A METHOD FOR GENERATING WHITE NOISE TIME HISTORIES WITH SKEWNESS > 0 AND KURTOSIS > 0

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Introduction

Certain structures may be subjected to random forcing or base excitation functions which have non-Gaussian probability density functions.

As an example, a launch vehicle may be subjected to a high-intensity acoustic pressure field during liftoff, followed by high-amplitude fluctuating pressure during transonic flight. In either case, the pressure time histories may be skewed, or have "high kurtosis" peaks. This phenomenon is discussed in Reference 1. Skewness and kurtosis formulas are given in Appendix A.

The effect of these non-Gaussian environments must be analyzed via a model of the launch vehicle or structure, such as a finite element model. The forcing function can then be applied to the model in the time domain via a model transient analysis. The resulting structural stresses can be analyzed. A rainflow fatigue count can also be performed, as discussed in Reference 2.

The purpose of this paper is to give a method for synthesizing non-Gaussian random time histories for modal transient analysis. The method is rather simple. The first step is to design a probability density function.

Example 1

Synthesize a white noise time history which has skewness = 0 and kurtosis ~ 4.7. The following examples are performed using Matlab script: witch.m.

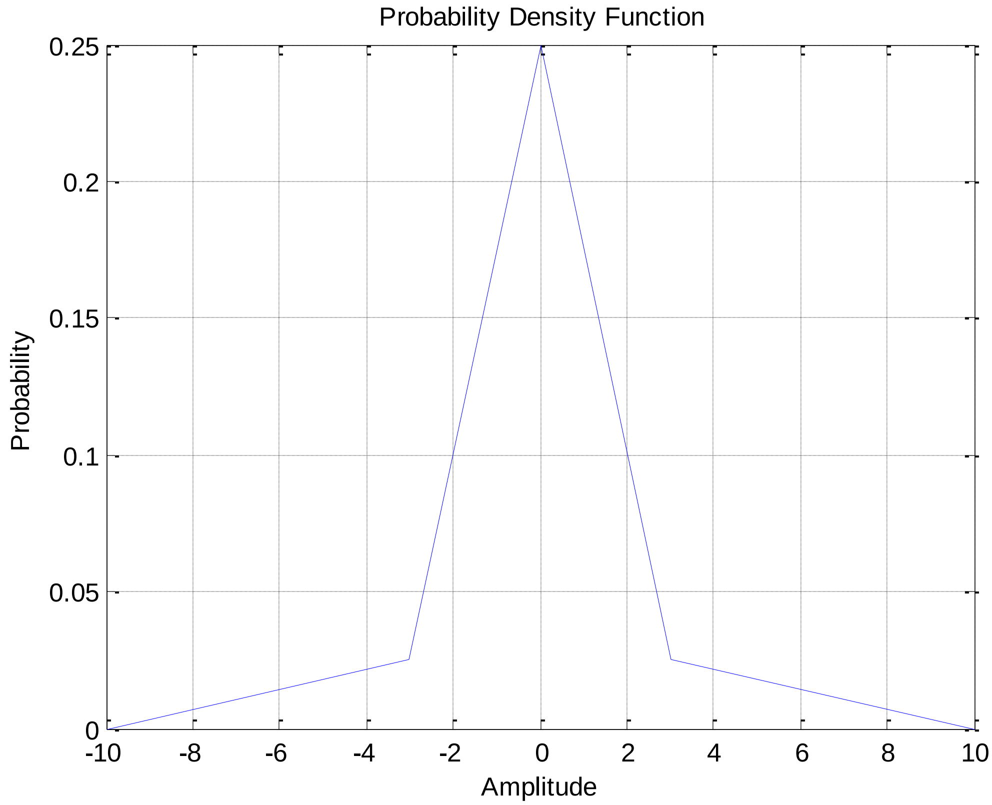

Figure 1.

A probability function with four segments is shown in Figure 1. The shape resembles a witch hat.

The next step is to scale the probability amplitude so that the area under the piecewise continuous curve is equal to one. This has already been done for the curve in Figure 1.

Next, random numbers are generated with a value between zero and one. Each random number corresponds to an area. The corresponding amplitude along the X-axis is then taken as the amplitude for the synthesized time history.

Note that this is the amplitude which is the upper limit for the area calculation. The lower limit is -10.

