Aircraft Stress, Equations

Fuselage Pressurization, Thin-Wall Cylinder

Cabin pressure differential Δp, radius $R = D/2$, skin thickness t, valid for $R/t > 10$

$$\sigma_h = \frac{\Delta p\, R}{t}, \qquad \sigma_L = \frac{\Delta p\, R}{2\, t}$$

von Mises equivalent stress, biaxial plane stress

$$\sigma_{vm} = \sqrt{\sigma_h^2 - \sigma_h \sigma_L + \sigma_L^2} = \frac{\sqrt{3}}{2}\, \sigma_h$$

Margins of safety are based on the maximum principal stress, $\sigma = \sigma_h$.

Wing Root Bending

Gross weight W, limit load factor n, semi-span s (root to tip). Lift per wing:

$$L = \frac{n\, W}{2}$$

Uniform lift distribution, $w = L/s$, spanwise station $\eta = y/s$

$$M(\eta) = \frac{L\, s}{2}\, (1 - \eta)^2, \qquad M_{root} = \frac{L\, s}{2}$$

Elliptical lift distribution, $w(\eta) = w_0 \sqrt{1 - \eta^2}$, $w_0 = 4L/(\pi s)$

$$M(\eta) = w_0 s^2 \left[\frac{(1 - \eta^2)^{3/2}}{3} - \eta \left(\frac{\pi}{4} - \frac{\eta \sqrt{1 - \eta^2} + \sin^{-1} \eta}{2} \right) \right]$$

$$M_{root} = \frac{4\, L\, s}{3\, \pi}, \qquad V_{root} = L$$

Root bending stress, area moment of inertia I, extreme fiber distance c

$$\sigma = \frac{M_{root}\, c}{I}$$

Wing weight (inertia relief), engine weight and tail load are neglected (conservative for the wing).

Margins of Safety

Stresses above are at limit load. Yield factor FS_y (typically 1.0), ultimate factor FS_u (typically 1.5)

$$MS_{yield} = \frac{F_{ty}}{FS_y\,\sigma} - 1, \qquad MS_{ult} = \frac{F_{tu}}{FS_u\,\sigma} - 1$$

$MS \geq 0$ is acceptable. Ultimate factor 1.5 per 14 CFR 25.303.

Units

English: in, lbf, psi; semi-span entered in ft and converted, $s = 12\,s_{ft}$ (in).

English: F_{ty}, F_{tu} in ksi, converted to psi.

Metric: gross mass m in kg, $W = m\,g$, $g = 9.80665$ m/s^2.

Metric: t and c in mm, I in cm^4 (10^{-8} m^4), Δp in kPa, stresses in MPa.

Material Values

Typical room-temperature sheet values used as presets, editable in the GUI:

Al 2024-T3: $F_{ty} = 47$, $F_{tu} = 64$, $F_{su} = 39$, $F_{bru} = 129$ ksi, $E_c = 10.7$ msi, $\nu = 0.33$

Al 7075-T6: $F_{ty} = 69$, $F_{tu} = 78$, $F_{su} = 47$, $F_{bru} = 148$ ksi, $E_c = 10.5$ msi, $\nu = 0.33$

Ti-6Al-4V annealed: $F_{ty} = 126$, $F_{tu} = 134$, $F_{su} = 79$, $F_{bru} = 252$ ksi, $E_c = 16.4$ msi, $\nu = 0.31$

F_{bru} is the ultimate bearing strength at edge distance $e/D = 2$.

Use design allowables from MMPDS for the actual product form, thickness and basis.

Metric: strengths in MPa, E_c in GPa (1 ksi = 6.894757 MPa, 1 msi = 6.894757 GPa).

Margin Summary and Report

The Index tab lists the lowest margin of each calculated analysis (load factor for the V-n diagram, tip deflection for wing deflection). PDF Report: a summary page, then one page per calculated analysis with its inputs, results and plot.

Skin Panel Buckling

Flat rectangular panel: width b (stringer pitch, across the load), length a (rib pitch, along the load), thickness t, compression modulus E_c, Poisson ratio ν. Elastic buckling stresses:

$$\sigma_{cr,e} = k_c \frac{\pi^2 E_c}{12\,(1-\nu^2)} \left(\frac{t}{b}\right)^2, \qquad \tau_{cr,e} = k_s \frac{\pi^2 E_c}{12\,(1-\nu^2)} \left(\frac{t}{d}\right)^2, \quad d = \min(a,b)$$

All edges simply supported, m = number of half-waves along a, $r = \min(a,b)/\max(a,b)$:

$$k_c = \min_m \left(\frac{m\,b}{a} + \frac{a}{m\,b}\right)^2, \qquad k_s = 5.35 + 4\,r^2$$

All edges clamped (long-plate value of k_c, conservative for $a/b < 3$):

$$k_c = 6.98, \qquad k_s = 8.98 + 5.6\,r^2$$

Plasticity, Johnson parabola with $F_{cy} \approx F_{ty}$ and $F_{sy} \approx F_{ty}/\sqrt{3}$ (approximate):

$$\sigma_{cr} = \sigma_{cr,e} \;\text{ if }\; \sigma_{cr,e} \le \frac{F_{cy}}{2}, \qquad \sigma_{cr} = F_{cy}\left(1 - \frac{F_{cy}}{4\,\sigma_{cr,e}}\right) \;\text{ otherwise}$$

Same form for τ_{cr} with F_{sy}.

