automobile_nvh_a: Equations

Automobile noise, vibration and harshness, ver 1.6 by Tom Irvine

1. Engine orders

An order is a multiple of a shaft rotation speed. With engine speed N in RPM, engine order O has the frequency

$$\text{Eq. 1: } f = \frac{O\,N}{60} \text{ Hz}$$

A four-stroke engine fires each cylinder once every two crankshaft revolutions, a two-stroke engine once every revolution. For n_c evenly firing cylinders and s strokes per cycle the firing order is

$$\text{Eq. 2: } O_f = \frac{2\,n_c}{s} \qquad O_f = \frac{n_c}{2} \text{ (4-stroke)} \qquad O_f = n_c \text{ (2-stroke)}$$

The firing order dominates the torque fluctuation (order 2 for an inline 4, order 3 for a V6, order 4 for a V8). The half order (camshaft) appears in a four-stroke engine with one cylinder that differs from the others (misfire, injector, compression). The app lists orders 1, 0.5 (4-stroke only), O_f and $2O_f$.

2. Driveline and tire orders

With transmission gear ratio g and final drive ratio g_d, the propshaft and wheel speeds and their orders, in engine-order terms, are

$$\text{Eq. 3: } N_p = \frac{N}{g} \qquad O_{p1} = \frac{1}{g} \qquad O_{p2} = \frac{2}{g}$$

$$\text{Eq. 4: } N_w = \frac{N}{g\,g_d} \qquad O_{t1} = \frac{1}{g\,g_d} \qquad O_{t2} = \frac{2}{g\,g_d}$$

Propshaft first order comes from imbalance; second order from a Cardan (U) joint working at an angle, whose output speed fluctuates twice per revolution. Tire/wheel first order comes from imbalance and radial force variation, second order from non-uniformity. The vehicle speed from the rolling radius r is

$$\text{Eq. 5: } V = \frac{2\pi\,r\,N_w}{60}$$

English: r in inches, V in in/sec, divided by 17.6 for mph. Metric: r in meters, V in m/sec, times 3.6 for km/h.

3. Campbell diagram

Each order is a straight line through the origin of the frequency versus engine-speed plot, Eq. 1. A vehicle mode of natural frequency f_n is a horizontal line. They cross, and the mode is excited by that order, at

$$\text{Eq. 6: } N_x = \frac{60\,f_n}{O}$$

The table of crossings lists those inside the Campbell speed range. Typical targets: idle firing frequency above the powertrain mount modes, and the body bending and torsion, steering column and first cabin acoustic modes away from the firing order over the common cruise speeds.

4. Powertrain mount isolation

The engine and transmission, mass m, on n_m mounts of vertical rate k_m each, is a single-degree-of-freedom bounce system with damping ratio ζ:

$$\text{Eq. 7: } f_n = \frac{1}{2\pi}\sqrt{\frac{n_m\,k_m}{m}} \qquad \delta_{st} = \frac{m\,g}{n_m\,k_m}$$

The force transmitted to the body per unit engine force, at the frequency ratio $r = f/f_n$, is the transmissibility

$$\text{Eq. 8: } T = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$T < 1$ (isolation) needs $r > \sqrt{2}$. The app evaluates T at the idle firing frequency $f = O_f N_{idle}/60$, and the lowest engine speed that is isolated:

$$\text{Eq. 9: isolation} = 100\,(1-T)\ \% \qquad = -20\log_{10} T\ \ \text{dB} \qquad N_{iso} = \frac{60\sqrt{2}\,f_n}{O_f}$$

Damping lowers the resonance peak but raises T in the isolation range. Rubber mounts have $\zeta \approx 0.05$ to 0.1. This is the bounce mode only; a full mount design also places the roll, pitch and fore-aft modes, with the torque roll axis.

5. Wheel imbalance and brake judder

At vehicle speed V and rolling radius r, the wheel rotation frequency and wheel order k are

$$\text{Eq. 10: } f_w = \frac{V}{2\pi\,r} \qquad f_k = k\,f_w$$

A chassis or steering mode f_n is excited by wheel order k at the speed

$$\text{Eq. 11: } V_x = \frac{2\pi\,r\,f_n}{k}$$

The rotating force from a residual imbalance U (mass times eccentricity) is

$$\text{Eq. 12: } F = U\,\omega^2 \qquad \omega = 2\pi f_w$$

English: U in oz-in, divided by 16 to lbm-in and by 386 to $\text{lbf}\,\text{sec}^2$, F in lbf. Metric: U in g-mm, times 10^{-6} to kg m, F in N. Imbalance and radial force variation excite the first order; tire non-uniformity and brake disc thickness variation or runout (brake judder) excite the first and second orders. Steering wheel shake appears where an order crosses the steering shimmy, suspension fore-aft or steering column mode.

