Vibrationdata Fatigue & Fracture — User Guide

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Overview

vibrationdata_fatigue_fracture.m (ver 1.5) is a MATLAB app for fatigue crack growth, damage tolerance, stress-life fatigue and random vibration fatigue, launched by typing `vibrationdata_fatigue_fracture`. Its calculations are in the open `+vdfat` package, which also runs from scripts. It covers the Forman–Newman–de Koning (FNK, NASGRO) equation with Newman closure, Paris, Walker and tabulated da/dN; twelve crack configurations with user β tables and weight-function stress gradients; constant-amplitude, spectrum, PSD, sine and duty-cycle loading with Willenborg, constant closure or the strip-yield (Newman) closure model; residual strength (LEFM, FAD, R-curve) with inspection intervals, Monte Carlo scatter and crack initiation; S-N fatigue with Marin factors; random vibration fatigue with six spectral methods, FDS, ERS and test tailoring; unit-load superposition; multiaxial fatigue (Findley, normal-stress, SWT and Fatemi–Socie critical planes, Crossland and Sines, von Mises stress PSD); rolling contact fatigue (Hertz line, point and elliptical contact, ball bearings, Dang Van, EHL film thickness); fatigue maps of finite-element models from Nastran punch and bulk data files; seam weld fatigue by structural stress, hot-spot and effective notch stress; ISO 281 ball bearing life; mission synthesis of random, sine dwell and sine sweep environments into a random test; fretting fatigue of a cylinder on a flat in partial slip; and Nastran punch import, in English or metric units.

- **Launch:** unzip the release into your MATLAB folder, change to the folder that holds `vibrationdata_fatigue_fracture.m` and type `vibrationdata_fatigue_fracture` (or `app = vibrationdata_fatigue_fracture;` for a handle). The launcher adds the package folder to the path. It is a programmatic uifigure app, tested in MATLAB R2026b (ver 1.3 and earlier: R2024a), and does not open in the App Designer editor.
- **Units:** ksi, inch, ksiVin, in/cycle throughout. Spectrum data in other units (moment, load) is converted with the scale factor on the Loading tab.
- **Plots** open in separate figure windows, so they can be edited, saved or copied into slides.

Workflow

Work through the Material, Geometry and Loading tabs (plus the Duty Cycle tab to combine events), then press **Run analysis**.

1. **Material.** Pick a library material or type your own constants (the dropdown switches to “User defined” when you edit a field). Choose the growth equation: FNK, Paris or Walker. The Kc label updates with thickness. **Plot da/dN curves** draws the rate curves at R = −1, 0, 0.5 and 0.7.
2. **Geometry.** Pick one of six crack geometries and enter width, thickness, hole diameter or constant β as the geometry needs (unused fields are greyed out). Enter the initial crack size a, plus c for surface and corner cracks. An optional final crack size stops the run early. **Plot β vs crack size** checks the K solution before running.
3. **Loading.** Choose constant amplitude (Smax, Smin, runout cycles), a spectrum file, or a variable from the MATLAB workspace. For spectra, set the data format, the turning-point counting method, the scale factor to ksi, the flight hours per block and the maximum number of blocks. Optionally turn on generalized Willenborg retardation. The text box summarizes the block: rows, cycles, peak stresses, RMS range and cycles per flight hour.
4. **Run & Results.** Set the integration tolerance (default 0.005) and run. A progress bar with Cancel appears. The results box lists the stop reason, cycles, blocks, flight hours, final crack size and Kmax. Buttons plot crack size against cycles or hours, plot Kmax against crack size, export to the workspace or CSV, and save or load the full model (including the spectrum) as a .mat file.

Crack growth equations

The default is the Forman–Newman–de Koning (FNK) equation, in which a crack-opening function f carries the stress-ratio effect, a threshold term slows growth near ΔKth, and a fracture term accelerates it near Kc.

```
\frac{da}{dN} = C\left[\frac{1-f}{1-R}\,\Delta K\right]^{n}\frac{\left(1-
\Delta K_{th}/\Delta K\right)^{p}}{\left(1-K_{max}/K_{c}\right)^{q}}
```

ΔK = Kmax − Kmin is the full range, including any negative part. A cycle with Kmax ⩽ 0 does no damage, and a cycle reaching Kmax ⩾ Kc is fracture.

Newman closure function. The constants come from the constraint factor α and the ratio Smax/σ0:

$$A_0 = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)\left[\cos\left(\frac{\pi S_{max}}{2\sigma_0}\right)\right]^{1/\alpha}, \qquad A_1 = \left(0.415 - 0.071\alpha\right)\frac{S_{max}}{\sigma_0}$$

$$A_3 = 2A_0 + A_1 - 1, \qquad A_2 = 1 - A_0 - A_1 - A_3$$

$$f = \begin{cases} \max\left(R,\; A_0 + A_1R + A_2R^2 + A_3R^3\right) & R \ge 0 \\ A_0 + A_1R & -2 \le R < 0 \\ A_0 - 2A_1 & R < -2 \end{cases}$$

Threshold. Here a0 is the intrinsic crack length (default 0.0015 in), which lowers the threshold for very small cracks. Cth uses Cth+ for R ⩾ 0 and Cth− for R < 0.

$$\Delta K_{th} = \Delta K_0\sqrt{\frac{a}{a+a_0}}\Bigg/\left[\frac{1-f}{(1-A_0)(1-R)}\right]^{1+C_{th}R}$$

Fracture toughness vs thickness. Kc rises above KIc in thin sections:

$$K_c = K_{Ic}\left[1 + B_k\,e^{-\left(A_k\, t/t_0\right)^2}\right], \qquad t_0 = 2.5\left(K_{Ic}/\sigma_{ys}\right)^2$$

Paris and Walker. These use their own constants Cp and np. The library fills them in to match the FNK curve at R = 0: Cp = C(1 − A0)^n.

- Paris: da/dN = Cp ΔK^np. Only the positive part of the cycle counts, so ΔK = Kmax for R < 0.
- Walker: da/dN = Cp [ΔK/(1 − R)^(1−γ)]^np for R ⩾ 0, with γ = 0 (ΔK = Kmax) for R < 0.
- Neither has a threshold. Both fracture at Kmax ⩾ Kc.

Geometry library

All solutions are for remote tension, with K = β S √(πa) for through cracks. Part-through cracks use the Newman-Raju form K = S √(πa/Q) F, evaluated at both crack tips.