Stress ratios at ultimate load, applied limit stresses σ (compression) and τ (shear):

$$R_c = \frac{FS_u\,\sigma}{\sigma_{cr}}, \qquad R_s = \frac{FS_u\,\tau}{\tau_{cr}}$$

Combined compression and shear, interaction $R_c + R_s^2 = 1$:

$$MS = \frac{2}{R_c + \sqrt{R_c^2 + 4\,R_s^2}} - 1, \qquad MS_c = \frac{1}{R_c} - 1, \quad MS_s = \frac{1}{R_s} - 1$$

Reference: Bruhn, Analysis and Design of Flight Vehicle Structures, Section C5.

Gust Load Factor and V-n Diagram

Wing loading W/S, mean geometric chord $\bar{c}$, lift curve slope a (per rad), density ρ at altitude (ISA)

$$\mu_g = \frac{2\,(W/S)}{\rho\,\bar{c}\,a\,g}, \qquad K_g = \frac{0.88\,\mu_g}{5.3 + \mu_g}$$

Gust load factor, derived gust velocity U_{de}, equivalent airspeed V_e, sea-level density $\rho_0 = 1.225$ kg/m^3

$$n = 1 \pm \frac{K_g\,\rho_0\,U_{de}\,V_e\,a}{2\,(W/S)}$$

English form, V in KEAS, U_{de} in ft/s, W/S in psf: $n = 1 \pm K_g\,U_{de}\,V\,a\,/\,(498\,W/S)$

Typical $U_{de} = 50$ ft/s at V_C and 25 ft/s at V_D (Pratt formula, 14 CFR 23.341 before Amdt 23-64).

Transport category airplanes now use discrete tuned gusts (14 CFR 25.341).

Maneuver envelope: stall lines, 1-g stall speed and maneuver speed

$$n = \frac{\rho_0\,V_e^2\,S\,C_{L,max}}{2\,W}, \qquad V_{S1} = \sqrt{\frac{2\,W/S}{\rho_0\,C_{L,max}}}, \qquad V_A = V_{S1}\sqrt{n_{max}}$$

Negative side: $n = -1$ up to V_C, then linear to 0 at V_D (14 CFR 25.333).

Design limit load factor = $\max(n_{max},\, n_{gust})$; it can be copied to the Wing Root tab.

ISA density: $\rho = 1.225\,(1 - 2.25577 \times 10^{-5}\,h)^{4.2559}$ for $h \le 11$ km; $\rho = 0.36392\,e^{-(h-11000)/6341.6}$ above.

Wing Deflection

Cantilever semi-span s, moment $M(y)$ from the lift distribution (page 1), $I(y) = I_r\,[1 - (1 - r_I)\,y/s]$

$$\theta(y) = \int_0^y \frac{M}{E_c\,I}\,d\eta, \qquad \delta(y) = \int_0^y \theta\,d\eta$$

Uniform load and uniform EI: $\delta_{tip} = Ls^3/(8EI)$, $\theta_{tip} = Ls^2/(6EI)$. Trapezoidal integration, 401 stations.

Wing Torsion Box, Single Cell

Box width w, depth h, enclosed area $A = w\,h$, cover thickness t_s, spar web thickness t_w

$$q_T = \frac{T}{2A}, \qquad q_V = \frac{V}{2h}, \qquad \tau_{skin} = \frac{q_T}{t_s}, \qquad \tau_{web} = \frac{q_T + q_V}{t_w}$$

$$\frac{d\theta}{dz} = \frac{T}{4A^2 G}\oint \frac{ds}{t} = \frac{T}{4A^2 G}\left(\frac{2w}{t_s} + \frac{2h}{t_w}\right), \qquad G = \frac{E_c}{2(1+\nu)}$$

$$MS_{yield} = \frac{F_{sy}}{FS_y\,\tau_{max}} - 1, \quad F_{sy} \approx \frac{F_{ty}}{\sqrt{3}}, \qquad MS_{ult} = \frac{F_{su}}{FS_u\,\tau_{max}} - 1$$

The two spar webs share V equally. Skin buckling is checked on the Buckling tab.

