iso2631_a: Equations

Whole-body vibration, ISO 2631-1, -2 and -4, ver 1.0 by Tom Irvine

1. Scope and units

The app evaluates whole-body vibration of people seated, standing or lying down by ISO 2631-1:1997 (health, comfort, perception and motion sickness), with the building weighting of ISO 2631-2:2003 and the rail vehicle comfort weighting of ISO 2631-4:2001. The standards are written in SI units: acceleration in m/sec^2, rotational acceleration in rad/sec^2, vibration dose values in m/sec$^{1.75}$ and motion sickness dose values in m/sec$^{1.5}$.

English: accelerations are entered in G and converted with $g = 9.80665$ m/sec^2. The criteria of the standards are always applied in SI units, and the summaries give the SI value with the G value. Metric: m/sec^2.

2. Frequency weightings, ISO 2631-1 Annex A

Each weighting is the product of a band-limiting high pass and low pass (two-pole Butterworth), an acceleration-velocity transition and an upward step. With $p = j\,2\pi f$ and $\omega_i = 2\pi f_i$:

(A.1) $$H_h = \frac{1}{1 + \sqrt{2}\,\omega_1/p + (\omega_1/p)^2}$$

(A.2) $$H_l = \frac{1}{1 + \sqrt{2}\,p/\omega_2 + (p/\omega_2)^2}$$

(A.3) $$H_t = \frac{1 + p/\omega_3}{1 + p/(Q_4\omega_4) + (p/\omega_4)^2}$$

(A.4) $$H_s = \frac{1 + p/(Q_5\omega_5) + (p/\omega_5)^2}{1 + p/(Q_6\omega_6) + (p/\omega_6)^2} \left(\frac{\omega_5}{\omega_6}\right)^2$$

(A.5) $H = H_h\,H_l\,H_t\,H_s$ Tables A.1 and A.2 (frequencies in Hz):

	f_1	f_2	f_3	f_4	Q_4	f_5	Q_5	f_6	Q_6	application
W_k	0.4	100	12.5	12.5	0.63	2.37	0.91	3.35	0.91	z-axis, seat, standing, recumbent
W_d	0.4	100	2.0	2.0	0.63	∞		∞		x- and y-axis
W_f	0.08	0.63	∞	0.25	0.86	0.0625	0.80	0.10	0.80	motion sickness
W_c	0.4	100	8.0	8.0	0.63	∞		∞		x-axis, seat-back
W_e	0.4	100	1.0	1.0	0.63	∞		∞		rotational, seat
W_j	0.4	100	∞	∞		3.75	0.91	5.32	0.91	vertical, under the head

A frequency of ∞ removes that term ($p/\infty = 0$); with $f_3 = f_4 = \infty$ the transition is 1, and with $f_5 = f_6 = \infty$ the step is 1.

W_b, ISO 2631-4:2001 Annex A: band limiting as (A.1) and (A.2) with $Q_1 = 0.71$ in place of $1/\sqrt{2}$, $f_1 = 0.4$, $f_2 = 100$, and the weighting

$$H_w = \frac{(p + \omega_3)\,(p^2 + \omega_5\,p/Q_5 + \omega_5^2)}{(p^2 + \omega_4\,p/Q_4 + \omega_4^2)\,(p^2 + \omega_6\,p/Q_6 + \omega_6^2)} \frac{K\,\omega_4^2\,\omega_6^2}{\omega_3\,\omega_5^2}$$

with $f_3 = f_4 = 16$ Hz, $Q_4 = 0.55$, $f_5 = 2.5$ Hz, $Q_5 = 0.90$, $f_6 = 4$ Hz, $Q_6 = 0.95$ and $K = 0.4$. W_m, ISO 2631-2:2003 Annex A: (A.1) and (A.2) with $f_1 = 10^{-0.1} = 0.794$ Hz and $f_2 = 100$ Hz, times $1/(1 + p/\omega_3)$ with $f_3 = 1/(0.028 \cdot 2\pi) = 5.684$ Hz.