6. Quarter-car ride model with seat

Sprung mass m_s (one quarter of the body) on suspension stiffness k_s and damping c_s; unsprung mass m_u (wheel, tire, brake, hub) on tire stiffness k_t; the driver m_d on the seat stiffness k_d and damping c_d; road displacement z_r. With $\mathbf{z} = \{z_s,\, z_u,\, z_d\}^T$,

Eq. 13: $\mathbf{M} = \begin{bmatrix} m_s & 0 & 0 \\ 0 & m_u & 0 \\ 0 & 0 & m_d \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} k_s + k_d & -k_s & -k_d \\ -k_s & k_s + k_t & 0 \\ -k_d & 0 & k_d \end{bmatrix}$

$\mathbf{M}\ddot{\mathbf{z}} + \mathbf{C}\dot{\mathbf{z}} + \mathbf{K}\mathbf{z} = \begin{Bmatrix} 0 \\ k_t\, z_r \\ 0 \end{Bmatrix} \qquad \mathbf{C}$ has the same form with c_s, c_d

With $m_d = 0$ the seat row and column are removed (2-DOF model). Uncoupled estimates, with the ride rate k_r:

Eq. 14: $k_r = \dfrac{k_s\, k_t}{k_s + k_t} \qquad f_{ride} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_r}{m_s}} \qquad f_{hop} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_s + k_t}{m_u}} \qquad f_{seat} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_d}{m_d}}$

The coupled natural frequencies are the roots of $\det(\mathbf{K} - \omega^2\mathbf{M}) = 0$; the app lists the 2-DOF values (no seat) and the 3-DOF values. Damping ratios and static deflection:

Eq. 15: $\zeta_s = \dfrac{c_s}{2\sqrt{k_s\, m_s}} \qquad \zeta_u = \dfrac{c_s}{2\sqrt{(k_s + k_t)\, m_u}} \qquad \delta_{st} = \dfrac{(m_s + m_d)\, g}{k_s}$

Typical passenger cars: f_{ride} = 1 to 1.5 Hz, f_{hop} = 10 to 15 Hz, ζ_s = 0.2 to 0.4, seat f_{seat} = 3 to 5 Hz.

7. Harmonic road excitation and transmissibility

A road undulation of wavelength λ at vehicle speed V excites the frequency

Eq. 16: $f = \dfrac{V}{\lambda}$

For harmonic road input $z_r e^{i\omega t}$, the displacement transmissibilities are the magnitudes of the solution of

Eq. 17: $\left(\mathbf{K} - \omega^2\mathbf{M} + i\,\omega\,\mathbf{C}\right)\mathbf{Z} = \begin{Bmatrix} 0 \\ k_t \\ 0 \end{Bmatrix} \qquad T_s = |Z_s| \qquad T_u = |Z_u| \qquad T_d = |Z_d|$

T_s peaks near f_{ride}; above about $\sqrt{2}\, f_{ride}$ the body is isolated until the wheel-hop resonance. The seat adds isolation above about $\sqrt{2}\, f_{seat}$ but amplifies near f_{seat}, where the body still moves.

8. Random road roughness (ISO 8608)

The road profile displacement PSD versus spatial frequency n (cycles/m), at the geometric mean of road class A (very good) to H, is

$$\text{Eq. 18: } G_d(n) = G_d(n_0)\left(\frac{n}{n_0}\right)^{-2} \qquad n_0 = 0.1 \text{ cycles/m}$$

$$G_d(n_0) = 16 \times 4^{k-1} \times 10^{-6} \text{ m}^3$$

with $k = 1$ for class A to 8 for class H (A 16, B 64, C 256, D 1024 $\times 10^{-6}$ m^3). At speed V (m/sec) the temporal PSD at the tire, with $n = f/V$, and the response PSDs and rms values are

$$\text{Eq. 19: } G_z(f) = \frac{1}{V}\,G_d\!\left(\frac{f}{V}\right) \qquad a_{rms} = \sqrt{\int_{f_1}^{f_2} (2\pi f)^4\,|Z_s|^2\,G_z\,df}$$

$$\text{Eq. 20: } s_{rms} = \sqrt{\int |Z_s - Z_u|^2\,G_z\,df} \qquad F_{rms} = k_t\sqrt{\int |Z_u - 1|^2\,G_z\,df}$$

s is the suspension travel and F the dynamic tire load, compared with the static wheel load $(m_s + m_u + m_d)g$; a ratio above about 1/3 (3 sigma near 1) means the tire can leave the road. Since $G_z \propto V$, the rms values grow as $\sqrt{V}$, and each road class doubles them. The seat acceleration is weighted with W_k (Section 9) for the comfort estimate.