Stringer Column with Effective Skin

Stringer area A_{st}, own inertia I_{st}, centroid at z from the skin; skin thickness t, stringer pitch b, rib pitch L

$$b_e = \min\left(b,\ 1.9\,t\sqrt{\frac{E_c}{\sigma_{cr}}}\right), \qquad A = A_{st} + b_e t, \qquad \bar{z} = \frac{A_{st}\,z}{A}$$

$$I = I_{st} + A_{st}(z-\bar{z})^2 + b_e t\,\bar{z}^2, \qquad \rho = \sqrt{I/A}, \qquad L' = \frac{L}{\sqrt{c}}$$

$$\sigma_e = \frac{\pi^2 E_c}{(L'/\rho)^2}; \qquad \sigma_{cr} = \sigma_e \ \text{ if } \ \sigma_e \le \frac{F_{cc}}{2}, \qquad \sigma_{cr} = F_{cc}\left(1 - \frac{F_{cc}}{4\,\sigma_e}\right) \ \text{ otherwise}$$

b_e depends on σ_{cr}, so the calculation is iterated to convergence. F_{cc} = crippling stress (F_{ty} if 0).

$$MS = \frac{\sigma_{cr}}{FS_u\,\sigma} - 1$$

End fixity $c = 1$ pinned, up to 4 fully fixed. Reference: Bruhn C7 (effective width) and C2 (Johnson-Euler).

Lug, Axial Load (simplified)

Lug width W, hole diameter D, edge distance e (hole center to edge), thickness t, pin shear strength $F_{su,p}$

$$P_u = FS_u \, FF \, P$$

$$P_{nt} = F_{tu}(W - D)\, t, \qquad P_{so} = 2\, F_{su}\, t \left(e - \frac{D}{2} \cos 40^\circ \right), \qquad P_{br} = F_{bru}\, D\, t, \qquad P_{pin} = 2\, \frac{\pi D^2}{4} F_{su,p}$$

$$MS_i = \frac{P_i}{P_u} - 1$$

No net-tension or bearing efficiency factors (Bruhn D1, Melcher-Grimes curves): preliminary sizing only.

F_{bru} is for $e/D = 2$; reduce it for $e/D < 2$. Fitting factor FF from the Material tab (1.15, 14 CFR 25.625).

Fastener Joint, Single-Shear Lap Splice

n fasteners of diameter d in n_r rows, sheet width w, thinner sheet thickness t, fastener shear strength $F_{su,f}$

$$P_{sh} = n\, \frac{\pi d^2}{4} F_{su,f}, \qquad P_{br} = n\, d\, t\, F_{bru}, \qquad P_{net} = F_{tu} \left(w - \frac{n}{n_r}\, d \right) t$$

$$MS_i = \frac{P_i}{FS_u \, FF \, P} - 1$$

Equal load sharing; the first-row net section carries the full load (conservative).

Use Hoop Load sets $P = \Delta p \, R \, w$ from the Fuselage tab (longitudinal splice).

Pressure Fatigue, Crack Initiation

Gross stresses σ_{max}, σ_{min} per cycle, fatigue notch factor K_f, one pressure cycle per flight

$$\sigma_a = K_f \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sigma_m = K_f \frac{\sigma_{max} + \sigma_{min}}{2}, \qquad \sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / F_{tu}} \quad \text{(Goodman)}$$

$$\sigma_{ar} = \sigma'_f \, (2N)^{-|b|} \quad \Rightarrow \quad N = \frac{1}{2} \left(\frac{\sigma_{ar}}{\sigma'_f} \right)^{-1/|b|}, \qquad MS_{life} = \frac{N/SF}{N_{design}} - 1$$

Defaults: 2024-T351, $\sigma'_f = 900$ MPa (130 ksi), $|b| = 0.102$ (Dowling, Mechanical Behavior of Materials).

Elastic K_f: if $K_f \, \sigma_{max} > F_{ty}$, use Neuber's rule. Crack growth: Damage Tolerance tab (page 9).

Material Factors (ver 1.8)

Fitting factor FF (Material tab, default 1.15, 14 CFR 25.625): Joint, Lug and Fastener Group tabs.

Temperature factors: $F \to k_T F$ for $F_{ty}, F_{tu}, F_{su}, F_{bru}$, and $E_c \to k_E E_c$; read k_T, k_E

from the MMPDS elevated-temperature curves (percent of room-temperature value).

Basis (typical, A, B, S) records which allowables were entered; it does not change the values.