3. Weighting a time history

The time history is weighted by the transfer function itself, magnitude and phase, as a causal digital filter. Each second-order factor of $H(p)$ is converted with the bilinear transform, every corner frequency pre-warped:

$$\text{Eq. 1: } s = \frac{2}{\Delta t}\,\frac{1 - z^{-1}}{1 + z^{-1}} \qquad \omega_i = \frac{2}{\Delta t}\,\tan(\pi f_i\,\Delta t)$$

The bilinear transform compresses the frequency axis near half the sample rate, so the filters run at L times the sample rate f_s, at least 40 times the upper band limit f_2 (100 Hz; 0.63 Hz for W_f):

$$\text{Eq. 2: } L = \left\lceil \frac{40\,\min(f_s/2,\,f_2)}{f_s} \right\rceil$$

The record and its mirror image form a continuous periodic sequence, which is interpolated by L with the FFT (band limited); every L-th filtered sample is kept. The digital weighting then follows Annex A within 0.02 dB in magnitude and 0.1 degree in phase up to 100 Hz. The filter starts at rest, as an instrument does, so the band-limiting high pass has a short start transient (a few seconds; about 20 sec for W_f).

4. Basic evaluation method, ISO 2631-1 6.1 and 6.2

$a_w(t)$ is the weighted acceleration times the multiplying factor k; T is the record length. The weighted rms acceleration and the crest factor are

$$\text{Eq. (1): } a_w = \left[\frac{1}{T}\int_0^T a_w^2(t)\,dt\right]^{1/2} \qquad \text{CF} = \frac{\max|a_w(t)|}{a_w}$$

For crest factors up to 9 the basic method is normally sufficient (6.2.2).

5. Additional methods, ISO 2631-1 6.3

Running rms with integration time τ (1 sec recommended) and the maximum transient vibration value:

$$\text{Eq. (2): } a_w(t_0) = \left[\frac{1}{\tau}\int_{t_0-\tau}^{t_0} a_w^2(t)\,dt\right]^{1/2} \qquad \text{Eq. (4): MTVV} = \max\, a_w(t_0)$$

The app uses the linear integration of Eq. (2), with $a_w = 0$ before the start of the record. Fourth power vibration dose value, the total of several periods, and the ratios above which the additional methods matter:

$$\text{Eq. (5): VDV} = \left[\int_0^T a_w^4(t)\,dt\right]^{1/4} \qquad \text{Eq. (6): VDV}_{total} = \left[\sum_i \text{VDV}_i^4\right]^{1/4}$$

$$\text{Eq. (7): } \frac{\text{MTVV}}{a_w} = 1.5 \qquad \text{Eq. (8): } \frac{\text{VDV}}{a_w\,T^{1/4}} = 1.75$$

When a ratio is exceeded, or the crest factor exceeds 9, the summary asks for MTVV and VDV to be reported with a_w. The app also gives the VDV extrapolated to the daily exposure T_d, VDV $(T_d/T)^{1/4}$, and the estimated vibration dose value of B.3.1 note 2 and C.2.2.2 note 2, with T_d in seconds:

$$\text{Eq. (B.5): eVDV} = 1.4\,a_w\,T_d^{1/4}$$

6. One-third octave band data, ISO 2631-1 6.4.2

a_i is the rms acceleration of band i and W_i the weighting factor of that band. The factors come from $|H(j2\pi f)|$ at the nominal band frequency, as Tables 3 and 4 of ISO 2631-1 and Table A.2 of ISO 2631-4 were calculated; Table A.1 of ISO 2631-2 (W_m) was calculated at the exact frequencies $10^{x/10}$ of band number x, so the app uses those for W_m. The factors then reproduce every table to its 0.01 dB rounding (W_b within 0.06 dB).

Eq. (9): $a_w = k \left[\sum_i (W_i \, a_i)^2 \right]^{1/2}$

7. Vibration in more than one axis and at more than one point

The vibration total value of the weighted rms accelerations of orthogonal axes at one point:

Eq. (10): $a_v = \left(k_x^2 \, a_{wx}^2 + k_y^2 \, a_{wy}^2 + k_z^2 \, a_{wz}^2 \right)^{1/2}$

Comfort (8.2.3): a_v is the point vibration total value at each point; the overall vibration total value is the root-sum-square of the point values. Rotational axes (k in m/rad) form their own point value, which is included in the overall value (8.2.3 note 2). An axis below 25 % of the largest at its point, or a point below 25 % of the largest point, may be excluded (note 3); the app marks them. Multiplying factors:

Seated, comfort (8.2.2.1)			**Seated, health (7.2.3)**	
seat x, y, z	W_d, W_d, W_k	1, 1, 1	seat x, y	W_d, $k = 1.4$
seat r_x, r_y, r_z	W_e	0.63, 0.4, 0.2 m/rad	seat z	W_k, $k = 1$
seat-back x	W_c	0.8	**Standing (8.2.2.2)**	
seat-back y, z	W_d	0.5, 0.4	floor x, y, z	W_d, W_d, W_k, $k = 1$
feet x, y, z	W_k	0.25, 0.25, 0.4	**Recumbent (8.2.2.3)**	
			horizontal, vertical	W_d, W_k, $k = 1$

8.2.3 note 4: when the seat-back is not measured, $k = 1.4$ may be used for the seat x- and y-axes to estimate comfort. Health (7.2.2): the assessment is made for each axis separately, with the highest of $1.4\,a_{wx}$, $1.4\,a_{wy}$ and a_{wz} on the seat pan; the vector sum is listed for comparable axes (7.2.2 note).

8. Comfort and perception, ISO 2631-1 Annex C

Approximate reactions to overall vibration total values in public transport (C.2.3); the ranges overlap and the standard sets no limits:

less than 0.315 m/sec^2	not uncomfortable	0.8 to 1.6 m/sec^2	uncomfortable
0.315 to 0.63 m/sec^2	a little uncomfortable	1.25 to 2.5 m/sec^2	very uncomfortable
0.5 to 1 m/sec^2	fairly uncomfortable	greater than 2 m/sec^2	extremely uncomfortable

Perception (C.3): half of alert, fit people can just detect W_k-weighted vibration with a peak of 0.015 m/sec^2 (interquartile range about 0.01 to 0.02 m/sec^2). The Time History page compares the W_k weighted peak with it.

9. Health, ISO 2631-1 Annex B

Two daily exposures are equivalent when (B.1), assuming the response is related to energy, or when (B.2), which other studies indicate:

$$\text{Eq. (B.1):}\ a_{w1}\,T_1^{1/2} = a_{w2}\,T_2^{1/2} \qquad \text{Eq. (B.2):}\ a_{w1}\,T_1^{1/4} = a_{w2}\,T_2^{1/4}$$

Several periods i of magnitude a_{wi} and duration T_i in a day are combined into the equivalent magnitudes (B.3) and (B.4), the 8-hour energy equivalent $A(8)$, and the total estimated vibration dose value from (B.5) and (6):

$$\text{Eq. (B.3):}\ a_{w,e} = \left[\frac{\sum a_{wi}^2\,T_i}{\sum T_i}\right]^{1/2} \qquad \text{Eq. (B.4):}\ a_{w,e} = \left[\frac{\sum a_{wi}^4\,T_i}{\sum T_i}\right]^{1/4}$$

$$A(8) = \left[\frac{\sum a_{wi}^2\,T_i}{8\ \text{h}}\right]^{1/2} \qquad \text{eVDV} = \left[\sum_i \left(1.4\,a_{wi}\,T_i^{1/4}\right)^4\right]^{1/4}$$

Health guidance caution zone, Figure B.1: below the zone health effects have not been clearly documented; in the zone caution is indicated; above it health risks are likely. The guidance is based mainly on exposures of 4 h to 8 h. The zone of (B.2) is given numerically: its bounds are eVDV = 8.5 and 17 m/sec$^{1.75}$ (B.3.1 note 2), so with T in seconds

$$\text{Eq. 3:}\ a_{lower} = \frac{8.5}{1.4\,T^{1/4}} \qquad a_{upper} = \frac{17}{1.4\,T^{1/4}} \qquad 0.47 \text{ and } 0.93\ \text{m/sec}^2 \text{ at } 8\ \text{h}$$