Geometry	Crack size definitions	Solution	Run stops at
Center through crack	a = half length, W = width	Tada: (1 − 0.025λ² + 0.06λ⁴)√sec(πλ/2), λ = 2a/W	a = 0.95 W/2

Geometry	Crack size definitions	Solution	Run stops at
Edge through crack	a from edge	Tada closed form, about 0.5% accurate for all a/W; β = 1.122 at small a	a = 0.95 W
Through crack at open hole	a from hole edge, D = hole diameter, hole centered	Bowie one-crack table (3.39 at a = 0, 1.37 at a/r = 1), long-crack asymptote beyond a/r = 10, times √sec(π(r + a)/W)	r + a = 0.95 W/2
Surface crack, semi-elliptical	a = depth, c = half surface length, t = thickness	Newman-Raju, 0 < a/c ⩽ 2, deepest and surface points	a = 0.95 t, then a through crack
Corner crack at plate edge	a = depth, c = length along face	Newman-Raju corner crack, both tips	a = 0.95 t, then a through crack
Corner crack at open hole	a = depth along the bore, c = length on the plate face from the hole edge, D, W, t	Compounded: Newman-Raju corner crack × Newman-Raju hole factor g2(λ), λ = 1/(1 + (c/r)cos 0.9φ), × √sec(π(r + c√(a/t))/W). Approximate	a = 0.95 t, then a through crack at the hole
Two through cracks at open hole	a = length of each crack from the hole edge, D, W	Bowie two-crack table (3.39 at a = 0, 1.45 at a/r = 1, → √((r + a)/a) for long cracks) × √sec(π(r + a)/W)	r + a = 0.95 W/2
Two corner cracks at open hole	a = depth along the bore, c = length on the face, for each crack	Single corner crack at hole × Bowie two-crack / one-crack ratio at c. Approximate	a = 0.95 t, then two through cracks at the hole
Surface crack in hole bore	a = radial depth from the bore, c = half-length along the bore (centered in t)	Compounded: Newman-Raju surface crack (bore as the free surface) × hole factor g2 × √sec(π(r + a)/W). Approximate	2c = 0.95 t, then a through crack at the hole of length a

Geometry	Crack size definitions	Solution	Run stops at
Infinite plate, constant β	a = crack length used in K	$K = \beta S \sqrt{(\pi a)}$	Kmax ≥ Kc or final size

Every geometry also stops at fracture (Kmax ≥ Kc), at net-section yield if that check is on, or at the optional final crack size. Surface and corner cracks grow both tips independently, so the aspect ratio a/c evolves; it is clipped to 0.02–2 inside the K equations.

Breakthrough. When a part-through crack reaches a = 0.95 t, the At breakthrough setting on the Geometry tab decides what happens:

- **Through crack, length = c** (default): a surface crack becomes a center through crack of half-length c, and a corner crack becomes an edge through crack of length c, and a corner crack at a hole becomes a through crack at the hole (Bowie) of length c, in the same width W. Growth continues in the same run with the same loading and retardation history.
- **Through crack, equal area:** the through-crack length is set so its area matches the part-through crack, πac/(4t). It starts shorter than c, so it gives slightly longer life.
- **Stop:** the run ends at breakthrough, as before.

The Results box reports the breakthrough cycle count, the depth and surface length at that point and the new through-crack length. The history plot marks breakthrough with a dashed line.

Net-section yield. With the check on (default), every cycle compares Smax × Agross/Anet with the flow stress (YS + UTS)/2. Net areas are W − 2a (center), W − a (edge), W − D − a (hole), Wt − πac/2 (surface) and Wt − πac/4 (corner). The edge-crack check ignores the bending from the offset ligament, so it is unconservative for pin-loaded or unrestrained edges.

In the surface-crack example (Ti-6Al-4V, t = 0.5 in, 50 ksi) the crack breaks through at 17,504 cycles with Kmax already at 79 of 85 ksi√in on the surface, so the through crack lasts only 1 cycle (length = c) or 107 cycles (equal area). Through-crack growth adds more life at lower stress or with a tougher material. For 304 stainless with a center crack at 30 ksi, net-section yield stops the run at a = 0.429 in, matching the hand value W/2 × (1 − 30/52.5) = 0.4286 in, where the run without the check went on to the 95% geometry limit.

Corner crack at a hole. The hole solution has no single published closed form, so the app compounds the Newman-Raju corner crack with the Newman-Raju hole factor g2(λ), which equals 3.11 at the bore for a very short crack (close to Kt = 3) and 1.0 far from the hole. With a very small hole it reproduces the plate corner crack to within 2%. In the 7075-T7351 example (t = 0.25 in, D = 0.25 in, a = c = 0.05 in, 20 ksi, R = 0.1) the crack runs ahead along the bore to a/c = 1.76, breaks through at 4,029 cycles and fractures at 10,080 (11,235 with equal area). Check the K against published tables or FEA before using it for design.

Comparing the hole cases. Same 7075-T7351 plate (W = 2 in, t = 0.25 in, D = 0.25 in), initial size 0.05 in, Smax 20 ksi, R = 0.1, breakthrough as a through crack of length c (or a for the bore crack), net-section check on:

Starting crack	Breakthrough (cycles)	Life (cycles)	Stop reason
One through crack	—	9,520	Fracture
Two through cracks	—	6,287	Net-section yield
One corner crack, a = c	4,029	10,080	Fracture
Two corner cracks, a = c	3,302	6,760	Net-section yield
Bore surface crack, a = c	2,212	10,653	Fracture

With two cracks the ligament on each side is cut at the same time, so net-section yield ends the run about a third sooner. Checks: the two-crack factor tends to the center-crack value $\sqrt{(\pi(r + a))}$ for long cracks (within 0.4% at a/r = 50), and with a very small hole the bore crack reproduces the plate surface-crack solution to within 0.5%.

Bending, pin-loaded holes and β tables

Bending. Enter the ratio Sb/St on the Geometry tab: the outer-fiber bending stress over the tension stress, applied in proportion to every cycle of the loading. It is active for three geometries:

- **Surface and corner cracks** use the Newman-Raju bending multiplier H, so K = K_tension × (1 + (Sb/St)·H). H is near 1 for shallow cracks and falls with depth; the deepest point can go negative. Corner cracks use the surface-crack H as an approximation.
- **Edge crack** uses the Tada in-plane bending solution (1.122 at small a/W, about 0.5% accurate), treated as bending of the strip in its own plane.

