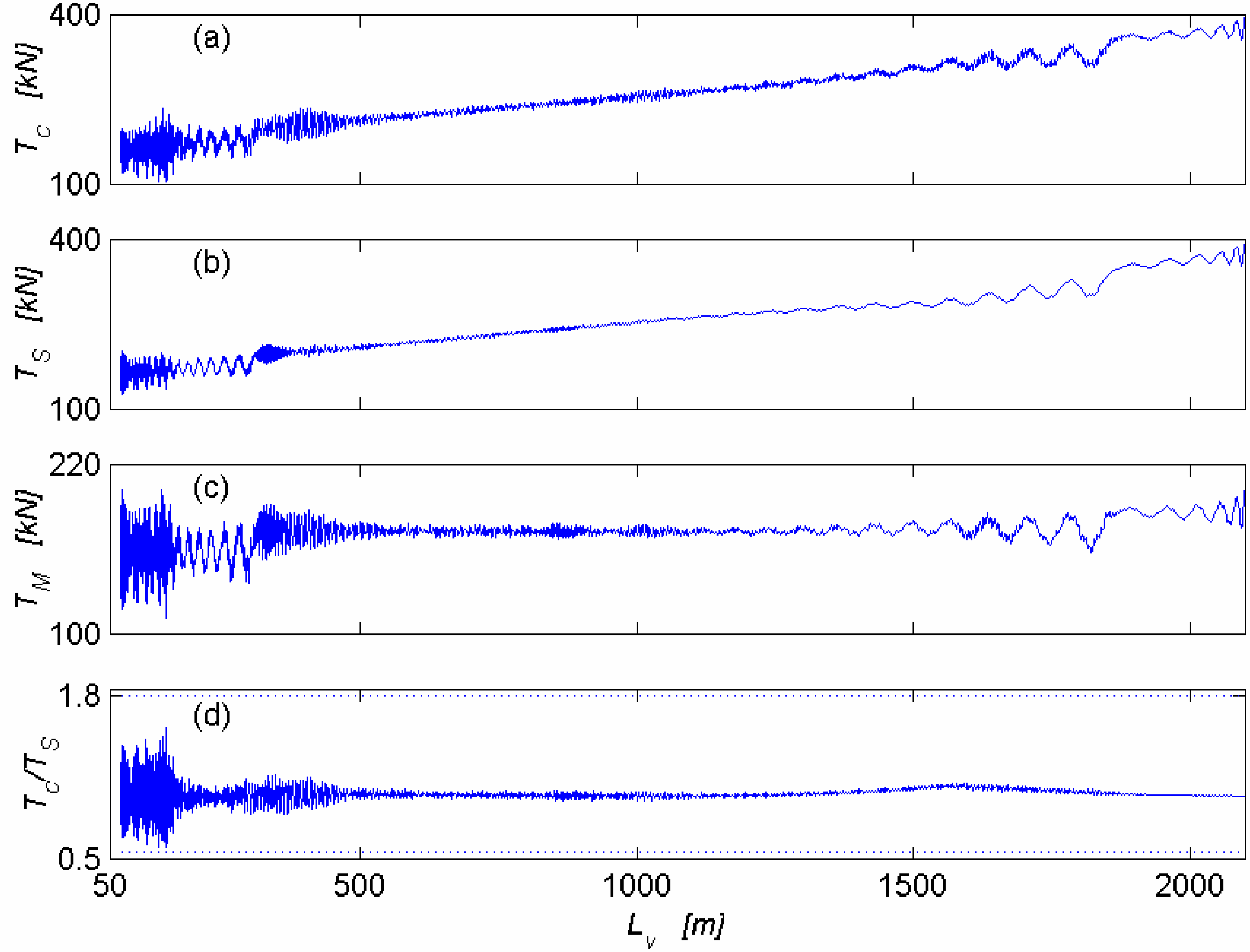


Frequency map: V = 19.5 m/s

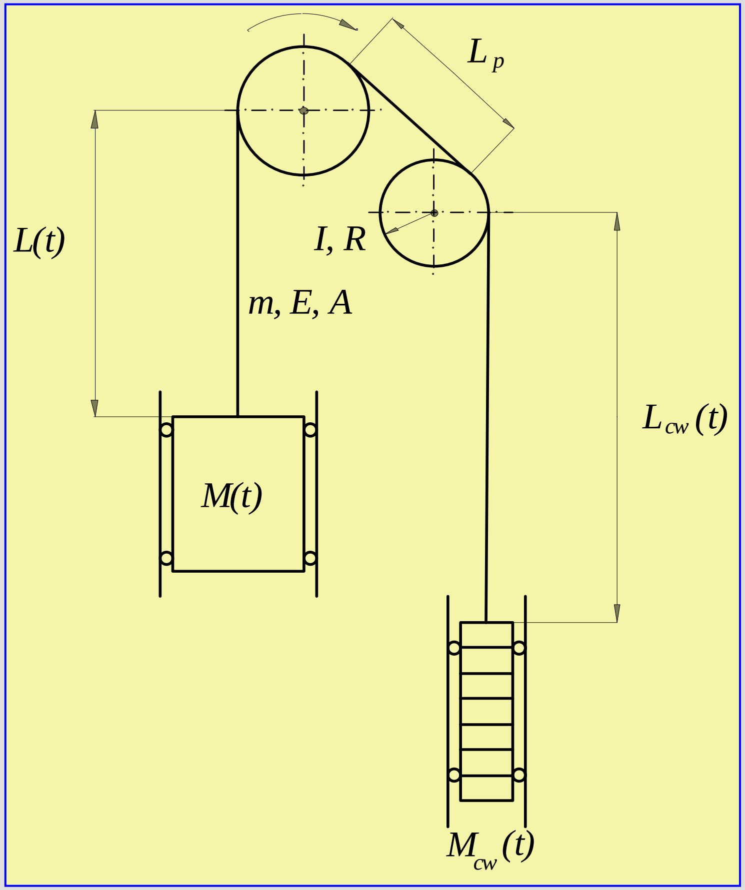


Displacement response: V = 19.5 m/s

Rope tensions: V = 19.5 m/s



Elevator suspension rope model



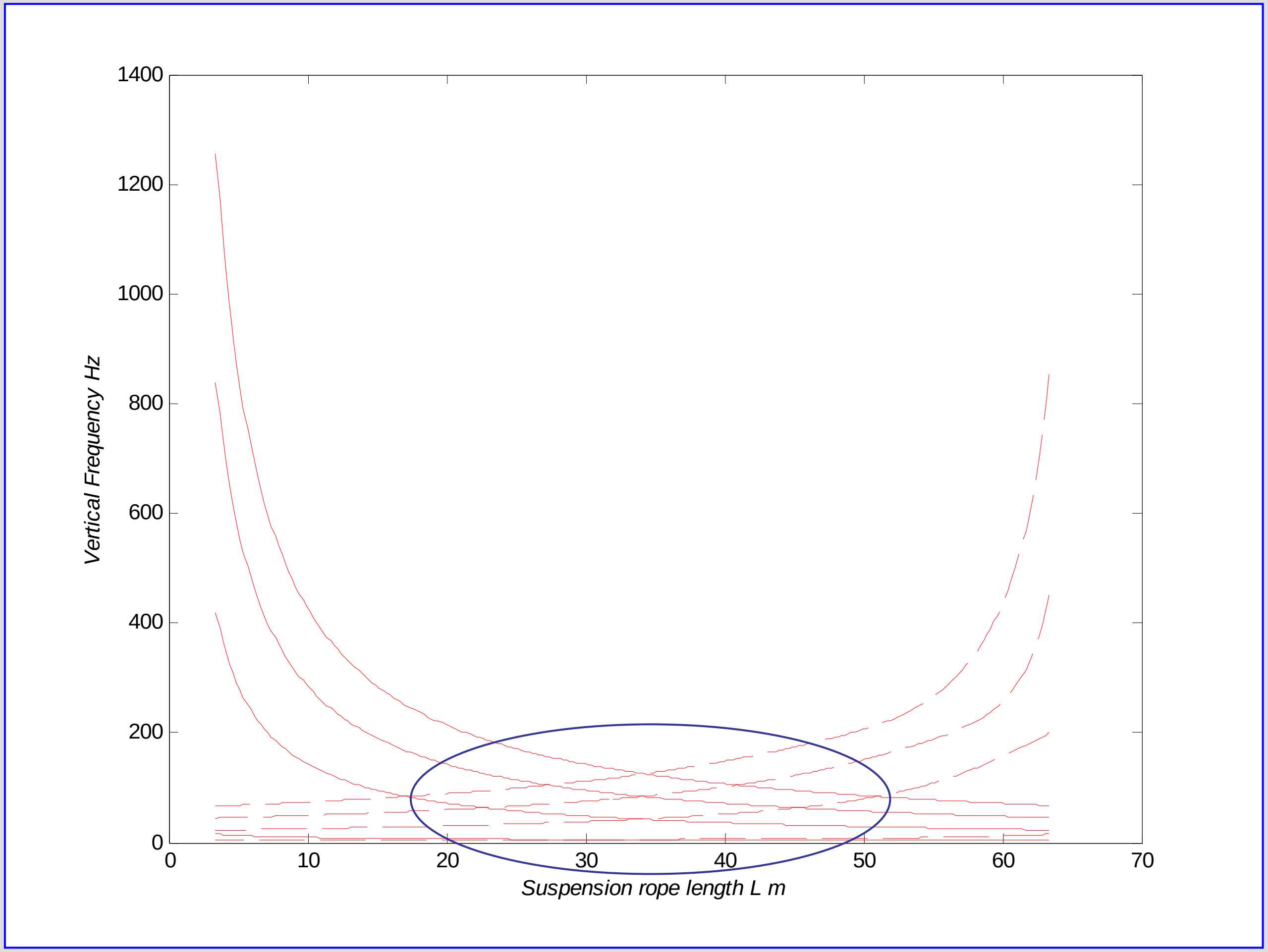
The system parameters

Car mass	P	*2000*	*kg*
Mass of rated load	Q	*1250*	*kg*
Balance	B	*40*	*%*
Equivalent mass of diverter pulley	M_e	*80*	*kg*
Rope length between traction sheave and pulley	L_p	*1.04*	*m*
Travel height	H	*60*	*m*
Well height	W	*70*	*M*
Car height	H	*3.2*	*m*
Hoist rope Young's modulus	E	*60.0*	kN/mm^2
Hoist rope diameter	d	*19*	*mm*
Hoist rope mass per unit length	m	*1.3*	*kg/m*
Compensating rope mass per unit length	m_c	*1.6*	*kg/m*