The zone of (B.1) is only drawn. The standard states that the two zones are the same from 4 h to 8 h, so the app draws the (B.1) bounds, $a\,T^{1/2}$ constant, through the (B.2) bounds at 5.66 h, the geometric middle of that range, and holds them constant below 10 min, as in Figure B.1:

$$\text{Eq. 4:}\ a(T) = a_{B.2}(T_m)\left(\frac{T_m}{\max(T,\ 10\ \text{min})}\right)^{1/2} \qquad T_m = \sqrt{32}\ \text{h}$$

This gives 0.43 to 0.86 m/sec^2 at 8 h and 3.0 to 5.9 m/sec^2 below 10 min, which agrees with Figure B.1 within its drawing accuracy. $a_{w,e}$ of (B.3) is compared with the (B.1) zone and the total eVDV (equivalently $a_{w,e}$ of (B.4)) with the (B.2) zone, at the total daily duration.

For reference only (not part of ISO 2631), EU Directive 2002/44/EC, Article 3, sets for whole-body vibration a daily exposure action value of 0.5 m/sec^2 $A(8)$ (or VDV 9.1 m/sec$^{1.75}$) and a limit value of 1.15 m/sec^2 $A(8)$ (or VDV 21 m/sec$^{1.75}$), with the highest of $1.4\,a_{wx}$, $1.4\,a_{wy}$ and a_{wz}. The app compares $A(8)$ and the total eVDV with them.

10. Motion sickness, ISO 2631-1 clause 9 and Annex D

Low-frequency z-axis motion (0.1 to 0.5 Hz), weighted with W_f. The motion sickness dose value from a record, or from a short measurement of a steady motion of rms a_w over the exposure T_0 (seconds; measure at least 240 sec):

$$\text{Eq. (D.1):}\ \mathrm{MSDV}_z = \left[\int_0^T a_w^2(t)\,dt\right]^{1/2} \qquad \text{Eq. (D.2):}\ \mathrm{MSDV}_z = a_w\,T_0^{1/2}$$

The percentage of people who may vomit is approximately $K_m\,\mathrm{MSDV}_z$, with $K_m = 1/3$ for a mixed population of unadapted male and female adults (D.2). The relationship is based on exposures from about 20 min to 6 h, with up to about 70 % vomiting; it may underestimate when a_w exceeds 0.5 m/sec^2. The exposure that gives P % is

$$\text{Eq. 5:}\ T_0 = \left(\frac{P}{K_m\,a_w}\right)^2$$

11. Frequency Weightings page

Lists $20\log_{10}|H|$ at each one-third octave band (band number x, nominal frequency of IEC 61260) next to the value in the standard, and plots the magnitude and phase of $H(j2\pi f)$. It confirms the transfer functions above against Tables 3 and 4 of ISO 2631-1:1997, Table A.2 of ISO 2631-4:2001 and Table A.1 of ISO 2631-2:2003.

12. Time history input file

A text file with two columns: time in seconds and acceleration (G, or m/sec^2 in metric; rad/sec^2 for W_e), with a constant time step (within 1 %). Header lines are skipped. The Demo signal is a reproducible vehicle seat record: random vibration near 4 Hz at 0.6 m/sec^2 rms with three decaying 4 Hz bumps. Its crest factor is just under 9, but MTVV/a_w and VDV/($a_w T^{1/4}$) exceed 1.5 and 1.75, so the additional methods apply (6.2.2 note).

References

1. ISO 2631-1:1997, Mechanical vibration and shock: Evaluation of human exposure to whole-body vibration, Part 1: General requirements.
2. ISO 2631-2:2003, Part 2: Vibration in buildings (1 Hz to 80 Hz).
3. ISO 2631-4:2001, Part 4: Guidelines for the evaluation of the effects of vibration and rotational motion on passenger and crew comfort in fixed-guideway transport systems.
4. ISO 8041, Human response to vibration: Measuring instrumentation.
5. IEC 61260, Electroacoustics: Octave-band and fractional-octave-band filters (band numbers and nominal frequencies).
6. Directive 2002/44/EC of the European Parliament and of the Council on the minimum health and safety requirements regarding the exposure of workers to the risks arising from physical agents (vibration), Article 3.
7. M.J. Griffin, Handbook of Human Vibration, Academic Press, 1990.

The app is a study and first-estimate aid. The guidance of Annexes B, C and D is informative; consult the standards and the applicable regulations for assessments.