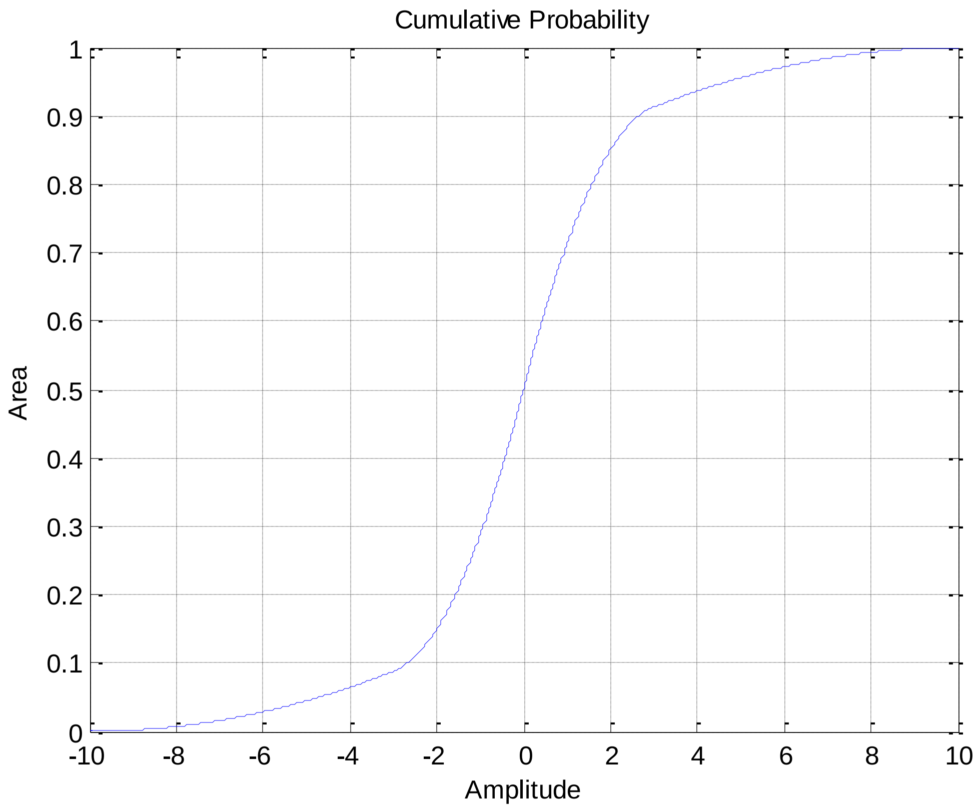

Figure 2.

As an example, assume that the random number is 0.3. Then the corresponding amplitude would be -1.

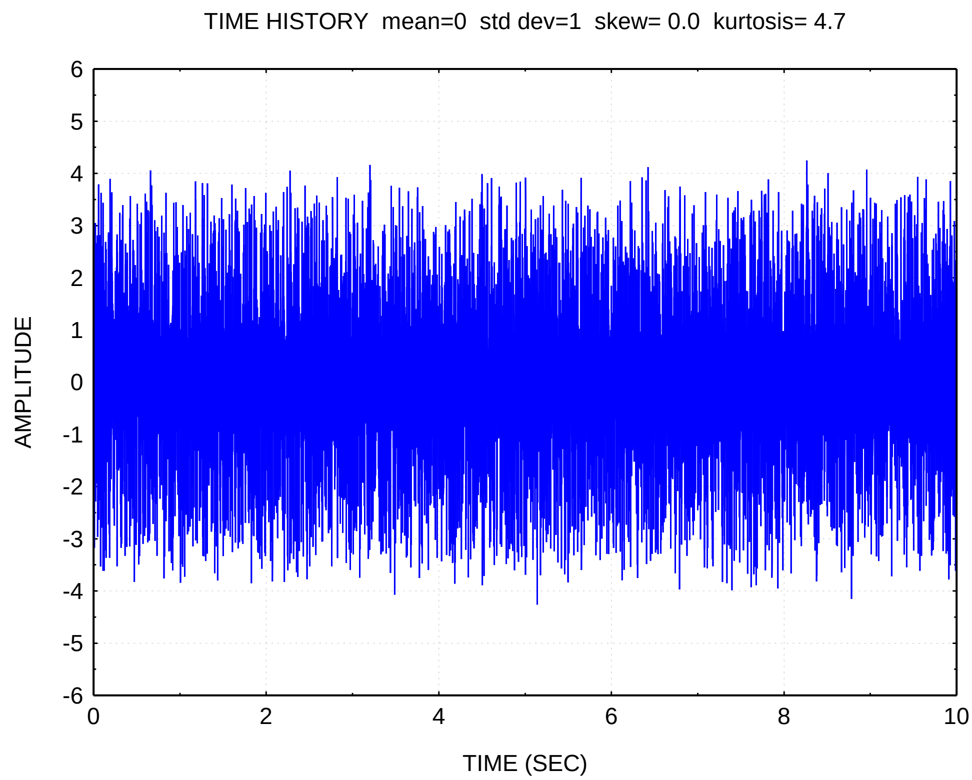

Figure 3a.