9. Seat vibration comfort (ISO 2631-1)

The frequency weightings are products of band-limiting, acceleration-velocity transition and upward-step filters, $s = i\,2\pi f$, $\omega_j = 2\pi f_j$:

$$\text{Eq. 21: } H_h = \frac{s^2}{s^2 + \frac{\omega_1}{Q_1}s + \omega_1^2} \qquad H_l = \frac{\omega_2^2}{s^2 + \frac{\omega_2}{Q_2}s + \omega_2^2}$$

$$H_t = \frac{1 + s/\omega_3}{1 + \frac{s}{Q_4\omega_4} + \frac{s^2}{\omega_4^2}} \qquad H_s = \frac{1 + \frac{s}{Q_5\omega_5} + \frac{s^2}{\omega_5^2}}{1 + \frac{s}{Q_6\omega_6} + \frac{s^2}{\omega_6^2}}\left(\frac{\omega_5}{\omega_6}\right)^2 \qquad W = |H_h H_l H_t H_s|$$

	f_1	f_2	$Q_{1,2}$	f_3	f_4	Q_4	f_5	Q_5	f_6	Q_6
W_k (vertical)	0.4	100	0.71	12.5	12.5	0.63	2.37	0.91	3.35	0.91
W_d (horizontal)	0.4	100	0.71	2.0	2.0	0.63	∞		∞	

Frequencies in Hz ($H_s = 1$ for W_d). The weighted rms acceleration of band levels a_i (m/sec^2) and the estimated dose value are

$$\text{Eq. 22: } a_w = \sqrt{\sum_i (W_i\,a_i)^2} \qquad eVDV = 1.4\,a_w\,T^{1/4}$$

a_w (m/sec^2)	< 0.315	0.315–0.63	0.5–1	0.8–1.6	1.25–2.5	> 2
reaction	not unc.	a little	fairly	uncomfortable	very	extremely

T in seconds; comfort reactions from ISO 2631-1 Annex C (the ranges overlap, and the app lists both names in an overlap). English inputs in G are multiplied by 9.81.

10. Tire cavity resonance

Tire size $w/AR\,\mathrm{R}D_{rim}$: section width w (mm), aspect ratio AR (%), rim diameter D_{rim} (in, times 25.4). The section height, outer diameter and cavity centreline diameter are

$$\text{Eq. 23: } h = \frac{w\,AR}{100} \qquad D_o = D_{rim} + 2h \qquad D_m = D_{rim} + h$$

The first acoustic mode of the air torus has a wavelength equal to the centreline circumference. Rotation at the wheel frequency f_r splits it into two peaks:

$$\text{Eq. 24: } f_0 = \frac{c}{\pi\,D_m} \qquad f_r = \frac{V}{\pi\,D_o} \qquad f_{\pm} = f_0 \pm f_r$$

c is the speed of sound of the cavity air (it rises with temperature). The unloaded outer diameter is used for f_r. Typical f_0 = 190 to 250 Hz.

11. Exhaust and intake silencer elements

The target is the firing frequency $f_t = O_f N/60$ (Eq. 2). Main pipe area S; plane waves, $k = 2\pi f/c$. A Helmholtz side-branch resonator with neck area S_n (radius a), neck length L and volume V:

$$\text{Eq. 25: } f_H = \frac{c}{2\pi}\sqrt{\frac{S_n}{V\,L_e}} \qquad L_e = L + 1.7\,a \qquad V = \frac{S_n}{L_e}\left(\frac{c}{2\pi f_t}\right)^2$$

$$\text{Eq. 26: } TL_H = 10\log_{10}\left[1 + \left(\frac{\sqrt{G\,V}/(2S)}{f/f_H - f_H/f}\right)^2\right] \qquad G = \frac{S_n}{L_e}$$

A closed quarter-wave side branch of area $S_b = S$ (end correction 0.85 a_p, pipe radius a_p) tuned to f_t:

$$\text{Eq. 27: } L_q = \frac{c}{4f_t} - 0.85\,a_p \qquad TL_Q = 10\log_{10}\left[1 + \left(\frac{S_b}{2S}\tan k(L_q + 0.85\,a_p)\right)^2\right]$$

A simple expansion chamber of length L_c and area ratio $m = S_c/S$:

$$\text{Eq. 28: } TL_E = 10\log_{10}\left[1 + \frac{1}{4}\left(m - \frac{1}{m}\right)^2 \sin^2 kL_c\right]$$

TL_E peaks at $f = (2j-1)\,c/(4L_c)$ and is zero at $f = j\,c/(2L_c)$. Resonators give a narrow, deep notch; the expansion chamber a broad band. Flow, losses and higher-order modes are not included. English: lengths in inches, c in ft/sec times 12; metric: mm divided by 1000, c in m/sec, volume in liters.