Longitudinal resonance frequencies

car side

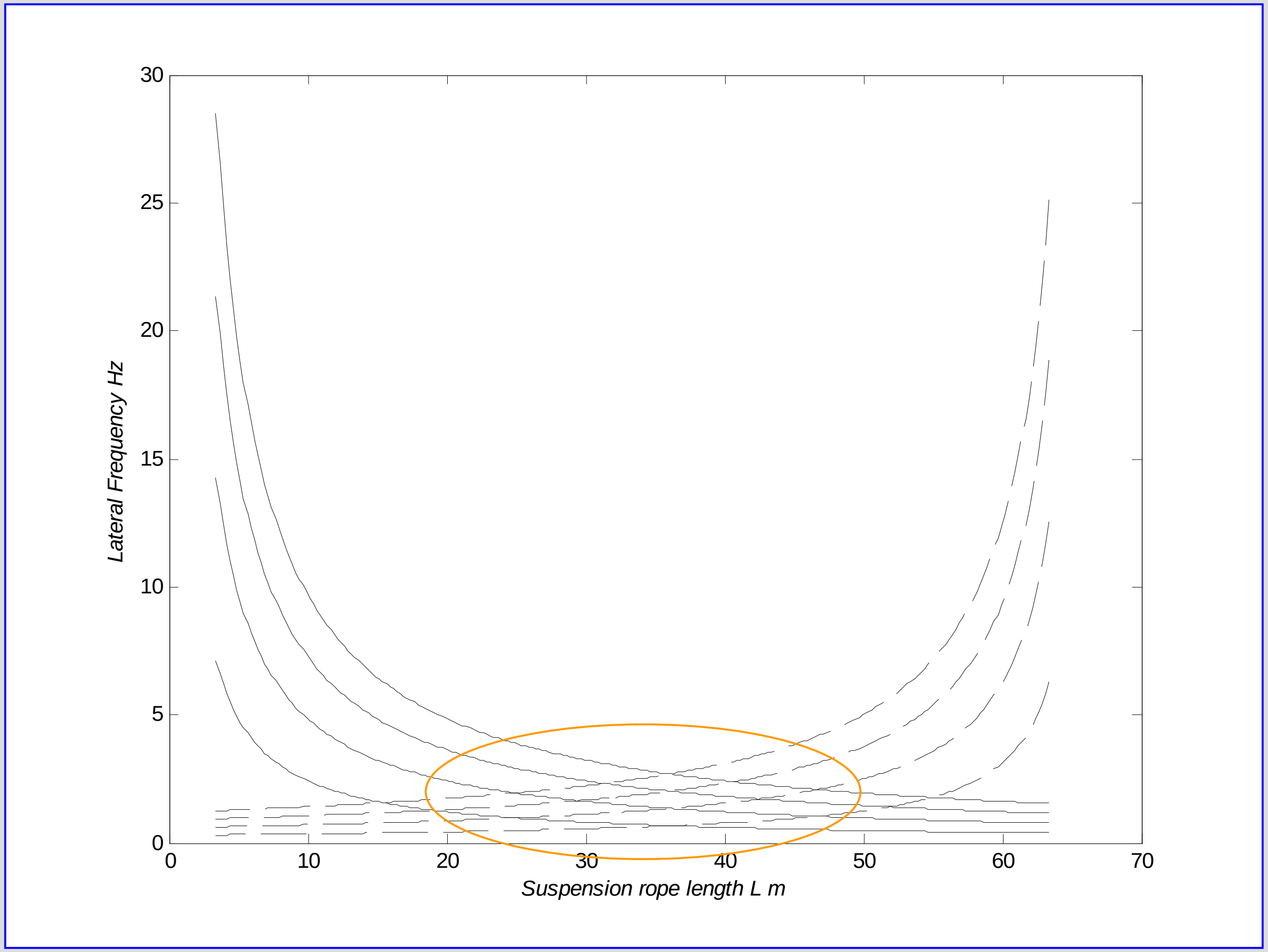
cw side



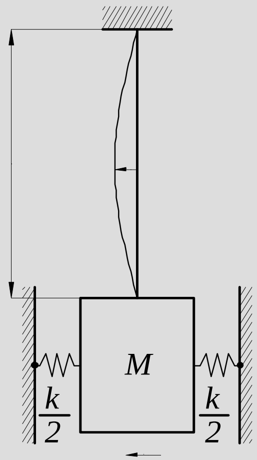
Lateral resonance frequencies (1)