- **After breakthrough** a part-through crack carries its out-of-plane bending as membrane stress, K × (1 + Sb/St), which is conservative. The net-section check uses the tension stress only.

Equal bending and tension (Sb/St = 1) leaves 13% of the tension-only life for a Ti-6Al-4V surface crack (t = 0.5 in, 30 ksi) and 7% for a 7075-T651 edge crack (W = 2 in, 20 ksi).

Pin-loaded hole (approximate). A single through crack in the net section of a pin-loaded hole, with the loading entered as the gross stress P/(W t). K uses Broek's superposition: half the open-hole solution (Bowie, gross stress) plus half the K from the crack-line force P acting near the hole. It ignores the lug edge distance and is conservative for a/r below about 0.3, where the second term treats the hole as part of the crack. At the same gross stress, a 7075-T7351 hole (W = 2 in, D = 0.5 in, a = 0.05 in, 10 ksi) gives 26,539 cycles pin-loaded against 36,143 open. For real lug designs, prefer a β table.

User β table. Load two columns [a, β] from a file or the workspace (Geometry tab, Load β table) and K = β S √(πa). β is interpolated with pchip, held at the first value below the table, and the run stops at the last a. Use it for lugs from published data (Schijve and Hoeymakers, Kirkby and Rooke), FEA results or any handbook solution.

Probabilistic life

The Probabilistic tab runs the current model many times with random scatter and reports the life distribution.

- **Inputs:** number of samples, random seed, and coefficients of variation for initial crack size (log-normal, a and c scaled together), C (log-normal, also scales Paris/Walker C), Kc (normal) and the load scale (log-normal, multiplies every stress in the block).
- **Results:** life at the 1, 5, 10, 50, 90 and 99th percentiles in cycles and hours, a log-normal fit (median and log10 standard deviation), runouts, the shortest-life sample with its random factors, and the probability of failure before a target life with a one-sided 95% Wilson upper bound.
- **Plots:** histogram, life on log-normal probability paper, and life against initial crack size. Export puts monte_carlo_results in the workspace.

For Example 1 (center crack, 7075-T651, 20 ksi) with COV 0.3 on a_0 and C, 0.1 on Kc and 0.05 on load, 400 samples gave a median of 16,883 cycles against the deterministic 16,347, a 1% life of 5,644 cycles and P(life < 10,000) = 0.158 (upper bound 0.190). Each sample is a full run, so spectrum and Willenborg cases take longer; Cancel keeps the samples completed so far.

Residual strength and inspection intervals

The Residual Strength tab finds the critical crack size at a limit stress and turns a crack growth run into an inspection interval.

- **Critical crack size:** the smaller of the sizes where Kmax reaches Kc and where the net section reaches the flow stress (YS + UTS)/2, at the limit stress (default: the largest stress in the block). Part-through cracks keep the initial a/c.
- **Residual strength curve:** far-field stress at failure against crack size, from both criteria, with the limit stress and the critical point marked.
- **Inspection interval:** growth time from the detectable size to the critical size, read from the last crack growth run, divided by a safety factor (default 2). The inspection plot marks the intervals on the crack growth curve.

For Example 1 the critical half-length at 20 ksi is 1.032 in (matching the run's fracture size); growth from a detectable 0.1 in takes 8,337 cycles, giving a 4,169-cycle interval.

Loading

Constant amplitude needs only Smax, Smin and a runout cycle limit. Spectrum loading is defined as one block that repeats until a stop condition.

Spectrum sources. Browse for a text file (.spc, .txt, .dat, .csv, .lb; header lines are skipped) or type the name of a numeric variable in the base workspace and press Read variable.

Data format	Columns	How it is used
Max, Min, Cycles	3 or more	Each row is applied as given (the first three columns)
Max, Min pairs	2	Each row is one cycle
Turning points	last column	Reduced to reversals, then counted (below)
Auto detect	any	3+ columns = Max/Min/Cycles; 2 columns with a monotonic first column = time + turning points; other 2 columns = Max/Min pairs; 1 column = turning points

Auto detect catches the time-column mistake: a two-column (time, value) file is never read as Max/Min pairs. The note line in the summary box says which reading it chose.

Turning-point counting.

- **Rainflow (closure order)** is ASTM E1049 three-point counting, with cycles applied in the order they close and residual half cycles at the end of the block. It is compact, but it moves the largest cycles toward the end of the block.
- **Sequential half cycles** applies each pair of consecutive turning points as a half cycle in time order. It keeps the load sequence, which matters for retardation and for where in the block fracture happens.

Scale factor and hours. Every load value is multiplied by the scale factor to give ksi (for example 0.0014 psi per in-lb becomes 1.4e-6 ksi per in-lb). Block duration is the flight hours one pass represents, so life in hours = blocks × block duration. Max blocks is the runout limit.

Nastran import

The Nastran Import tab reads results straight from a punch file, so a SOL 112 moment history or a SOL 111 stress PSD goes into the analysis without a separate extraction script.

1. **Open punch file.** The file is indexed once in 256 MB chunks: every run of `$` header lines starts a block, and the table lists each block's kind, ID, item code, line count and header RMS. Re-scan after the file changes.
2. **Pick the block and the data.** For SORT2 element records (a first line plus `-CONT-` lines), set the record item code: the word number in each record, where word 1 is time and word 5 is the plane-2 bending moment at end A for a CBEAM. XYPUNCH blocks (index, x, y per line) need no item code. Treat as sets time history or PSD when the header doesn't say PSDF.
3. **Scale to ksi.** Enter a factor, or enter c and I and press the button to set c / I / 1000 for a bending moment in lbf-in. PSDs are multiplied by the scale squared.
4. **Extract** reads the block in 128 MB chunks and reports points, sample rate or frequency range, raw and scaled extremes and RMS. **Plot** shows it in a separate figure.
5. **Send** a time history to the Loading tab (as turning points) or the Rainflow tab, or a PSD to the Loading tab as a stress PSD. **Export** puts nastran_x, nastran_y (unscaled) and the block record in the workspace.

Rank selected by damage reads each selected block (all blocks if one or none is selected) and fills the Damage index column: Σ n·ΔS^m per second from rainflow for time histories, or the Dirlik equivalent for PSDs, using the scaled values. Use it to find the critical element before running crack growth on it.