12. Cabin acoustic modes

The passenger compartment is approximated by a rectangular box with rigid walls, length L (fore-aft), width W and height H. The natural frequencies, for integers $l, m, n \geq 0$ (not all zero), are

$$\text{Eq. 29: } f_{lmn} = \frac{c}{2}\sqrt{\left(\frac{l}{L}\right)^2 + \left(\frac{m}{W}\right)^2 + \left(\frac{n}{H}\right)^2}$$

c = speed of sound, 1125 ft/sec or 343 m/sec in air at 20°C. A mode is *axial* with one non-zero index, *tangential* with two, *oblique* with three. The first longitudinal mode $f_{100} = c/(2L)$, typically 50 to 70 Hz in a car, causes boom. The number of modes below f (Morse) and the Schroeder frequency, above which the modes overlap, are

$$\text{Eq. 30: } N(f) = \frac{4\pi V f^3}{3c^3} + \frac{\pi S f^2}{4c^2} + \frac{L_e f}{8c} \qquad f_s = 2000\sqrt{\frac{T_{60}}{V}}$$

$V = LWH$, S = total surface area, $L_e = 4(L + W + H)$ total edge length. In f_s, V is in m^3 and the reverberation time T_{60} in seconds (about 0.05 to 0.1 sec in a trimmed car).

13. Sound pressure level and addition

Sound pressure level of rms pressure p, and the energy sum of levels L_i (bands, or uncorrelated sources):

$$\text{Eq. 31: } L_p = 20\log_{10}\frac{p}{p_{ref}} \qquad L = 10\log_{10}\sum_i 10^{L_i/10}$$

$p_{ref} = 20\ \mu$Pa $= 2.901\times10^{-9}$ psi. Two equal levels add to 3 dB more; the app reports the overall rms pressure in psi (English) or Pa (metric).

14. A- and C-weighting (IEC 61672-1)

$$\text{Eq. 32: } R_A(f) = \frac{12194^2\, f^4}{(f^2+20.6^2)\sqrt{(f^2+107.7^2)\,(f^2+737.9^2)}\,(f^2+12194^2)}$$

$$A = 20\log_{10} R_A + 2.00$$

$$\text{Eq. 33: } R_C(f) = \frac{12194^2\, f^2}{(f^2+20.6^2)\,(f^2+12194^2)} \qquad C = 20\log_{10} R_C + 0.06$$

The weighted band level is $L_i + A(f_i)$, and dBA is the energy sum of the weighted band levels (Eq. 31). Values at the nominal octave band centre frequencies:

Hz	31.5	63	125	250	500	1000	2000	4000	8000
A (dB)	-39.5	-26.2	-16.2	-8.7	-3.2	0.0	1.2	1.0	-1.1
C (dB)	-3.0	-0.8	-0.2	0.0	0.0	0.0	-0.2	-0.8	-3.0

15. Bands, noise rating and speech interference

Narrowband lines at frequency f are added (Eq. 31) into the one-third octave band k, and any band into the octave band j, with base-10 centre frequencies:

$$\text{Eq. 34: } k = \operatorname{round}\left(10 \log_{10} \frac{f}{1000}\right) \qquad f_k = 1000 \cdot 10^{k/10} \qquad j = \operatorname{round}\left(\frac{10}{3} \log_{10} \frac{f}{1000}\right)$$

The noise rating curves (ISO R1996) are straight lines $L = a + b\,NR$ in each octave band; the rating is the lowest curve not exceeded by any band. The speech interference levels (ANSI S3.14) are arithmetic means of octave band levels:

$$\text{Eq. 35: } NR = \left\lceil \max_j \frac{L_j - a_j}{b_j} \right\rceil \qquad SIL4 = \frac{L_{500} + L_{1k} + L_{2k} + L_{4k}}{4}$$

$$PSIL = \frac{L_{500} + L_{1k} + L_{2k}}{3}$$

Hz	31.5	63	125	250	500	1000	2000	4000	8000
a	55.4	35.5	22.0	12.0	4.8	0.0	-3.5	-6.1	-8.0
b	0.681	0.790	0.870	0.930	0.974	1.000	1.015	1.025	1.030

NR uses the octave bands from 31.5 Hz to 8 kHz that are present; SIL4 and PSIL need all of their bands, otherwise the app reports n/a.