—— car side

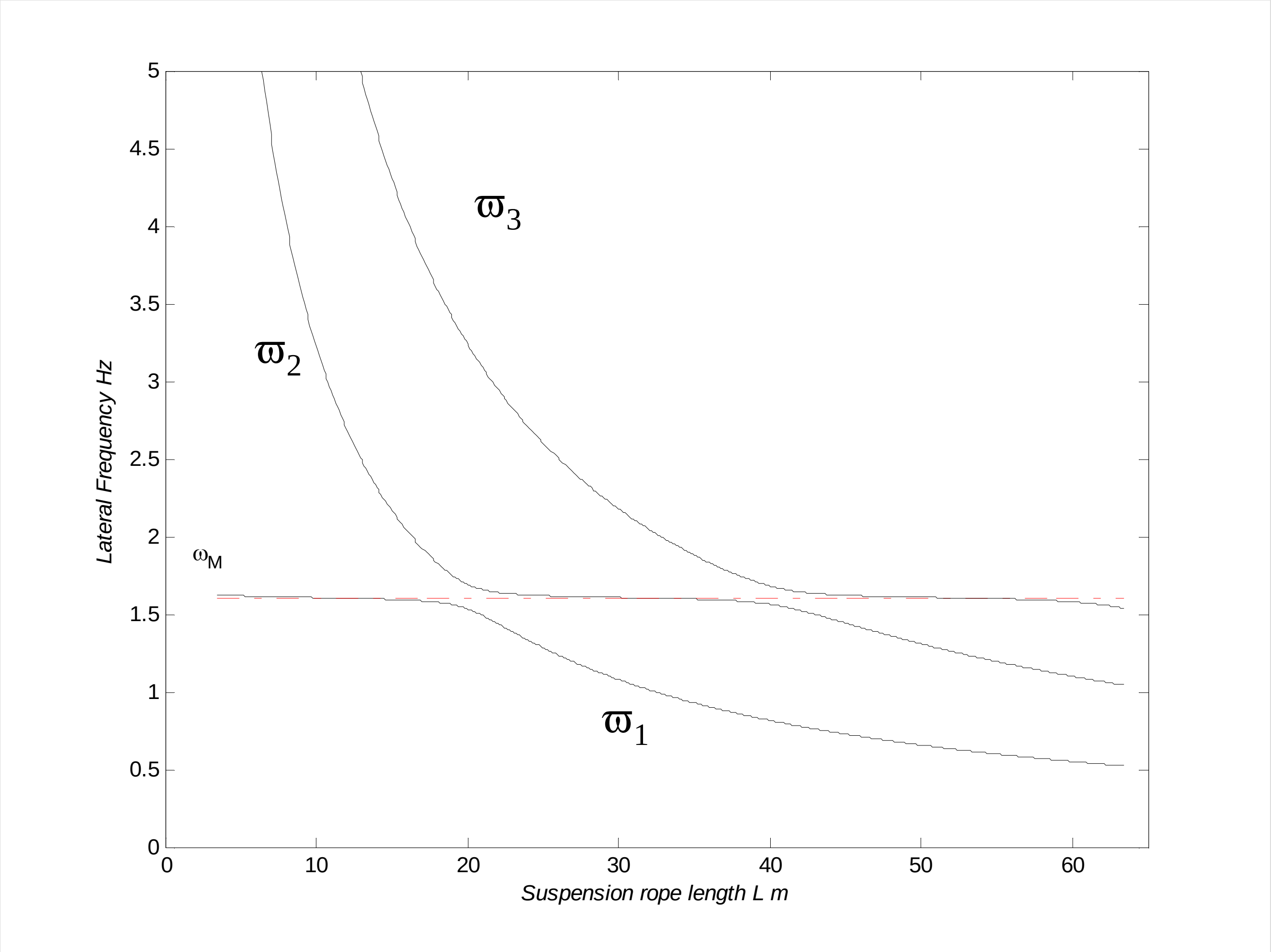
------ cw side