Columns 73–80 (the punch sequence number, which overflows to asterisks) are always ignored. On a test punch the reader recovered SORT2 word 5 and word 100 exactly, and gave identical results with chunk sizes from 37 kB to 1 MB; indexing ran at about 220 MB/s in testing.

Retardation

Generalized Willenborg reduces the effective stress ratio after an overload. Each overload leaves a plastic zone ahead of the tip, and later cycles slow until the crack grows through that zone. Retardation applies to spectrum loading only; under constant amplitude it has no effect.

On each cycle the app computes the current zone size and the K needed to reach the boundary of the largest zone so far:

```
Z = \frac{1}{\pi\,\alpha_c}\left(\frac{K_{max}}{\sigma_{ys}}\right)^2, \qquad
K_{req} = \sigma_{ys}\sqrt{\pi\,\alpha_c\,(a_{OL} + Z_{OL} - a)}
```

If Kreq > Kmax, both Kmax and Kmin drop by a reduction KR, and growth is computed from the reduced values with the FNK equation:

```
K_R = \phi\,(K_{req} - K_{max}), \qquad \phi = \frac{1 - K_{max,th}/K_{max}}
{R_{so} - 1}, \qquad K_{max,th} = \frac{\Delta K_{th}}{1-R}
```

- **Rso** is the shut-off overload ratio: growth arrests when Kreq/Kmax reaches Rso. Typical values are 2 to 3; a lower Rso gives stronger retardation.
- **Constraint αc** sets the zone size: 1 for plane stress, up to 3 for plane strain. A larger αc gives a smaller zone and less retardation.
- A positive Kmin is reduced but not below zero; a negative Kmin is left unchanged.
- Each growing tip (a and c for part-through cracks) keeps its own overload history.

Willenborg runs cycle by cycle, so it is much slower than non-interaction. Expect roughly 1–10 µs per cycle in MATLAB. The strip-yield model below is slower again, a few milliseconds per model update.

Constant closure. The third Retardation option uses one crack-opening stress for the whole spectrum, Sop = Cf × Smax, where Smax is the largest stress in the block. Every cycle grows with ΔKeff = Kmax − max(Kop, Kmin) in the FNK equation, with the threshold

and fracture terms unchanged. Cycles that stay below Sop do not grow. It needs the FNK equation. This follows the NASGRO constant closure model; it is not a copy of NASGRO's implementation.

- **Cf = 0 (automatic)**: the Newman opening function at the spectrum ratio R = lowest Smin / highest Smax. For constant amplitude this gives exactly the non-interaction FNK result.
- **Cf entered**: any value from −1 to 0.99.
- **Calibrate Cf** (Run tab): enter a measured spectrum life in blocks and the app finds the Cf that reproduces it. Life rises with Cf, so the answer is unique. Check a calibrated Cf on a second spectrum before taking credit for it.

In a test block of 99 cycles at 20/2 ksi and one 30/3 ksi overload (7075-T7351 edge crack), automatic constant closure gives more than 1.2 times the non-interaction life, because the overload's opening level shields the small cycles.

Strip-yield (Newman). The fourth Retardation choice computes the crack-opening stress So from the plastic wake, cycle by cycle, with Newman's strip-yield model (the model behind FASTRAN). Rigid-perfectly plastic bar elements sit in the plastic zone ahead of the tip (intact: yield at ασ0 in tension and −σ0 in compression, σ0 = (σys + σu)/2) and in the wake behind it (broken: compression only). At each update the model is loaded to Smax and unloaded to Smin; every element meets displacement compatibility with its residual stretch, and yielded elements take a new length. The contact stresses in the wake at Smin give So:

```
S_o = S_{min} - \frac{2}{\pi}\sum_{wake}\sigma_j\left[\sin^{-1}\frac{b_{2j}}
{c} - \sin^{-1}\frac{b_{1j}}{c}\right], \qquad \Delta K_{eff} = K_{max} -
\max\left(K_o, K_{min}\right)
```

- **Inputs:** the Constraint box is the constraint factor α (1 plane stress, 3 plane strain; the material's FNK α is a typical value). FNK growth equation only.
- **Updates:** after 1% of the current plastic zone of growth, using the largest Smax and smallest Smin since the last update, and at once on a cycle above the last Smax or below the last Smin, so overloads and underloads act on that cycle.
- **Results:** the Run & Results text gives α, the flow stress, the number of updates and the range of So; **Plot opening stress** draws So and the crack size against cycles.
- **Checks:** the influence functions match the point-load solution, the crack-tip opening matches Dugdale, the reversed plastic zone matches theory within one element, and the constant-amplitude So/Smax is within 6% of Newman's closure equations (α = 1 to

3, R = −1 to 0.5). Constant-amplitude life is within 10% of FNK with the Newman function. A 1.8x overload every 1000 cycles more than triples the life of a 2024-T3 sheet center crack; an underload after each overload gives about a third of that back.

- **Limits:** infinite-plate opening stress with the geometry's K; fixed α (no constraint-loss transition); surface and corner cracks use the surface length c and one So for both tips; a transition to a through crack starts a new wake.

PSD and sine loading

The Loading tab also turns a stress PSD or a sine environment into one block of counted cycles. Each event has a duration in seconds; set Block duration to the flight hours that block represents.

Load type	Inputs	How the cycles are counted
Stress PSD	PSD file or workspace [f P] (ksi²/Hz after scale²), duration, mean stress	Dirlik, narrowband Rayleigh, time-domain synthesis or a custom range PDF
Sine dwell	Frequency, amplitude, mean, duration	f × T cycles at one range
Sine sweep	Start and end frequency, rate (oct/min), amplitude, mean, number of sweeps	60(f2 − f1)/(rate · ln 2) cycles per sweep
Sine on random	The PSD above plus a table of tones (frequency, amplitude)	Time-domain synthesis, then rainflow

- **PSD moments.** Breakpoints are interpolated log-log, and m0, m1, m2, m4 give the RMS, the zero-crossing rate ν0+ = √(m2/m0), the peak rate νp = √(m4/m2) and the irregularity factor γ = m2/√(m0 m4).
- **Dirlik and Rayleigh.** The range PDF is binned to 8 σ amplitude (default 200 bins). Cycle count = νp T for Dirlik and ν0+ T for Rayleigh. The bins are spread through the block in 100 sub-blocks, so the largest ranges are not all applied at the end.
- **Time-domain synthesis.** A Gaussian history is built by inverse FFT with random phases at 10 × fmax, optionally gated, rainflow counted, and the counts are scaled from the synthesis length to the event duration.