16. Order tracking of test data

The signal x (sample rate f_{sa}) is cut into frames of N points with overlap, each multiplied by the periodic Hann window $w_n = 0.5 - 0.5\cos(2\pi n/N)$ and transformed, $X_k = \sum_n w_n x_n e^{-i2\pi kn/N}$, $\Delta f = f_{sa}/N$. The spectral map shows the sine amplitude at the line centres; the order amplitude is found from the mean square in a band:

$$\text{Eq. 36: } A_k = \frac{2|X_k|}{\sum w_n} \qquad A_O = \sqrt{2 \sum_{k \in B} \frac{2|X_k|^2}{N \sum w_n^2}}$$

$$B : \frac{O\,N_{min}}{60} - 2\Delta f \le f_k \le \frac{O\,N_{max}}{60} + 2\Delta f$$

N_{min}, N_{max} are the lowest and highest engine speeds in the frame, so the band follows the order while the speed changes and the window leakage is included. A_O is plotted against the mean frame speed (the order cut). A fixed-frequency component inside the band (a resonance where the order crosses it) adds to A_O. The workspace array is $n \times 3$: time (sec), RPM, signal.

17. Transfer path analysis (classical, mount stiffness method)

The noise at a target (the driver's ear) at one engine order is the sum of the contributions of the structure-borne paths j, each a mount or isolator between the active (source) side and the passive (body) side. The rms accelerations measured on both sides are phasors $a = |a|\, e^{i\phi}$ at the order frequency, and the relative displacement across the mount is

$$\text{Eq. 37: } f = \frac{O\,N}{60} \qquad \omega = 2\pi f \qquad \Delta x_j = -\frac{a_{A,j} - a_{P,j}}{\omega^2}$$

The mount dynamic stiffness k_j (at the order frequency) and loss factor η_j give the path force. The noise transfer function H_j (Pa/N, magnitude and phase) from the passive-side mount point to the target is measured with the source removed, by a hammer or shaker at the mount point or reciprocally with a volume source at the target:

$$\text{Eq. 38: } F_j = k_j\,(1 + i\,\eta_j)\,\Delta x_j \qquad p_j = H_j\,F_j \qquad p = \sum_j p_j$$

English inputs: a in G is multiplied by 386 to in/sec^2, k is in lbf/in, and F in lbf is multiplied by 4.448 to N before Eq. 38. The levels re $p_{ref} = 20$ μPa (Eq. 31), A-weighted with $A(f)$ of Eq. 32, are

$$\text{Eq. 39: } L_j = 20\log_{10}\frac{|p_j|}{p_{ref}} \qquad L = 20\log_{10}\frac{|p|}{p_{ref}} \qquad L_A = L + A(f)$$

Because the contributions add with their phases, the path with the highest level is not always the largest contributor. The share is the component of p_j in phase with the total; the shares add to 100 %, and a negative share is a path that cancels part of the total. The energy sum (phases ignored) is shown for comparison:

$$\text{Eq. 40: } s_j = 100\,\frac{\mathrm{Re}\,(p_j\,\overline{p})}{|p|^2}\ \% \qquad \sum_j s_j = 100 \qquad L_{inc} = 10\log_{10}\sum_j 10^{L_j/10}$$

When a measured total level L_m is entered, the app shows $L - L_m$: a large difference means a missing path (airborne noise, another mount axis) or an error in the stiffness or NTF data.

What-if (source, path, contribution)

Factors α_j on the mount stiffness and β_j on the NTF give the modified contributions and the change at the target, with the accelerations held at their measured values ($\beta_j = 0$ removes a path):

$$\text{Eq. 41: } {p_j}' = \alpha_j\,\beta_j\,p_j \qquad p' = \sum_j {p_j}' \qquad \Delta L = 20\log_{10}\frac{|p'|}{|p|}$$

Holding the accelerations assumes the mount is much softer than the source and body at the mount points, so the relative displacement does not change with k. A stiffer mount also changes Δx, so a large factor overstates the change. Each path is one direction; enter the three axes of a mount as three paths.