The synthesized time history is shown in Figure 3a.

The initial sample rate was 2000 samples per second.

Note that a cubic spline fit was performed on the data, which increased the sample rate to 20,000 samples per second.

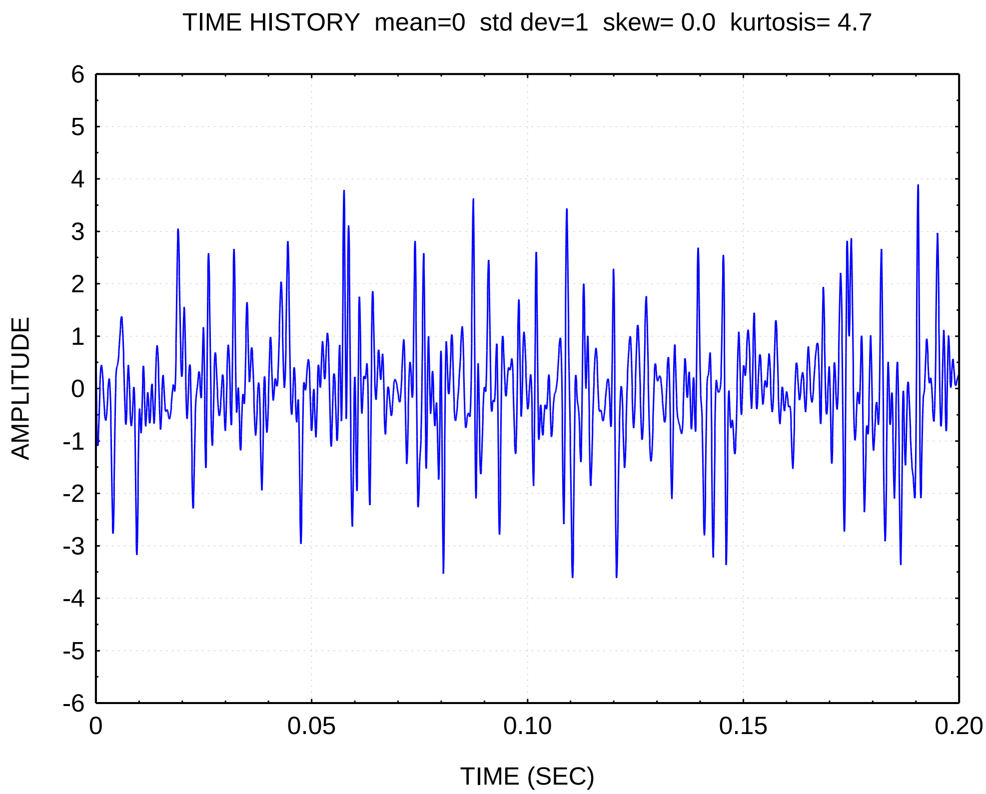

Figure 3b.

The close-up view shows the effect of the “high kurtosis” peaks.