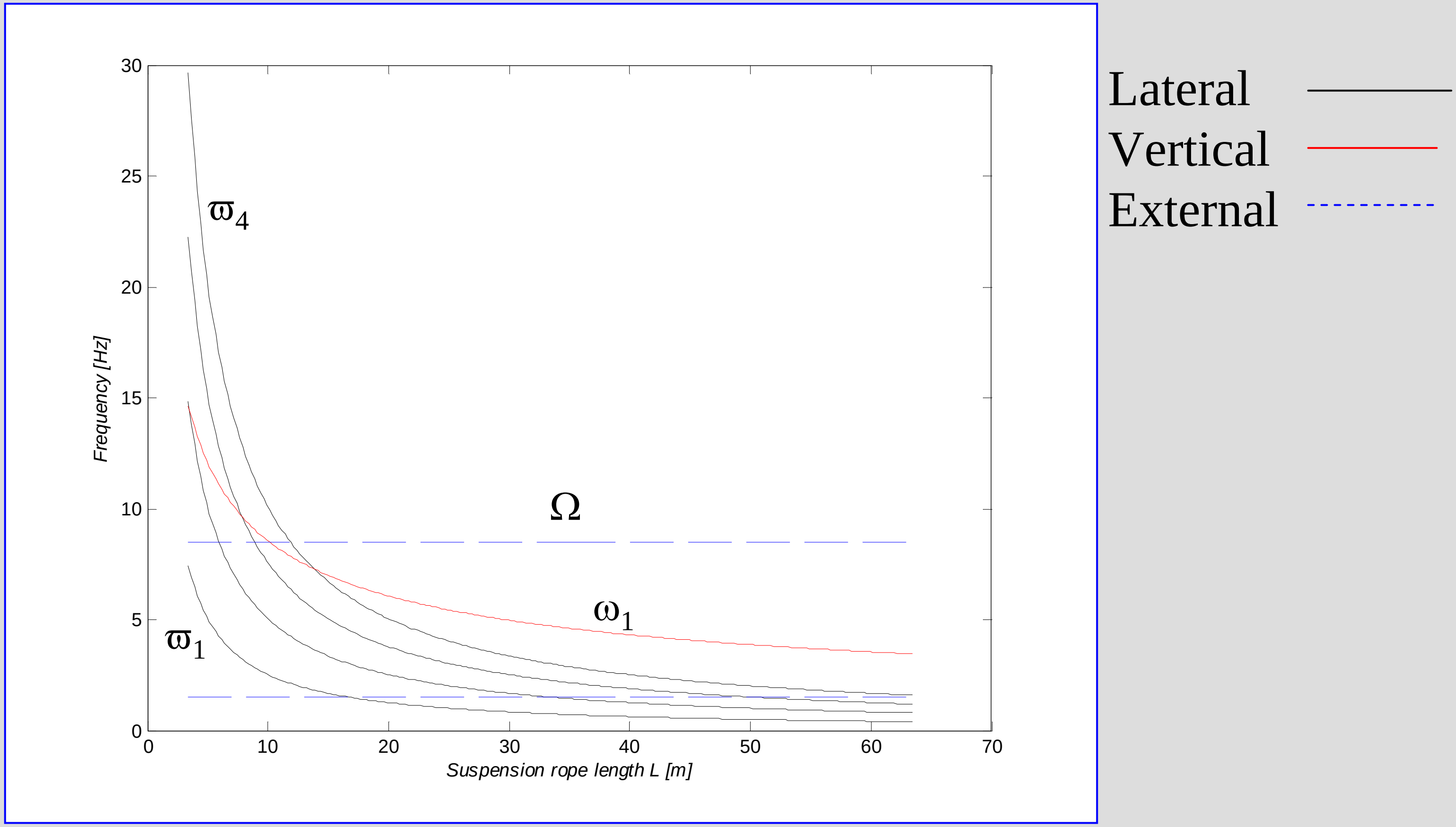
Lateral resonance frequencies (2)