Example, the default English inputs: order 2 at 3000 RPM, f = 100 Hz, A = -19.1 dB. Engine mount RH, k = 1500 lbf/in, 0.50 G active, 0.020 G passive: $|\Delta x|$ = 0.000471 in, $|F|$ = 0.714 lbf; with $|H|$ = 0.040 Pa/N, $|p_1|$ = 0.1270 Pa. Five paths: L = 72.7 dB, 53.5 dBA. What-if, RH mount α = 0.7: 51.3 dBA (-2.2 dB).

18. Engine balance: shaking forces and moments

Cylinder i has its crank pin at angle ϕ_i and its axis at bank angle θ_i from vertical, at axial position z_i. With crank radius r (half the stroke) and connecting rod length l, the piston position from the crank centre and the inertia force of the reciprocating mass m along the cylinder axis are

Eq. 42: $$x = r\cos\alpha + \sqrt{l^2 - r^2\sin^2\alpha} \qquad \alpha = \omega t + \phi_i - \theta_i$$

Eq. 43: $$F_i = -m\,\omega^2\frac{d^2x}{d\alpha^2} \approx m\,r\,\omega^2\left(\cos\alpha + \lambda\cos 2\alpha\right) \qquad \lambda = \frac{r}{l}$$

Eq. 44: $$F_v = \sum_i F_i\cos\theta_i \qquad F_l = \sum_i F_i\sin\theta_i \qquad M_p = \sum_i F_i\cos\theta_i\,z_i$$

$$M_y = \sum_i F_i\sin\theta_i\,z_i \qquad T = \sum_i F_i\frac{dx}{d\alpha}$$

The app uses the exact $d^2x/d\alpha^2$ and takes the 1st, 2nd and 4th order amplitudes by FFT over one revolution. The exact 2nd order coefficient is $A_2 = \lambda + \lambda^3/4 + 15\lambda^5/128 + \ldots$. In units of $mr\omega^2$ (forces) and $mr\omega^2 s$ (moments, s = cylinder spacing): inline 3, 1st and 2nd order moments $\sqrt{3}$ and $\sqrt{3}A_2$; inline 4, 2nd order vertical force $4A_2$; inline 6 and boxer 6, none; V8 cross-plane, 1st order rotating couple $\sqrt{10}$ (cancelled by the crank counterweights); V8 flat-plane, 2nd order lateral force $4\sqrt{2}A_2$; boxer 4, 2nd order yaw moment $2A_2$. The rotating masses are assumed balanced by the counterweights; balance shafts are not included.

19. Electric motor orders

With motor speed N (RPM), p pole pairs and Q stator slots, an order O of the motor shaft has frequency $ON/60$. The orders are 1 (rotor imbalance), p (electrical frequency f_e), $2p$ (main radial force), $6p$ and $12p$ (torque ripple), Q (slot passing), $\mathrm{lcm}(Q, 2p)$ (cogging), z_1 and $z_3 z_1/z_2$ (gear mesh). Sine-triangle PWM at switching frequency f_{sw} adds current harmonics, and magnetic noise, at the sidebands of Eq. 46:

Eq. 45: $$f_e = \frac{p\,N}{60} \qquad i = \frac{z_2}{z_1}\,\frac{z_4}{z_3} \qquad V = \frac{2\pi\,r\,N}{60\,i}$$

Eq. 46: $$f = |f_{sw} \pm 2f_e| \qquad f = |2f_{sw} \pm f_e|$$

r is the tire rolling radius and i the overall reduction ratio. The sidebands move away from f_{sw} and $2f_{sw}$ as the speed rises; a sideband crossing a structural or cabin mode gives a whine at that speed.

20. Panel sound transmission

A panel of modulus E, density ρ, Poisson ratio ν and thickness h has surface mass m (kg/m^2), bending stiffness D and coincidence frequency f_c, where the bending wave speed equals the speed of sound c:

$$\text{Eq. 47: } m = \rho\, h \qquad D = \frac{E h^3}{12(1-\nu^2)} \qquad f_c = \frac{c^2}{2\pi}\sqrt{\frac{m}{D}}$$

Single panel, Sharp's method with the field-incidence mass law and loss factor η; between $f_c/2$ and f_c the transmission loss is interpolated linearly in $\log f$:

$$\text{Eq. 48: } TL = 20\log_{10}(f\, m) - 47 \quad f < f_c/2$$

$$TL = 20\log_{10}(f\, m) + 10\log_{10}\frac{2\eta f}{\pi f_c} - 47 \quad f \geq f_c$$