- **Custom function.** Enter the name of a function on the MATLAB path with the signature `[p, rate] = fname(Sr, m0, m1, m2, m4)`, returning the range PDF in 1/ksi and the cycle rate in cycles/s. Use it to plug in your own spectral method.
- **Gate.** Spectrum files, synthesized histories and the Rainflow tab all accept a hysteresis gate: reversals smaller than the gate are removed before counting.

Duty cycle

The Duty Cycle tab combines several loading events into one repeating block. Set up an event on the Loading tab and press **Add to duty cycle**; the event is stored as counted cycles at that moment.

- Edit an event's name or Repeats in the table; Move up and Move down set the order.
- A constant-amplitude event is stored as one cycle, so its Repeats is the cycle count.
- Repeats are applied by repeating the sequence (the default) or by multiplying the counts. Events whose repeated rows would exceed 2 million fall back to multiplying.
- Duty cycle duration is the flight hours of one pass. With **Use duty cycle for analysis** ticked, crack growth, initiation and the Rainflow tab all use the duty cycle instead of the single Loading-tab event.

Rainflow tool

The Rainflow tab counts a time history from a file, a workspace variable or the current loading block, without running crack growth.

- **Inputs:** gate (ksi), scale factor, range and mean bins for the plots, and the S-N slope m for the damage-weighted histogram. A two-column file with a monotonic first column is read as time and value.
- **Summary:** turning points, cycles, maximum, mean and RMS range, cycle-weighted mean, peak and valley, cycle rate, and the share of cycles that carries 90% of the m-slope damage.
- **Plots:** range histogram, range-mean rainflow matrix, range exceedance, level crossings, damage-weighted histogram and the time history.
- **Outputs:** a Max/Min/Cycles .spc file, `rainflow_cycles` and `rainflow_turning_points` in the workspace, or **Use as loading spectrum** to send the cycles to the Loading tab.

Crack initiation and total life

The Initiation tab computes Miner damage per block for the same loading, with an elastic Kt (or Kf) on the nominal stress. Initiation life plus the crack growth life from the Run tab gives total life when both use the same loading.

- **Stress-life:** Basquin, σar = σf′(2N)^b, with Goodman, Gerber, Morrow, SWT, Walker or no mean-stress correction.
- **Strain-life:** Coffin-Manson-Basquin with Morrow, SWT or no correction. Local stresses and strains come from Neuber's rule on the Ramberg-Osgood cyclic curve, using the doubled (Masing) curve for the range and the cyclic curve for the peak.
- **Constants:** filled from the material's UTS and E with the Uniform Material Law (Bäumel and Seeger) when you pick a material. Steels use σf′ = 1.5 Su, b = −0.087, c = −0.58; the other families use the aluminum/titanium set, σf′ = 1.67 Su, b = −0.095, εf′ = 0.35, c = −0.69. K′ and n′ follow from compatibility. These are estimates; replace them with measured data where you have it.
- **Endurance cutoff:** cycles with predicted life beyond the cutoff do no damage (0 = no cutoff).
- **Outputs:** damage per block, initiation life in blocks, cycles and hours, the most damaging row, the total life line, a life-curve plot and a damage-by-range plot.

Weld S-N

Choose **Weld S-N** as the method on the Initiation tab to get the fatigue life of a welded detail from its stress range. The result is life to failure, which already includes crack growth, so it is not added to the Run tab result.

```
N = 2\times10^{6}\left(\frac{FAT}{\Delta\sigma}\right)^{m_1} \quad (N \le
N_{knee}), \qquad N = N_{knee}\left(\frac{\Delta\sigma_{knee}}
{\Delta\sigma}\right)^{m_2} \quad (N > N_{knee})
```

Standard	Detail classes (FAT, MPa)	m1	Knee	Below the knee
IIW steel	160 to 36	3	10^7	Haibach m2 = 5, fatigue limit, or m2 = 22
IIW aluminum	71 to 12	3	10^7	Same choices as IIW steel

Standard	Detail classes (FAT, MPa)	m1	Knee	Below the knee
Eurocode 3 steel	160 to 36	3	5×10^6 ($\Delta\sigma D$ = 0.737 $\Delta\sigma C$)	m2 = 5 to the 10^8 cutoff ($\Delta\sigma L$ = 0.549 $\Delta\sigma D$)
Custom	any FAT	any	any	any m2 (0 = fatigue limit) and cutoff

- **Stress range:** the full range Smax − Smin of each cycle, in nominal or hot-spot stress to match the detail class. Kt and the mean-stress corrections are not applied.
- **Thickness:** above the reference thickness (default 25 mm), FAT is multiplied by (tref/t)^n, with n = 0.3 by default (IIW uses 0.2 for some details).
- **Residual stress:** High (as-welded) uses no R effect. Moderate and Low (stress relieved) apply the IIW enhancement f(R) to the range: up to 1.3 and 1.6 respectively at $R \leqslant -1$.
- **Miner limit:** the damage sum at failure, 1.0 by default. IIW recommends 0.5 for steel under variable amplitude.
- **Plot life curve** draws the S-N curve in MPa with the FAT point at 2×10^6 cycles.

Checks: FAT 90 at a 68.9 MPa range gives 4.448×10^6 cycles, matching the closed form. The Eurocode 3 curve built from the knee and cutoff reproduces the standard's 0.737 and 0.549 ratios.

Numerical method and outputs

Non-interaction runs integrate block by block with K frozen over each chunk of cycles and a Heun predictor-corrector step. Willenborg runs go cycle by cycle.

- **Chunking.** Within a window of up to 20,000 rows, growth rates are computed at the current crack size and summed. The chunk ends as soon as accumulated growth reaches tolerance × a (or × c), splitting a row of many cycles if needed.
- **Corrector.** Rates are recomputed at the predicted end size and averaged with the start. With the default tolerance of 0.005, the Paris closed-form check agrees to 0.02%.
- **Constant amplitude** is treated as one block of N runout cycles, so the same step control applies.
- **Fracture** is checked on every row: the run stops at the first cycle with $K_{max} \geqslant K_c$, at the crack size it has reached.

- **No growth.** If a full block (or the full runout) adds no growth, the run reports infinite life below threshold.