$$\omega_M = \sqrt{\frac{k}{M}}$$

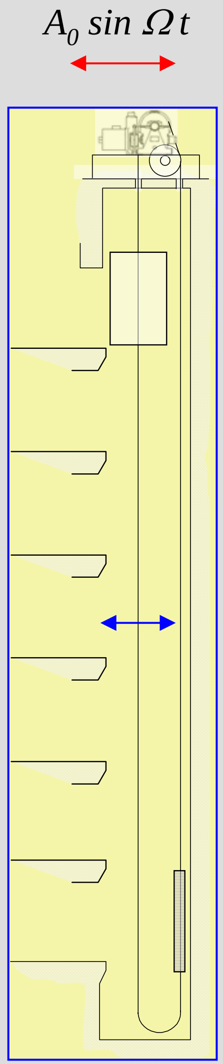


External, the lateral and the 1st longitudinal natural frequencies (car side)

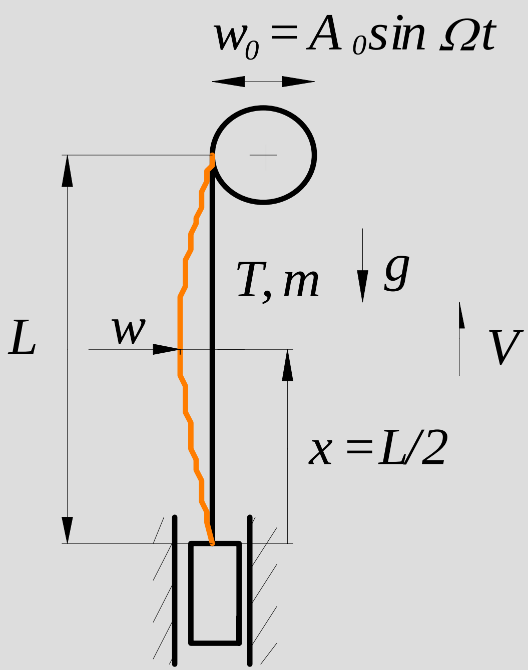


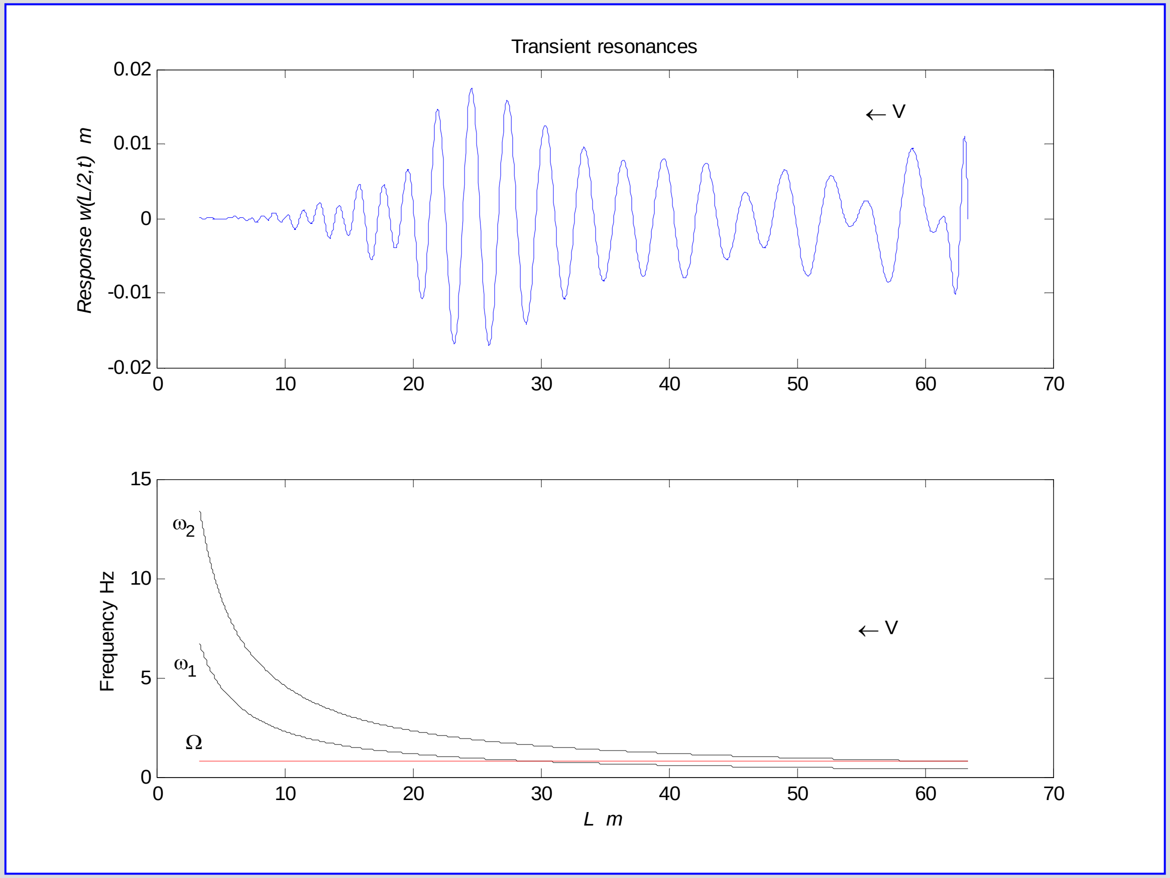
Resonance: building sway effect

- Building often excited near its own natural frequency;
- Periodic sway at low frequencies results;
- Elevator ropes strongly affected;
- Resonance conditions occur during elevator travel;
- *Whirling* motions may result.