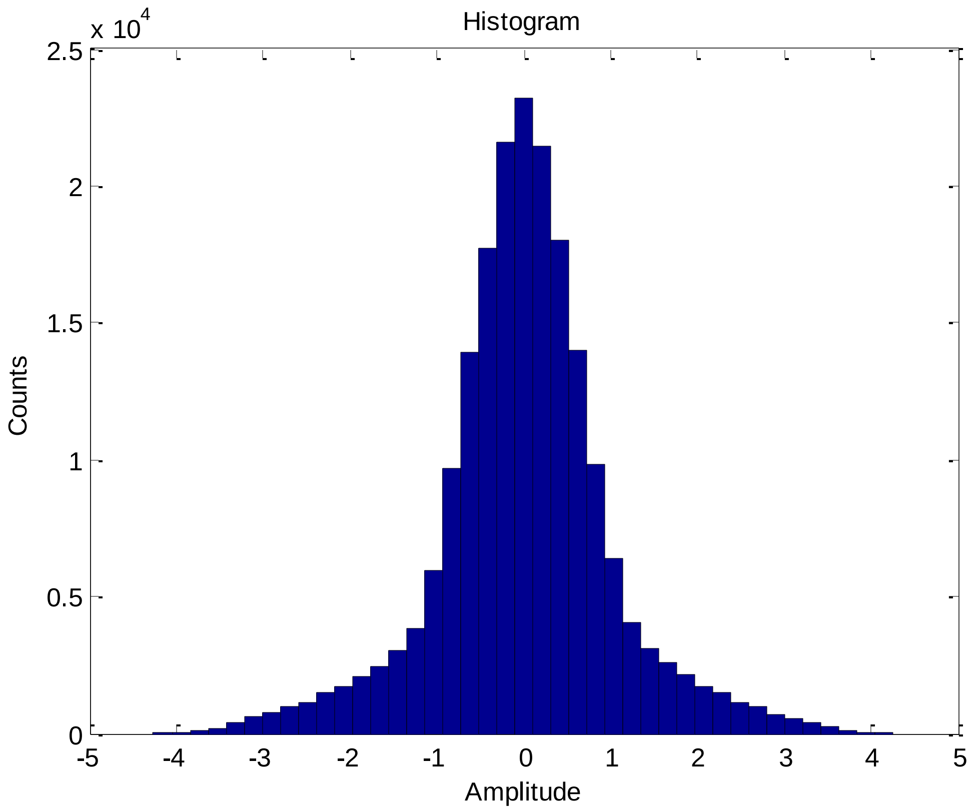

Figure 4.

This is the histogram for the time history in Figure 3. Its shape resembles a witch hat.

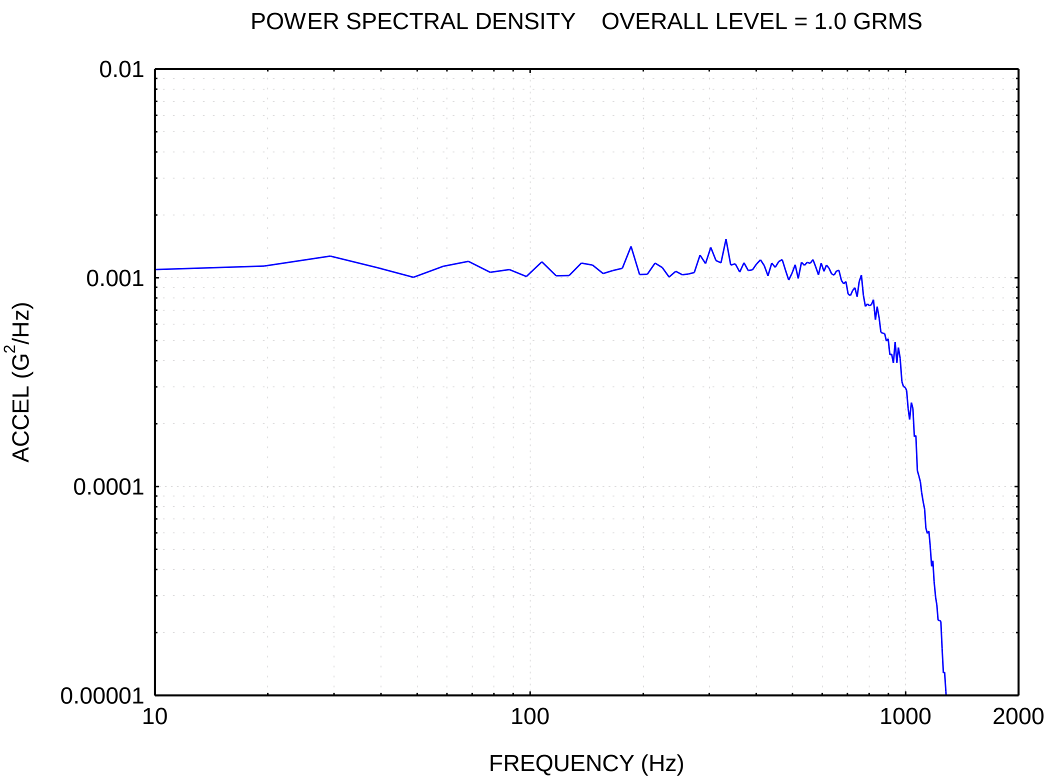

Figure 5.

The PSD in Figure 5 is calculated from the synthesized time history in Figure 3a.

The PSD is flat over the domain extending to about 700 Hz.

The cut-off frequency is 1000 Hz which is the Nyquist frequency of the original sample rate.

Example 2

Synthesize a white noise time history which has skewness = 1.4 and kurtosis ~ 5.2.