Pressure Bulkhead / Dome, Membrane Stresses

Shell of revolution, pressure p, thickness t, principal radii R_1 (meridian) and R_2 (normal to the axis)

$$\sigma_\phi = \frac{p\,R_2}{2\,t}, \qquad \sigma_\theta = \frac{p\,R_2}{t}\left(1 - \frac{R_2}{2\,R_1}\right)$$

Hemisphere, radius a: $R_1 = R_2 = a$, $\sigma = p\,a/(2t)$. Spherical cap, base radius a, height h:

$$R_s = \frac{a^2 + h^2}{2\,h}, \qquad \sigma = \frac{p\,R_s}{2\,t}, \qquad F_{ring} = \frac{p\,R_s}{2}\sqrt{1 - \left(\frac{a}{R_s}\right)^2}\,a$$

Ellipsoid, semi-axes a (radius) and b (depth), x = distance from the axis:

$$R_2 = \frac{\sqrt{a^4 - x^2(a^2 - b^2)}}{b}, \qquad R_1 = \frac{R_2^3\,b^4}{a^4}$$

Hoop stress at the edge is compressive when $a/b > \sqrt{2}$. Edge (discontinuity) bending is not included.

Cutout in a Skin, Elliptical Hole (Inglis)

Remote stresses σ_x, σ_y; cutout width a (along x) and height b (along y); infinite plate

$$\sigma_A = \sigma_y\left(1 + \frac{2a}{b}\right) - \sigma_x \;\;(\text{sides}), \qquad \sigma_B = \sigma_x\left(1 + \frac{2b}{a}\right) - \sigma_y \;\;(\text{top, bottom})$$

Circular hole ($a = b$): $3\sigma_y - \sigma_x$. Doubler thickness t_d, land width w_d each side:

$$\sigma_{peak} \approx \max(|\sigma_A|, |\sigma_B|)\,\frac{t}{t + t_d}, \qquad \text{area ratio} = \frac{2\,w_d\,t_d}{a\,t} \;\text{ and }\; \frac{2\,w_d\,t_d}{b\,t} \;\geq 1$$

Margins on the peak elastic stress (conservative for static strength of ductile skins).

Damage Tolerance, Center-Cracked Panel

Half-crack length a, panel width W, gross stress σ, finite-width factor (Feddersen)

$$K = \beta\,\sigma\sqrt{\pi a}, \qquad \beta = \sqrt{\sec\frac{\pi a}{W}}$$

Residual strength, fracture toughness K_c (plane stress), with net-section yield:

$$\sigma_R(a) = \min\left(\frac{K_c}{\beta\sqrt{\pi a}},\ F_{ty}\left(1-\frac{2a}{W}\right)\right)$$

Critical half-crack a_c: $\sigma_R(a_c) = \sigma_{limit}$. Crack growth, Paris law, $\Delta\sigma = \sigma_{max} - \max(\sigma_{min}, 0)$:

$$\frac{da}{dN} = C\,\Delta K^m, \quad \Delta K = \beta\,\Delta\sigma\sqrt{\pi a}, \qquad N = \int_{a_0}^{a_c}\frac{da}{C\,\Delta K^m}$$

$$MS_{residual} = \frac{\sigma_R(a_0)}{\sigma_{limit}} - 1, \qquad MS_{life} = \frac{N}{SF\,N_{required}} - 1, \qquad \text{inspection threshold} = \frac{N}{SF}$$

Units: English ksi, in, ksi$\sqrt{\text{in}}$, C in in/cycle; metric MPa, mm (m in K), MPa$\sqrt{\text{m}}$, C in m/cycle.

Conversion: $C_{SI} = 0.0254\,C_{Eng}/1.0988^m$ (1 ksi$\sqrt{\text{in}}$ = 1.0988 MPa$\sqrt{\text{m}}$).

The default C, m and K_c are illustrative only. Use test data or NASGRO for the alloy, thickness, environment and stress ratio. No retardation, no R-ratio correction, constant amplitude.

Integration: 2000 log-spaced points, trapezoidal rule.

Eccentric Fastener Group

Rectangular pattern, n fasteners at (x_i, y_i) from the centroid; load P along $-y$ at offset e along x

$$J = \sum_i (x_i^2 + y_i^2), \qquad F_{x,i} = \frac{P\,e\,y_i}{J}, \qquad F_{y,i} = -\frac{P}{n} - \frac{P\,e\,x_i}{J}$$

$$R_{max} = \max_i \sqrt{F_{x,i}^2 + F_{y,i}^2}, \qquad MS_{shear} = \frac{F_{su,f}\,\pi d^2/4}{FS_u\,FF\,R_{max}} - 1, \qquad MS_{br} = \frac{F_{bru}\,d\,t}{FS_u\,FF\,R_{max}} - 1$$

Elastic method, equal fastener stiffness (Bruhn D1).