Results. The text box lists the stop reason, cycles, blocks, flight hours, final a and c, Kmax at the block peak stress, history points and run time.

- **Plot crack size vs cycles / vs hours**: a (and c) history.
- **Plot K vs crack size**: Kmax at the block peak stress against crack size, with Kc marked.
- **Export to workspace** creates `crack_growth_results` (N, a, c, blocks, hours, Nf, mode and more) and `crack_growth_model` (all inputs, including the processed block).
- **Export history CSV** writes Cycles, Blocks, Hours, a_in and c_in.
- **Save / Load model** stores every input, plus the raw spectrum, in a .mat file.

Materials library

The 51 built-in materials, listed alphabetically, are representative values for demonstration and teaching. They are not taken from any commercial materials database and are not design allowables. Replace them with your program's approved data before using results for design.

Material	UTS (ksi)	YS (ksi)	KIc (ksi√in)	C	n	p	q	ΔK0 (ksi√in)	α	Kc at t = 0.25 in
15-5PH H1025	155	145	90	3.0e-09	2.8	0.25	0.25	5	2.5	174.1
17-4PH H1150	135	105	100	3.0e-09	2.8	0.25	0.25	5	2.5	198.8
2014-T651 plate (L-T)	70	63	22	2.0e-08	3	0.5	1	2.4	1.9	33.2
2024-T3 sheet (L-T)	65	47	34	8.0e-09	3.2	0.5	1	2.8	2	83.2
2024-T351 plate (L-T)	68	50	34	8.0e-09	3.2	0.5	1	2.8	2	66.4
2050-T84 Al-Li plate (L-T)	74	69	30	1.0e-08	3	0.5	1	2.6	1.9	52.7
2099-T83 Al-Li extrusion (L-T)	80	74	27	1.0e-08	3	0.5	1	2.6	1.9	42.4

Material	UTS (ksi)	YS (ksi)	KIc (ksi√in)	C	n	p	q	ΔK0 (ksi√in)	α	Kc at t = 0.25 in
2124-T851 plate (L-T)	70	64	26	1.2e-08	3.1	0.5	1	2.6	1.9	44.0
2195-T8 Al-Li plate (L-T)	82	77	30	1.0e-08	3	0.5	1	2.6	1.9	49.4
2219-T87 plate (L-T)	63	51	30	1.0e-08	3	0.5	1	2.4	2	57.6
2524-T3 sheet (L-T)	63	45	37	6.0e-09	3.2	0.5	1	2.8	2	91.3
300M steel, 280 ksi UTS	280	240	55	6.0e-09	2.6	0.25	0.25	4	2.5	56.5
304 stainless, annealed	75	30	150	4.0e-09	3	0.25	0.25	6	2.5	300.0
4130 steel, 125 ksi UTS	125	100	90	5.0e-09	2.8	0.25	0.25	6	2.5	178.6
4340 steel, 200 ksi UTS	200	180	80	4.0e-09	2.7	0.25	0.25	5	2.5	141.9
4340 steel, 260 ksi UTS	260	217	45	6.0e-09	2.6	0.25	0.25	4	2.5	45.2
6013-T6 sheet (L-T)	56	52	28	1.2e-08	3	0.5	1	2.2	2	57.8
6061-T6 plate (L-T)	42	36	26	1.5e-08	3	0.5	1	2	2	51.1
7050-T7451 plate (L-T)	76	66	30	1.5e-08	3	0.5	1	2.5	1.9	53.7
7055-T7751 plate (L-T)	92	88	26	2.5e-08	3	0.5	1	2.4	1.9	33.0
7075-T6 sheet (L-T)	83	73	27	2.5e-08	3	0.5	1	2.4	1.9	50.7
7075-T651 plate (L-T)	82	72	27	2.5e-08	3	0.5	1	2.5	1.9	43.3
7075-T7351 plate (L-T)	70	58	30	1.5e-08	3	0.5	1	2.5	1.9	56.1
7085-T7651 plate (L-T)	82	76	32	1.3e-08	3	0.5	1	2.6	1.9	55.3

Material	UTS (ksi)	YS (ksi)	KIc (ksi√in)	C	n	p	q	ΔK0 (ksi√in)	α	Kc at t = 0.25 in
7150-T7751 plate (L-T)	89	83	26	2.0e-08	3	0.5	1	2.4	1.9	35.2
7175-T7351 forging (L-T)	74	63	32	1.3e-08	3	0.5	1	2.5	1.9	59.5
9Ni-4Co-0.30C steel	230	200	90	4.5e-09	2.7	0.25	0.25	5	2.5	160.5
A-286 (AMS 5737)	146	95	120	3.0e-09	3	0.25	0.25	6	2.5	239.5
A357-T6 casting	45	36	17	2.0e-08	3	0.5	1	2	2	30.9
AerMet 100	285	250	115	4.0e-09	2.7	0.25	0.25	4.5	2.5	207.0
AZ31B-H24 magnesium sheet	42	32	22	6.0e-08	3	0.5	1	1.8	2	43.0
AZ91E-T6 magnesium casting	40	21	11	8.0e-08	3	0.5	1	1.5	2	20.6
CP titanium Grade 2	50	40	60	1.5e-09	3.2	0.5	0.5	3.5	2.5	119.9
Custom 465 H950	245	230	70	3.5e-09	2.8	0.25	0.25	5	2.5	91.8
Haynes 188 cobalt, room temp	140	67	120	3.0e-09	3	0.25	0.25	6	2.5	239.9
Inconel 625 annealed, room temp	120	60	120	3.0e-09	3	0.25	0.25	6	2.5	239.9
Inconel 718, room temperature	200	170	90	2.5e-09	3	0.25	0.25	6	2.5	169.2
Inconel X-750, room temperature	175	115	100	2.5e-09	3	0.25	0.25	6	2.5	198.3
Maraging 250 (C250), aged	260	255	80	5.0e-09	2.7	0.25	0.25	4.5	2.5	108.5
PH13-8Mo H1000	215	205	80	3.0e-09	2.8	0.25	0.25	5	2.5	132.0
Ti-10V-2Fe-3Al STA	175	165	45	1.2e-09	3.3	0.5	0.5	3.5	2.5	52.4