Double wall: panels m_1, m_2 with an air gap d containing absorbent, air density $\rho_0 = 1.21$ kg/m^3. The mass-air-mass resonance f_0 and the cavity limiting frequency f_l divide three regions:

$$\text{Eq. 49: } f_0 = \frac{1}{2\pi}\sqrt{\frac{\rho_0 c^2 (m_1 + m_2)}{d\, m_1 m_2}} \qquad f_l = \frac{c}{2\pi d}$$

$$\text{Eq. 50: } TL = 20\log_{10}\left[f(m_1 + m_2)\right] - 47 \quad f < f_0$$

$$TL = TL_1 + TL_2 + 20\log_{10}\frac{4\pi f d}{c} \quad f_0 \leq f < f_l \qquad TL = TL_1 + TL_2 + 6 \quad f \geq f_l$$

TL_1 and TL_2 are the single panel values of Eq. 48. With $c = 343$ m/sec the middle term is Sharp's $20\log_{10}(fd) - 29$; the form above is continuous at f_l for any c. The method assumes field incidence, panels large compared with the wavelength, and no structural connection (studs, clips) between the panels; leaks and flanking paths through holes and seals usually set the limit in a vehicle. A limp barrier (heavy layer) is entered with a very low modulus.

Example, the default metric inputs: 0.8 mm steel, $m_1 = 6.28$ kg/m^2, $f_c = 15323$ Hz; 1.5 mm barrier, $m_2 = 2.70$ kg/m^2, on a 20 mm gap: $f_0 = 309$ Hz, $f_l = 2730$ Hz. At 1000 Hz: $TL_1 = 29.0$ dB, $TL_2 = 21.6$ dB, double wall 47.9 dB.

21. Powertrain mount system, 6 degrees of freedom

The powertrain is a rigid body with coordinates $\mathbf{q} = \{u_x, u_y, u_z, \theta_x, \theta_y, \theta_z\}$ at its centre of gravity: x forward, y left, z up. Mount i at position $\mathbf{r}_i$ from the CG has stiffnesses $\mathbf{k}_i = \mathrm{diag}(k_x, k_y, k_z)$ along the vehicle axes. For small motions its deflection is

$$\text{Eq. 51: } \mathbf{u}_i = \mathbf{u} + \theta \times \mathbf{r}_i = \mathbf{S}_i\,\mathbf{q} \qquad \mathbf{S}_i = [\mathbf{I} \quad -\tilde{\mathbf{r}}_i] \qquad \mathbf{K} = \sum_i \mathbf{S}_i^T\,\mathbf{k}_i\,\mathbf{S}_i$$

$\tilde{\mathbf{r}}$ is the cross-product matrix ($\tilde{\mathbf{r}}\mathbf{a} = \mathbf{r} \times \mathbf{a}$). With mass m and the inertia tensor $\mathbf{J}$ about the CG (products of inertia $I_{xy} = \int x\,y\,dm$), the natural frequencies and mode shapes follow from

$$\text{Eq. 52: } \mathbf{M} = \begin{bmatrix} m\mathbf{I} & 0 \\ 0 & \mathbf{J} \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \qquad \left(\mathbf{K} - \omega^2\mathbf{M}\right)\phi = 0$$

The kinetic energy distribution of mode j gives the share of each coordinate k; a decoupled mode has nearly 100 % in one coordinate. With products of inertia a share can be slightly negative; the shares add to 100 %:

$$\text{Eq. 53: } KE_{kj} = 100\,\frac{\phi_{kj}\,(\mathbf{M}\phi_j)_k}{\phi_j^T\mathbf{M}\,\phi_j}\ \%$$

A free rigid body under a torque about the crankshaft axis $\mathbf{e}_c$ rotates about the torque roll axis (TRA) through the CG. The static response to the weight and to the reaction torque T (engine torque times the gear ratio) is

$$\text{Eq. 54: } \mathbf{e}_{TRA} \propto \mathbf{J}^{-1}\mathbf{e}_c \qquad \mathbf{q} = \mathbf{K}^{-1}\{0,\ 0,\ -mg,\ 0,\ 0,\ 0\}^T \qquad \mathbf{q} = \mathbf{K}^{-1}\{0,\ T\mathbf{e}_c\}^T$$

The mount forces on the powertrain are $-\mathbf{k}_i\mathbf{S}_i\mathbf{q}$. A good mount system has all six modes below the idle firing frequency divided by $\sqrt{2}$ (Eq. 9), a decoupled torque roll mode, and an elastic rotation axis under torque close to the TRA, so the torque pulses at idle excite only the torque roll mode. The model assumes the mount principal axes are along the vehicle axes and ignores mount damping and the frequency dependence of rubber stiffness.