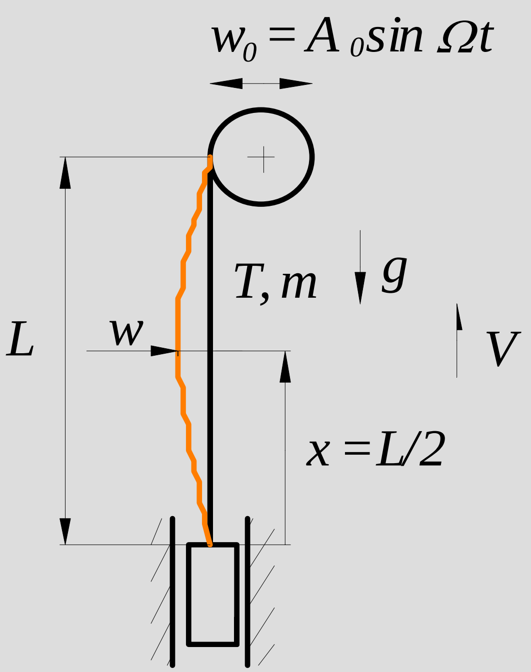


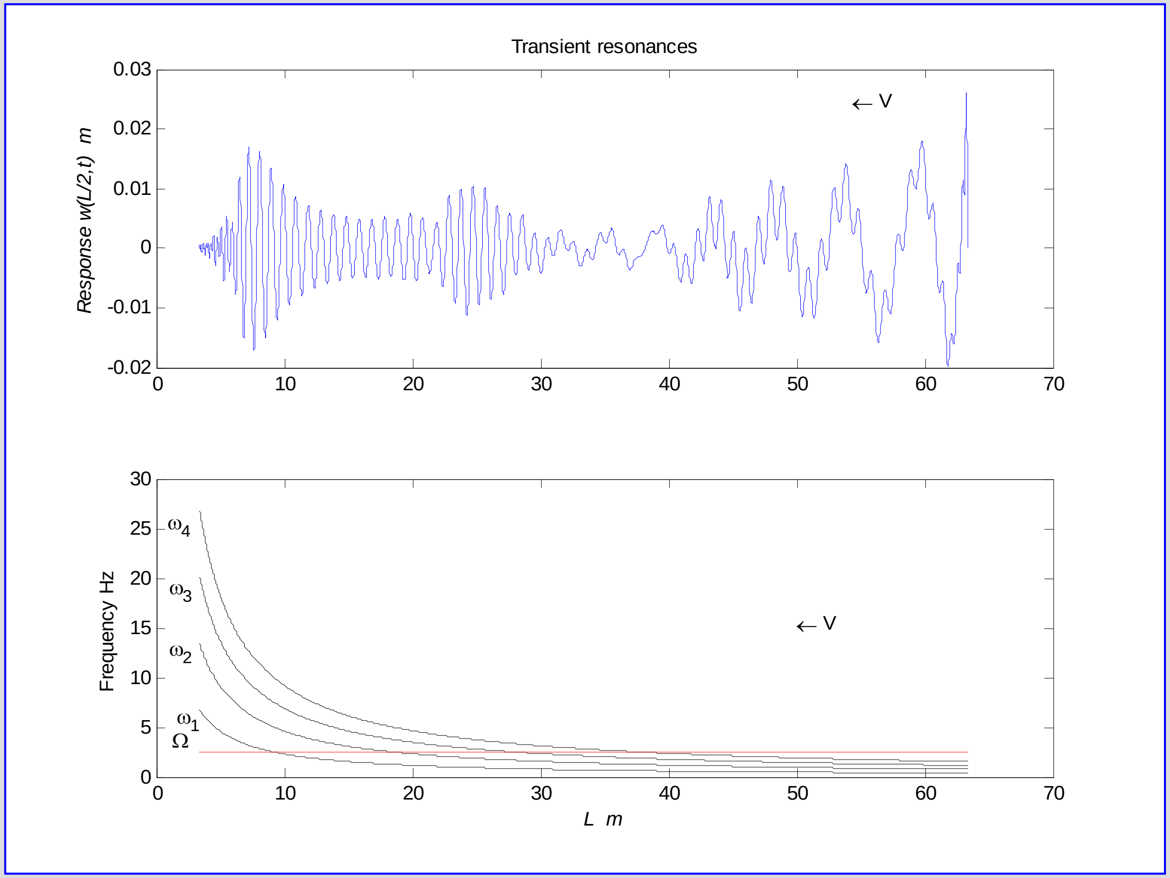
Passage through lateral resonance





Multiple transient resonances

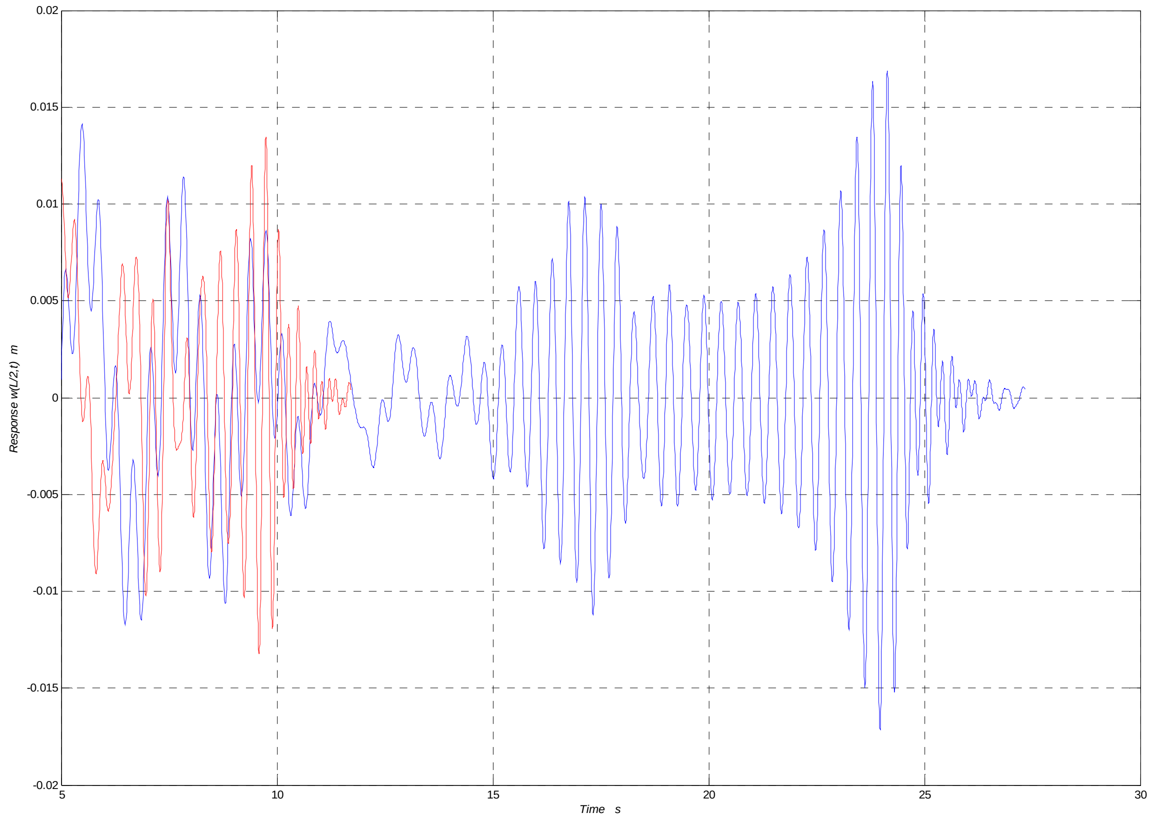




Passage through resonance at various speeds

V = 2.5 m/s

V = 10 m/s



Conclusions

- Despite recent developments in new technologies elastic suspension components in vertical transport systems are susceptible to oscillations;
- The non-linear and non-stationary nature of long moving ropes in transport installations is often responsible for adverse dynamic behaviour of the entire system;
- Large dynamic responses occur due to the transient resonance phenomena;
- The prediction of resonance conditions is of primary importance in the design of vertical transport installations;
- Subsequently, a suitable control strategy can be sought to minimize the resonance effects;
- The resonance effects can be reduced/shifted through the speed and/or acceleration changes.

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