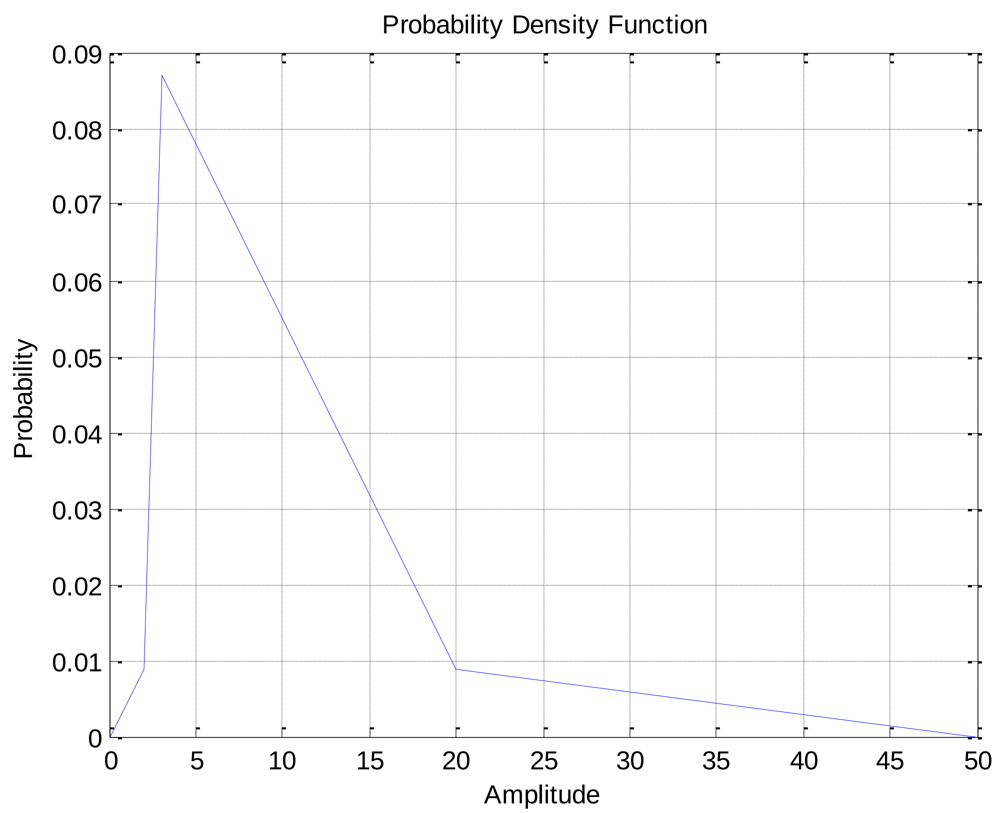

Figure 6.

The probability density function is shown in Figure 6. The area under the curve is equal to one.

The shape is a skewed witch hat.

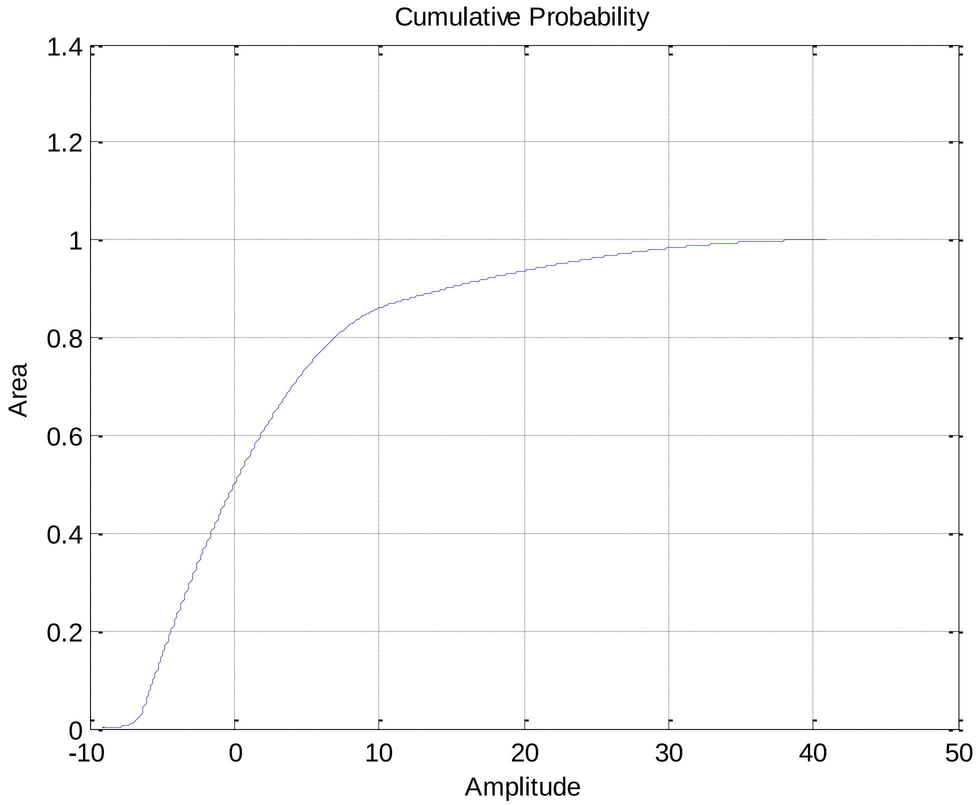
Cumulative Probability
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Area
-10
0
10
20
30
40
50
Amplitude