Material	UTS (ksi)	YS (ksi)	KIc (ksi√in)	C	n	p	q	ΔK0 (ksi√in)	α	Kc at t = 0.25 in
Ti-15V-3Cr-3Sn-3Al STA	165	155	45	1.2e-09	3.3	0.5	0.5	3.5	2.5	56.0
Ti-5Al-2.5Sn ELI	115	105	70	1.0e-09	3.2	0.5	0.5	4	2.5	136.5
Ti-6Al-2Sn-4Zr-2Mo duplex annealed	145	135	55	1.0e-09	3.3	0.5	0.5	4	2.5	93.3
Ti-6Al-4V annealed plate	140	130	60	1.0e-09	3.3	0.5	0.5	4	2.5	108.1
Ti-6Al-4V ELI beta-annealed	125	115	75	8.0e-10	3.2	0.5	0.5	4.5	2.5	146.0
Ti-6Al-4V STA	160	145	40	1.2e-09	3.3	0.5	0.5	3.5	2.5	47.1
Ti-6Al-6V-2Sn STA	170	160	35	1.3e-09	3.3	0.5	0.5	3.5	2.5	35.4
Waspaloy, room temperature	185	115	100	2.5e-09	3	0.25	0.25	6	2.5	198.3
WE43-T6 magnesium casting	36	25	15	6.0e-08	3	0.5	1	1.8	2	28.9
ZK60A-T5 magnesium extrusion	50	40	26	5.0e-08	3	0.5	1	1.8	2	50.6

C is in in/cycle with ΔK in ksi√in. All entries share Ak = 1, Bk = 1 (1.5 for 2024-T3, 2524-T3 and 7075-T6 sheet; 1.2 for 6013-T6 sheet), Smax/σ0 = 0.3, Cth+ = 1.5, Cth− = 0.1, a0 = 0.0015 in and Walker γ = 0.5. Kc depends on thickness, so it is recomputed from the Geometry tab's t at run time.

Use LEFM with care for the low-yield, high-toughness entries: 304 stainless, CP titanium Grade 2, Inconel 625, A-286 and Haynes 188. Their KIc values are nominal, and the Kc formula reaches its 2 × KIc cap, so elastic-plastic fracture or net-section yield will usually govern before Kmax reaches Kc.

The four magnesium alloys grow roughly 5 times faster than aluminum at the same ΔK, mainly because of their lower modulus, and the AZ91E and WE43 castings have low toughness. Treat their constants as rough placeholders: magnesium crack growth data is

sparse and varies strongly with product form and environment.

To add a permanent material, add a row to `+vdfat/+mat/materials.csv` and put the reference in its `source` column. Every shipped row is marked `representative - not verified`.

Verification and limitations

The engine was checked against closed-form and published values before release. Details are in the worked examples deck.

Check	Result
Paris, constant amplitude, infinite plate, a = 0.05 → 1.0 in	21,528 cycles vs 21,524 closed form (0.02%)
Integration tolerance 0.005 vs 0.001 (FNK, center crack)	Same life to 0.00%
Semicircular surface crack, a/t = 0.01	F = 0.663 deepest, 0.729 surface (Newman-Raju)
Quarter-circular corner crack	F = 0.723 both tips
Edge crack, small a	β = 1.122
Rainflow, ASTM E1049 example sequence	Ranges and counts match the standard
Willenborg on constant amplitude	Within 0.5% of non-interaction

New-feature check	Result
Flat PSD, 1 ksi RMS: synthesized RMS	1.0000 ksi
Damage (m = 5) relative to synthesis	Dirlik 0.94, Rayleigh 1.25
Crack growth, 2.66 ksi RMS PSD (Example 6)	Dirlik 0.463 hr, synthesis 0.435 hr, Rayleigh 0.340 hr
Sine on random, 1 ksi RMS + 2 ksi tone	1.733 ksi RMS vs √3 = 1.732
Sweep 10–2000 Hz at 1 oct/min	172,258 cycles, matches closed form
Hysteresis gate, 1 ksi on a test sequence	Correct reversals kept

New-feature check	Result
Basquin and Goodman lives	Match hand calculations exactly
Neuber product $\Delta\sigma\Delta\varepsilon$	Equals $(K_t \Delta S)^2/E$ to 5 digits

Not included (vs commercial crack growth codes):

- Exact lug solutions with edge-distance effects (use a β table), bending for hole geometries, and holes off the plate centerline
- Elastic-plastic fracture beyond the FAD Option 1 curve (J-integral, Options 2 and 3)
- Stress gradients for hole geometries, and a check of the weight functions against published results or FEA
- A sourced materials database (the shipped constants are representative values)
- Multiaxial strain-life with local plasticity (SWT and Fatemi–Socie use elastic strains), thermomechanical fatigue; fretting critical distance and short-crack growth; spot weld methods; composites; roller bearing life; thermal and starved EHL

Tabular da/dN, stress gradients and residual strength

Three fracture features beyond the closed-form equations, each verified against an independent result where one exists.

Tabular da/dN. Choose **Table** as the growth equation and load a text file of three columns: R, ΔK, da/dN, one curve per R value. Rates are interpolated log-log in ΔK and linearly in R (in log rate). Below a curve's first ΔK the rate is zero; above its last point the last segment is extrapolated, and fracture still occurs at Kmax = Kc. Outside the R range the nearest curve is used, and for R < 0 with no negative-R curve ΔK is taken as Kmax. Monte Carlo scatter on C also scales the tabulated rates.

Stress gradients. On the Geometry tab, **Load stress gradient** reads two columns: distance x from the crack mouth, and stress divided by the applied stress S. x runs from the crack center (center crack), the plate edge (edge crack), or the loaded surface through the thickness (surface and corner cracks). The gradient replaces the bending ratio. A through-thickness gradient is carried as its peak value after breakthrough, as bending is.

Geometry	Weight function
Center crack	Exact infinite-plate solution times the Tada finite-width factor
Edge crack	Glinka universal form, fitted to Tada tension and in-plane bending
Surface and corner crack, both tips	Glinka universal forms, fitted to Newman-Raju tension and bending

The fitted forms reproduce their two reference solutions exactly, and the center crack matches the exact results for linear and quadratic face stress. Other profiles are not yet checked against published weight-function results or FEA.