Bonded Single-Lap Joint, Volkersen Shear Lag

Load P, width w, overlap L, adherends Et_1, Et_2, adhesive thickness t_a and shear modulus G_a

$$\lambda^2 = \frac{G_a}{t_a}\left(\frac{1}{Et_1} + \frac{1}{Et_2}\right), \qquad \psi = \frac{t_1}{t_2}, \qquad -\frac{L}{2} \le x \le \frac{L}{2}$$

$$\tau(x) = \frac{P\,\lambda}{2\,w}\left[\frac{\cosh \lambda x}{\sinh(\lambda L/2)} + \frac{1-\psi}{1+\psi}\,\frac{\sinh \lambda x}{\cosh(\lambda L/2)}\right]$$

Balanced joint: $\tau_{max}/\tau_{avg} = (\lambda L/2)\coth(\lambda L/2)$, $\tau_{avg} = P/(wL)$.

$$MS_{adhesive} = \frac{\tau_{allow}}{FS_u\,\tau_{max}} - 1, \qquad MS_{adherend} = \frac{F_{tu}}{FS_u\,P/(w\,t_{min})} - 1$$

Adhesive linear elastic; peel stresses (Goland-Reissner) and adherend bending not included.

Spanwise Wing Loads, Schrenk Approximation

Trapezoidal wing, root chord c_r, tip chord c_t, taper $\lambda = c_t/c_r$, $\eta = y/s$, lift per wing $L = nW/2$

$$w(\eta) = \frac{L}{s} \cdot \frac{1}{2}\left[\frac{4}{\pi}\sqrt{1-\eta^2} + \frac{2\,(1-(1-\lambda)\eta)}{1+\lambda}\right]$$

$$V(\eta) = s\int_{\eta}^{1} w\,d\eta', \qquad M(\eta) = s\int_{\eta}^{1} V\,d\eta'$$

Mean of the elliptical and planform (chord) distributions. Numerical integration, 4001 stations.

Schrenk on the Wing Root or Deflection tab uses this taper ratio with that tab's weight, n and span.

Thermal Stress

Temperature change ΔT; member 1 ($E_1 = E_c$, α_1, A_1), member 2 (E_2, α_2, A_2); + tension

Member 1 restrained, restraint factor k (1 = fully restrained):

$$\sigma_r = -k\,E_1\,\alpha_1\,\Delta T$$

Members 1 and 2 joined, equal length, free ends (equilibrium $A_1\sigma_1 + A_2\sigma_2 = 0$):

$$\sigma_1 = \frac{E_1\,E_2\,A_2\,(\alpha_2-\alpha_1)\,\Delta T}{E_1A_1 + E_2A_2}, \qquad \sigma_2 = -\sigma_1\,\frac{A_1}{A_2}$$

Margins on member 1 with the Material tab allowables. Defaults: 2024 and Ti-6Al-4V typical α.

Wing Vibration and Aeroelastics (preliminary)

Uniform cantilever, semi-span L, EI, GJ ($G = E_c/2(1+\nu)$), mass per length m, $I_\theta = m\,(r\,c)^2$

$$\omega_{b1} = 1.8751^2\sqrt{\frac{EI}{mL^4}}, \qquad \omega_{b2} = 4.6941^2\sqrt{\frac{EI}{mL^4}}, \qquad \omega_\theta = \frac{\pi}{2L}\sqrt{\frac{GJ}{I_\theta}}$$

Strip theory, lift curve slope a, elastic axis $e = (e/c)\,c$ aft of the aerodynamic center. Divergence:

$$q_D = \frac{\pi^2\,GJ}{4\,L^2\,c\,a\,e}, \qquad V_D^{div} = \sqrt{\frac{2\,q_D}{\rho_0}} \ \text{(EAS)}$$

Typical section with the cantilever frequencies: $k_h = m\,\omega_{b1}^2$, $k_\theta = I_\theta\,\omega_\theta^2$, $S = m\,x_\theta\,c$; quasi-steady lift $q\,c\,a\,\theta$

$$\begin{vmatrix} m\Omega + k_h & S\Omega + q\,c\,a \\ S\Omega & I_\theta\Omega + k_\theta - q\,c\,a\,e \end{vmatrix} = 0, \qquad \Omega = -\omega^2$$

Flutter (coalescence) at the lowest q where the two roots Ω become complex: discriminant $B^2 - 4AC < 0$.

$$MS_{div} = \frac{V^{div}}{1.15\,V_D} - 1, \qquad MS_{flutter} = \frac{V_F}{1.15\,V_D} - 1 \quad \text{(14 CFR 25.629)}$$

No unsteady aerodynamics (Theodorsen), compressibility or structural damping: a classroom check, not a flutter clearance. Reference: Hodges and Pierce, Introduction to Structural Dynamics and Aeroelasticity.