Figure 7.

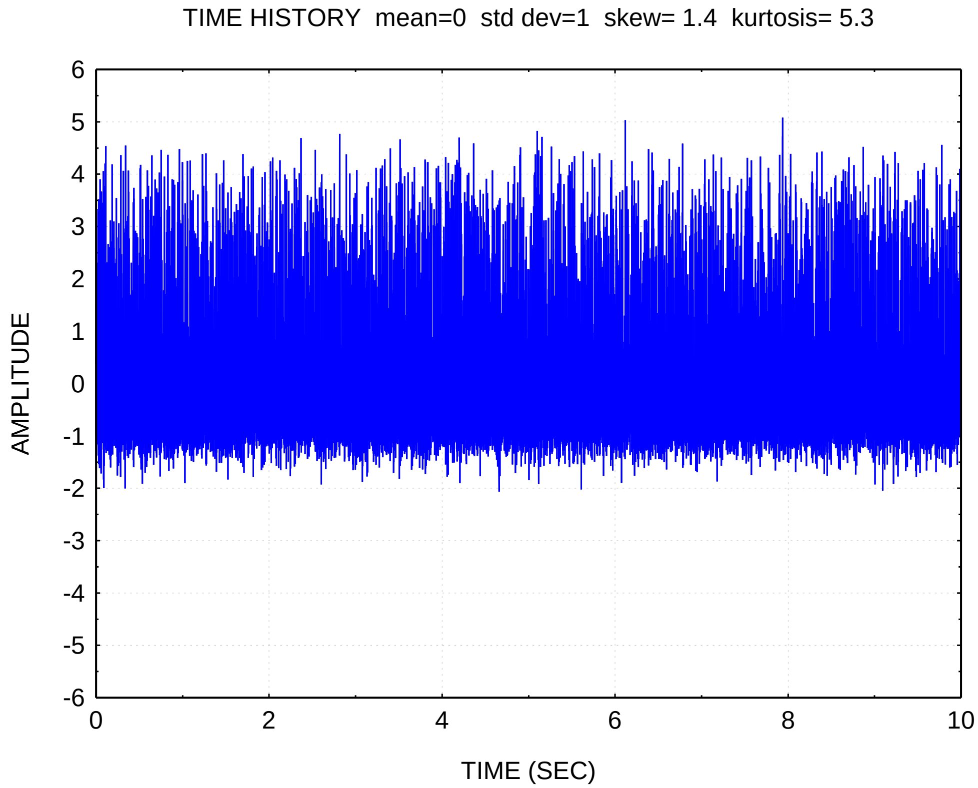

Figure 8a.

The synthesized time history is shown in Figure 8a.

The initial sample rate was 2000 samples per second.

Note that a cubic spline fit was performed on the data, which increased the sample rate to 20,000 samples per second.