22. Units

	English	*metric*
mass	lbm, divided by 386 to $\mathrm{lbf\,sec^2/in}$	kg
stiffness	lbf/in	N/m
damping	$\mathrm{lbf\,sec/in}$	$\mathrm{N\,sec/m}$
vehicle speed V	mph, times 17.6 to in/sec	km/h, divided by 3.6 to m/sec
road wavelength λ	ft, times 12 to in	m
gravity g	$386\ \mathrm{in/sec^2}$	$9.81\ \mathrm{m/sec^2}$
static deflection, travel	in	mm
force	lbf	N
imbalance U	oz-in, divided by 16 to lbm-in	g-mm, times 10^{-6} to $\mathrm{kg\,m}$
acceleration	G (ISO 2631: times 9.81 to $\mathrm{m/sec^2}$)	$\mathrm{m/sec^2}$
cabin, pipe dimensions	in	m (cabin), mm (pipes)
speed of sound c	ft/sec, times 12 to in/sec	m/sec
resonator volume	$\mathrm{in^3}$	liters
tire size	mm, %, in (tire code)	mm, %, in (tire code)
sound pressure	psi	Pa
TPA: k, a, NTF	lbf/in, G, Pa/N (force times 4.448)	$\mathrm{N/m}$, $\mathrm{m/sec^2}$, Pa/N
balance: m; stroke, rod	lbm (divided by 386); in	kg; mm
balance: force; moment	lbf; $\mathrm{lbf\,in}$	N; $\mathrm{N\,m}$
panel: E; ρ; h, d	psi; $\mathrm{lbm/in^3}$; in	GPa; $\mathrm{kg/m^3}$; mm
panel surface mass	$\mathrm{lbm/ft^2}$ ($\mathrm{kg/m^2}$ divided by 4.882)	$\mathrm{kg/m^2}$
mount system: I; position	$\mathrm{lbm\,in^2}$ (divided by 386); in	$\mathrm{kg\,m^2}$; mm

Road PSD (ISO 8608) is defined in SI units; the app converts the speed to m/sec and the displacement PSD to $\mathrm{in^2/Hz}$ (English) or $\mathrm{m^2/Hz}$ (metric). The ISO 2631 weighted acceleration a_w is always reported in $\mathrm{m/sec^2}$, with G in brackets.

23. Example: the default English inputs

Inline 4-cylinder four-stroke engine at 3000 RPM, gear 1.0, final drive 3.55, rolling radius 12 in:

- firing order 2, $f = 2(3000)/60 = 100.0$ Hz; tire 1st order $1/3.55 = 0.2817$, $f = 14.08$ Hz; $V = 2\pi(12)(845.1)/60/17.6 = 60.3$ mph;
- the firing order crosses the 24 Hz body bending mode at $N_x = 60(24)/2 = 720$ RPM.

Powertrain 400 lbm on 3 mounts of 900 lbf/in, $\zeta = 0.08$: $f_n = 8.12$ Hz; idle 750 RPM firing 25 Hz, $r = 3.08$, $T = 0.131$, isolation 87%.

Quarter car, 900 lbm sprung, 90 lbm unsprung, $k_s = 130$ lbf/in, $c_s = 9$ lbf·sec/in, $k_t = 1200$ lbf/in:

- $k_r = 130(1200)/1330 = 117.29$ lbf/in, $f_{ride} = 1.129$ Hz, $f_{hop} = 12.02$ Hz, $\zeta_s = 0.258$;
- 165 lbm driver on a 270 lbf/in seat: $f_{seat} = 4.00$ Hz; $\delta_{st} = (900 + 165)/130 = 8.19$ in; 60 mph over a 50 ft wavelength: $f = 1.76$ Hz.

Tire 225/45R17: $h = 101.25$ mm, $D_m = 431.8 + 101.25 = 533.0$ mm; $c = 1130$ ft/sec $= 344.4$ m/sec, $f_0 = 205.7$ Hz.

Exhaust, 4-cylinder at 2000 RPM: $f_t = 66.7$ Hz; with $c = 1600$ ft/sec, a 1.5 in by 3 in neck needs $V = 868$ in^3; a 6 in chamber on a 2 in pipe gives $m = 9$, $TL_{max} = 13.2$ dB.

Cabin 110 by 55 by 45 in, $c = 1125$ ft/sec: $f_{100} = 61.4$ Hz; $V = 4.46$ m^3, $T_{60} = 0.08$ sec: $f_s = 268$ Hz.

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