Shear Web, Kuhn Incomplete Diagonal Tension

Ultimate shear $V_u = FS_u V$, web depth h_e, thickness t, stiffener spacing d, stiffener area A_u, radius ρ

$$\tau = \frac{V_u}{h_e\, t}, \qquad k = \tanh\left(\frac{1}{2}\log_{10}\frac{\tau}{\tau_{cr}}\right) \;\; (\tau > \tau_{cr}), \qquad \sigma_u = \frac{k\,\tau\,\tan\alpha}{A_u/(d\,t) + \frac{1}{2}(1-k)}$$

τ_{cr}: simply supported panel $d \times h_e$, $k_s = 5.35 + 4\,(d_{min}/d_{max})^2$, Johnson correction; $\alpha = 45°$.

$$L_e = \frac{h_e}{\sqrt{1 + k^2(3 - 2d/h_e)}} \;\; (d < 1.5h_e), \qquad q_{rivet} = \tau\, t\,(1 + 0.414\,k)$$

Stiffener column allowable: Johnson-Euler on L_e/ρ. Web allowable F_{su} (Kuhn's chart reduces it as k grows).

No C_1, C_2 stress concentration or forced crippling. Reference: Bruhn C11, NACA TN 2661.

Crippling, Needham Method

Formed section split into angle units with legs a_i, b_i, $b' = (a_i + b_i)/2$, thickness t

$$\frac{F_{cs}}{\sqrt{E_c F_{cy}}} = \frac{C_e}{(b'/t)^{0.75}}, \qquad F_{cs} \le F_{cy}, \qquad F_{cc} = \frac{\sum F_{cs,i} A_i}{\sum A_i}$$

$C_e = 0.316$ (angle, two free edges), 0.342 (one free edge: channel, Z), 0.366 (no free edge); $F_{cy} \approx F_{ty}$.

$$MS = \frac{F_{cc}}{FS_u\,\sigma} - 1$$

Reference: Bruhn C7. Use Fcc in Stringer copies F_{cc} to the Stringer tab as the column cut-off.

Composite Laminate, Classical Laminate Theory

Ply E_1, E_2, G_{12}, ν_{12}; $Q_{11} = E_1/\Delta$, $Q_{22} = E_2/\Delta$, $Q_{12} = \nu_{12}E_2/\Delta$, $Q_{66} = G_{12}$, $\Delta = 1 - \nu_{12}\nu_{21}$

$$[A, B, D] = \sum_k \bar{Q}_k \left[(z_k - z_{k-1}),\ \frac{z_k^2 - z_{k-1}^2}{2},\ \frac{z_k^3 - z_{k-1}^3}{3} \right], \qquad \left\{ \begin{array}{c} N \\ 0 \end{array} \right\} = \left[\begin{array}{cc} A & B \\ B & D \end{array} \right] \left\{ \begin{array}{c} \varepsilon^0 \\ \kappa \end{array} \right\}$$

Ply strains at mid-ply $\varepsilon = \varepsilon^0 + z\,\kappa$, rotated to ply axes; stresses σ_1, σ_2, τ_{12}. Tsai-Wu, strength ratio R:

$$(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_1\sigma_2)R^2 + (F_1\sigma_1 + F_2\sigma_2)R - 1 = 0$$

$F_1 = 1/X_t - 1/X_c$, $F_{11} = 1/(X_t X_c)$, same for Y; $F_{66} = 1/S^2$, $F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$.

Maximum stress: $R = \min(X/|\sigma_1|,\ Y/|\sigma_2|,\ S/|\tau_{12}|)$, with X_t or X_c and Y_t or Y_c by sign.

$$MS = \frac{R_{min}}{FS_u} - 1 \quad \text{(first ply failure, loads at limit)}$$

Defaults: T300/5208 carbon/epoxy (Tsai; Daniel and Ishai). Layup notation: [0/45/-45/90]2s, +-45, 0_2.

Load Cases

Store saves every input of every tab under a name. Run All calculates each analysis for each stored case, then lists the worst result of each analysis (lowest margin; largest load factor, deflection or moment) and the case that gives it. The inputs and results on the tabs are restored after the run.

Lug, Bruhn D1 Method (ver 2.0)

Ultimate load $P_u = FS_u\, FF\, P$ at angle θ from the axis: $P_a = P_u \cos\theta$, $P_{tr} = P_u \sin\theta$

$$P_{tu} = K_t\, F_{tu}\, (W - D)\, t, \qquad P_{bru} = K_{br}\, F_{tu}\, D\, t, \qquad P_{tru} = K_{tru}\, F_{tu}\, D\, t$$

K_t, K_{br}, K_{tru}: efficiency factors read from Bruhn Figs. D1.12, D1.13 and D1.15 (inputs).

$$R_a = \frac{P_a}{\min(P_{tu}, P_{bru})}, \qquad R_{tr} = \frac{P_{tr}}{P_{tru}}, \qquad MS = \frac{1}{\left(R_a^{1.6} + R_{tr}^{1.6}\right)^{1/1.6}} - 1$$

Pin double shear $2\,(\pi D^2/4)\, F_{su,p}$ as before. Pin bending and bushings are not included.