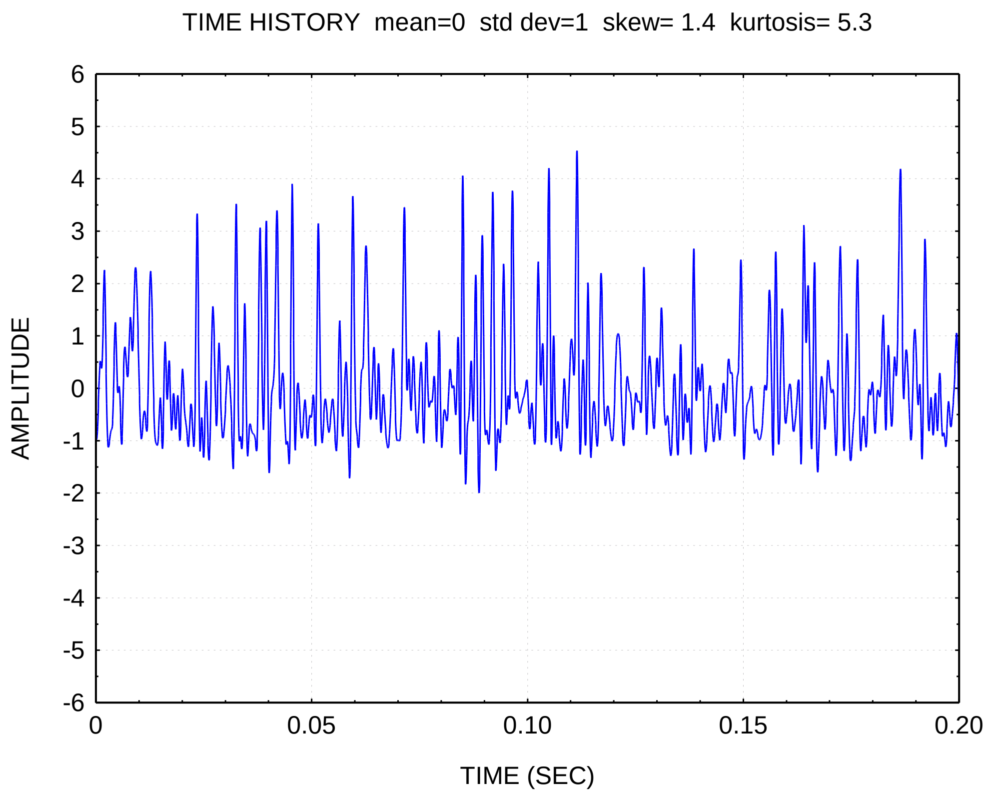
TIME HISTORY mean=0 std dev=1 skew= 1.4 kurtosis= 5.3
AMPLITUDE
TIME (SEC)
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
0
0.05
0.10
0.15
0.20

Figure 8b.

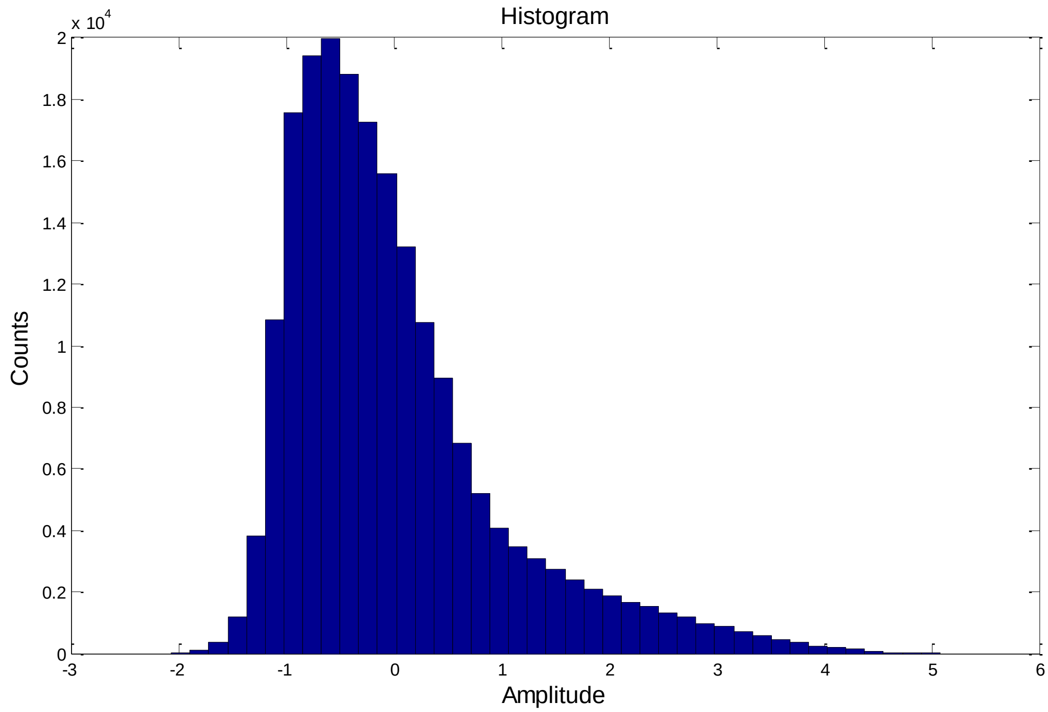

Figure 9.