Fastener Joint, Huth Flexibility (ver 2.0)

Huth (1986), single shear, sheets t_1, t_2 (E_c), fastener E_f; riveted Al $a = 2/5$, $b = 2.2$; bolted $a = 2/3$, $b = 3.0$

$$C = \left(\frac{t_1 + t_2}{2d}\right)^a b \left(\frac{1}{t_1 E} + \frac{1}{t_2 E} + \frac{1}{2 t_1 E_f} + \frac{1}{2 t_2 E_f}\right)$$

Row i load P_i (row compliance C/n_{row}), sheet stiffness $k_j = E\, t_j\, w/s$ between rows at spacing s:

$$\frac{C}{n_{row}}\,(P_{i+1} - P_i) = \frac{\sum_{j \le i} P_j}{k_2} - \frac{P - \sum_{j \le i} P_j}{k_1}, \qquad \sum_i P_i = P$$

Most loaded fastener $\max(P_i)/n_{row}$ for shear and bearing; the row 1 net section carries P.

Fatigue: Neuber Notch Plasticity and Miner's Rule (ver 2.0)

Cyclic stress-strain curve (Ramberg-Osgood), H' and n'; local maximum and local range by Neuber's rule:

$$\sigma\left(\frac{\sigma}{E}+\left(\frac{\sigma}{H'}\right)^{1/n'}\right)=\frac{(K_f S_{max})^2}{E},\qquad \Delta\sigma\left(\frac{\Delta\sigma}{E}+2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}\right)=\frac{(K_f\,\Delta S)^2}{E}$$

Local amplitude $\sigma_a=\Delta\sigma/2$, mean $\sigma_m=\sigma_{max}-\sigma_a$; Goodman and Basquin as on page 7.

Each flight: one ground-air-ground cycle, plus n_g gust cycles of gross amplitude S_g about S_{max}:

$$D=\frac{1}{N_{GAG}}+\frac{n_g}{N_{gust}},\qquad N_{life}=\frac{1}{D},\qquad MS=\frac{N_{life}/SF}{N_{design}}-1$$

Defaults: 2024-T351, $H'=662$ MPa (96 ksi), $n'=0.070$ (Dowling). Elastic option: $\sigma=K_f S$.

Damage Tolerance: Geometries and Growth Laws (ver 2.0)

Edge crack (Tada), $q=a/W$: $\beta=1.12-0.231q+10.55q^2-21.72q^3+30.39q^4$, net section $1-a/W$

Crack of length a at an open hole of radius r (Bowie fits), with a secant width correction:

$$\beta_1=0.6762+\frac{0.8733}{0.3245+a/r},\qquad \beta_2=0.9439+\frac{0.6865}{0.2772+a/r}$$

Stress ratio $R=\sigma_{min}/\sigma_{max}$ (compression ignored). Walker and Forman laws (Forman C scaled by K_c):

$$\frac{da}{dN}=C\left(\frac{\Delta K}{(1-R)^{1-\gamma}}\right)^m,\qquad \frac{da}{dN}=\frac{C\,K_c\,\Delta K^m}{(1-R)K_c-\Delta K}$$

Discrete Tuned Gust, 14 CFR 25.341(a) (ver 2.0)

Gust $U = \frac{U_{ds}}{2}\left(1 - \cos\frac{\pi s}{H}\right)$, $0 \le s \le 2H$, $H = 30$ to 350 ft; $U_{ds} = U_{ref}\, F_g\, (H/350)^{1/6}$

U_{ref} (EAS) at V_C: 56 ft/s at sea level, 44 ft/s at 15,000 ft, 20.86 ft/s at 60,000 ft (linear); at V_D: half.

$$F_g = \frac{1}{2}(F_{gz} + F_{gm}) \text{ (sea level)}, \quad F_{gz} = 1 - \frac{Z_{mo}}{250000}, \quad F_{gm} = \sqrt{R_2 \tan\frac{\pi R_1}{4}}$$

F_g rises linearly to 1.0 at Z_{mo}. Check F_{gm} against the current eCFR text of 25.341(a)(6).

Rigid airplane in plunge, quasi-steady lift (no Kussner lag), true airspeed V, vertical velocity w:

$$\dot{w} = \frac{\rho V S a}{2m}(U - w), \qquad \Delta n = \frac{\rho V S a}{2mg}(U - w)$$

The largest Δn over H at V_C and V_D gives the tuned gust load factor. 25.341(a)(1) requires a dynamic analysis with unsteady aerodynamics and structural modes: this tab is a rigid-airplane estimate only.

Wing Root Inertia Relief (ver 2.0)

Wing structure and fuel W_w spread uniformly along s, engine W_e at y_e, all multiplied by n:

$$M_{net}(y) = M_{lift}(y) - n\left[\frac{W_w}{s}\frac{(s-y)^2}{2} + W_e\,\langle y_e - y\rangle\right], \qquad V_{net} = L - n\,(W_w + W_e)$$

Curved Panels and Cylinders, NASA SP-8007 (ver 2.0)

Unstiffened cylinder, radius R, thickness t; axial compression with the SP-8007 knock-down factor:

$$\sigma_{cr} = \gamma \, \frac{E\,t}{R\sqrt{3(1-\nu^2)}}, \qquad \gamma = 1 - 0.901\left(1 - e^{-\phi}\right), \qquad \phi = \frac{1}{16}\sqrt{\frac{R}{t}}$$

Torsion, moderate length L (frame pitch a), Donnell form, 0.67 knock-down on the stress (conservative):

$$\tau_{cr} = 0.67 \times 0.747 \, \frac{E}{(1-\nu^2)^{5/8}} \left(\frac{t}{R}\right)^{5/4} \left(\frac{R}{L}\right)^{1/2}$$

Curved panel: the larger of the flat-panel value (page 3) and the cylinder value (an approximation to the Gerard-Becker curves). Curvature parameter $Z = \frac{b^2}{Rt}\sqrt{1-\nu^2}$ is listed for reference.

Shear Web, Kuhn: Angle, Peak Shear, Forced Crippling (ver 2.0)

$$\tan^2\alpha = \frac{\varepsilon - \varepsilon_F}{\varepsilon - \varepsilon_S}, \quad \varepsilon = \frac{\tau}{E}\left[\frac{2k}{\sin 2\alpha} + \sin 2\alpha\,(1-k)(1+\nu)\right], \quad \varepsilon_S = -\frac{\sigma_U}{E}, \quad \varepsilon_F = -\frac{\sigma_F}{E}$$

$$\sigma_U = \frac{k\,\tau\tan\alpha}{A_U/(dt) + \tfrac{1}{2}(1-k)}, \qquad \sigma_F = \frac{k\,\tau\cot\alpha}{2A_F/(ht) + \tfrac{1}{2}(1-k)}$$

$$\tau_{max} = \tau\,(1 + k^2 C_1)(1 + k\,C_2), \quad C_1 = \frac{1}{\sin 2\alpha} - 1, \qquad \sigma_0 = 26{,}000\,k^{2/3}\left(\frac{t_U}{t}\right)^{1/3}\ \text{psi}$$

C_2 (Bruhn Fig. C11.17) and the web allowable (C11.19) are chart inputs; σ_0 for 2024-T3.

Flutter, Theodorsen Unsteady Aerodynamics (ver 2.0)

Typical section: $b = c/2$, $a = 2(e/c) - 1/2$, $x_\alpha = 2(x/c)$, $r_\alpha = 2(r/c)$, $\mu = m/(\pi\rho b^2)$, $\sigma = \omega_h/\omega_\alpha$

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i\,H_0^{(2)}(k)}, \quad l_h = 1 - \frac{2iC}{k}, \quad l_\alpha = \frac{1}{2} - \frac{i(1+2C)}{k} - \frac{2C}{k^2}, \quad m_h = \frac{1}{2}, \quad m_\alpha = \frac{3}{8} - \frac{i}{k}$$

$$\begin{vmatrix} \mu(1-\sigma^2\Omega) + l_h & \mu x_\alpha + l_\alpha - (\frac{1}{2}+a)l_h \\ \mu x_\alpha + m_h - (\frac{1}{2}+a)l_h & \mu r_\alpha^2(1-\Omega) + m_\alpha - (\frac{1}{2}+a)(l_\alpha + m_h) + (\frac{1}{2}+a)^2 l_h \end{vmatrix} = 0$$

$\Omega = (\omega_\alpha/\omega)^2(1+ig)$; for each k: ω, g and $V = \omega b/k$ (k method, V-g and V-f).

Flutter where the required damping g of a mode rises through the structural damping g_s.

Check, independent p-k solver: $\mu = 20$, a = -0.2, $x_\alpha = 0.1$, $r_\alpha^2 = 0.24$, $\sigma = 0.4$ gives $V_F = 2.184\,b\omega_\alpha$.

Laminate: Moments, Thermal and Moisture, Buckling (ver 2.0)

$$\begin{Bmatrix} N + N^{T,H} \\ M + M^{T,H} \end{Bmatrix} = [ABD] \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix}, \qquad N^{T,H} = \sum_k \bar{Q}_k \left(\bar{\alpha}_k \Delta T + \bar{\beta}_k \Delta C\right)(z_k - z_{k-1})$$

Ply stress $\sigma = R\,\sigma_{mech} + \sigma_{res}$: Tsai-Wu solved for R (residual fixed), both faces of each ply.

Panel buckling, simply supported, $D_{16} = D_{26} = 0$, compressive N_x, N_y (Jones):

$$\lambda = \min_{m,n} \frac{\pi^2 \left[D_{11}(m/a)^4 + 2(D_{12} + 2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4\right]}{(m/a)^2(-N_x) + (n/b)^2(-N_y)}$$