The histogram in Figure 9 is calculated from the synthesized time history in Figure 8a.

The resulting histogram shape is a skewed witch hat.

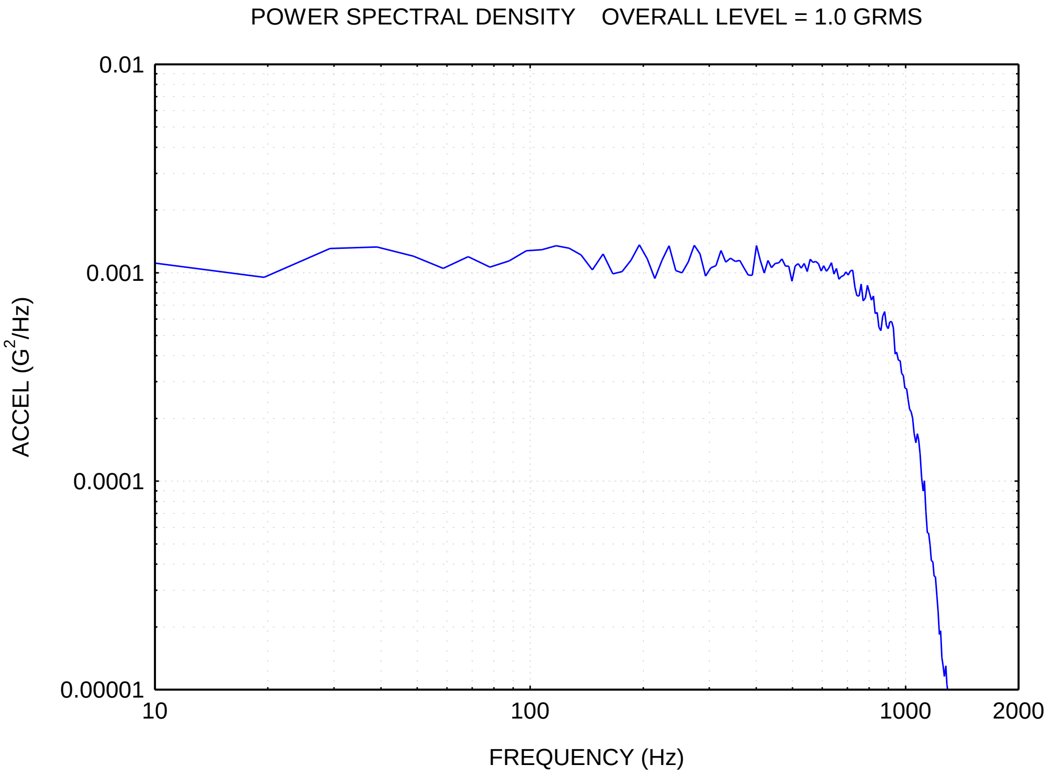

Figure 10.

The PSD in Figure 10 is calculated from the synthesized time history in Figure 8a.

The PSD is flat over the domain extending to about 700 Hz.

The cut-off frequency is 1000 Hz which is the Nyquist frequency of the original sample rate.

References

1. T. Irvine, Non-Gaussian Acoustics Pressure Amplitudes in High-Intensity Sound Fields, Revision A, Vibrationdata, 2010.

2. T. Irvine, Rainflow Cycle Counting in Fatigue Analysis, Revision A, Vibrationdata, 2010.

APPENDIX A

Skewness

Skewness is a measure of the asymmetry of the probability density function.

The skewness for a time series Y_i is

$$\text{Skewness} = \frac{\sum_{i=1}^{n} [Y_i - \mu]^3}{n\sigma^3} \qquad \text{(A-1)}$$

where

μ	=	mean
σ	=	standard deviation
n	=	number of samples

The skewness is zero if the probability density is perfectly symmetric.

Kurtosis

Kurtosis is a parameter that describes the shape of a random variable's histogram or its equivalent probability density function (PDF).

The kurtosis for a time series Y_i is

$$\text{Kurtosis} = \frac{\sum_{i=1}^{n} [Y_i - \mu]^4}{n\sigma^4} \qquad \text{(A-2)}$$

The term in the numerator is the "fourth moment about the mean."

A pure sine time history has a kurtosis of 1.5.

A time history with a normal distribution has a kurtosis of 3.

Some alternate definitions of kurtosis subtract a value of 3 so that a normal distribution will have a kurtosis of zero.

A kurtosis larger than 3 indicates that the distribution is more peaked and has heavier tails than a normal distribution with the same standard deviation.