Residual strength. The Residual Strength tab has a **Failure criterion** selector:

- **LEFM + net section**: Kmax = Kc, or net-section stress = (YS + UTS)/2. This is the previous behavior.
- **FAD (R6 / BS 7910 Option 1)**: Kr = K/Kc, Lr = net-section stress / YS, with the Option 1 curve and cut-off Lr max = (YS + UTS)/(2 YS).
- **R-curve tangency** (through cracks): load Δa and KR. Instability is the first maximum of KR(Δa)/K(a + Δa), and the stable tearing at that point is reported.

```
f(L_r) = \left(1 + \tfrac{1}{2}L_r^2\right)^{-1/2}\left[0.3 + 0.7\,e^{-0.6 L_r^6}\right], \quad L_r \le L_{r,\max}
```

The FAD uses the net-section stress as the reference stress, which simplifies BS 7910 for surface cracks. Check the curve and reference stress against the current edition of the standard before design use.

S-N fatigue

The **S-N Fatigue** tab defines one stress-life curve in stress amplitude, used by this tab and by the Vibration Fatigue and Superposition tabs. **Compute S-N damage** applies it to the Loading-tab block (or duty cycle) or to the last Rainflow-tab count, and reports damage per block and life in blocks, cycles and hours.

Input	Meaning
Basquin: Sf, b	Sa = Sf (2N)^b
Two points	Straight line on log-log through (Sa1, N1) and (Sa2, N2)
Knee N, slope beyond knee	Endurance limit at the knee (slope 0) or a second slope m2
Cutoff N	Cycles beyond which damage is zero
Mean stress correction	Goodman, Gerber (need Su), Morrow (uses Sf), SWT, Walker (γ)
Marin factors	kf = ka kb kc kd ke / Kf; allowable amplitude = kf × curve

The surface (ka), size (kb) and reliability (ke) factors are the usual textbook estimates for steels, entered from memory. Check them against your source, and prefer measured S-N data for the actual surface and size.

Vibration fatigue

The **Vibration Fatigue** tab has three panels.

Stress PSD fatigue. Compare spectral methods applies the S-N tab curve to the Loading-tab stress PSD with every method and lists damage and life side by side, relative to Dirlik.

Method	Cycle distribution	Rate
Dirlik	Exponential + two Rayleigh terms	Peak rate
Zhao-Baker	Weibull + Rayleigh mixture (method 1)	Peak rate
Tovo-Benasciutti	Mixture with the 2005 b coefficient	Between zero-crossing and peak rate
Lalanne	Rice distribution of positive peaks, each peak an amplitude	Peak rate
Narrowband	Rayleigh amplitudes	Zero-crossing rate
Steinberg	68.3% at 1σ, 27.1% at 2σ, 4.33% at 3σ	Zero-crossing rate

Dirlik, Zhao-Baker and Tovo-Benasciutti were checked against the FLife package. A **kurtosis** above 3 applies the Hermite non-Gaussian amplitude transform for a symmetric process; h4 is solved so the model kurtosis matches exactly.

Fatigue damage spectrum and ERS. Load a base acceleration PSD (G²/Hz) or copy the Loading-tab PSD. **Compute FDS & ERS** gives the pseudo-damage (G^b, amplitude, A = 1) of SDOF absolute response acceleration at each natural frequency for each b, the same convention as fds_psd_engine, and the expected largest peak $\sigma\sqrt{(2\ln(\nu_0 T))}$.

1. Compute the FDS of each environment and press **Add FDS to envelope**. Natural frequencies, Q, b and duration must match.
2. **Test PSD from envelope** finds the PSD of the test duration whose FDS covers the envelope for every b, starting from the Miles inversion and correcting six times. It is saved as test_psd.
3. **Compress to test time** scales the base PSD by (Tfield/Ttest)^(2/b), which keeps the FDS unchanged, and saves compressed_psd for the smallest b.

Smooth a derived test PSD into straight-line segments, and check compressed levels against the ERS and the hardware's strength.

SOL 111 transfer functions. Open a SORT1 frequency-response punch file, give the result header (ELEMENT STRESSES, ACCELERATION, ...) and the two words that hold magnitude and phase (or real and imaginary), and a scale to stress. **Response & damage per ID** forms |H|² × base PSD for every ID and ranks them by S-N damage; **Export table**

writes all IDs to CSV. The reader is checked on a real SOL 111 grid-acceleration file; element-stress word positions depend on the element type and are not yet checked on a real file.

Unit-load superposition

The **Superposition** tab computes damage element by element from a linear static FE run with one unit load per subcase. Each element's stress history is the sum over load channels of its unit-load stresses times that channel's load history; it is then rainflow counted and assessed with the S-N tab curve.

- **Unit-load stresses**: five columns per row — element ID, channel, sx, sy, txy (stress per unit load).
- **Load histories**: one column per channel, in channel order, optionally after a time column.
- **Stress measure**: critical plane on normal or shear stress, maximum absolute principal, signed von Mises, or sx.

The results table also gives the **biaxiality ratio** a = s2/s1 (0 uniaxial, −1 shear, +1 equibiaxial) and the spread of the principal angle. An element whose angle spread is 5° or more, or whose ratio varies by 0.1 or more, is marked non-proportional: use a critical-plane measure for it.

Multiaxial fatigue

The Multiaxial tab assesses 3D stress states, from a stress history or from stress transfer functions under a base-drive PSD. It uses the S-N curve, Marin factors and mean stress correction from the S-N Fatigue tab.

Stress history. Load stress history reads sx sy txy (plane stress) or sx sy sz txy tyz txz, one row per time point, optionally after a time column. The criteria are:

- **Critical plane: Findley.** On every plane and shear direction the shear stress is rainflow counted, and $f = \tau_a + k\,\sigma_{n,max}$ is converted to an equivalent uniaxial amplitude $2f / (k + \sqrt{(1 + k^2)})$, so a uniaxial history gives its uniaxial damage. k is about 0.2 to 0.3 for steels; k = 0 is Tresca shear. $\sigma_{n,max}$ is the largest normal stress on the plane during each counted cycle. Ver 1.3 took it over the whole history, which is weak for non-proportional loading; the $\sigma_{n,max}$ list still offers that choice.

- **Critical plane: SWT (strain-life).** On every plane the elastic normal strain is rainflow counted, and each cycle's Smith–Watson–Topper parameter σn,max εa gives the life from the Initiation tab strain-life constants (σf′, b, εf′, c). It suits tensile (Mode I) cracking.
- **Critical plane: Fatemi–Socie (strain-life).** On every plane and shear direction the shear strain γ = τ/G is rainflow counted, and γa (1 + kfs σn,max / Sy) gives the life, with shear constants estimated from the axial ones (τf′ = σf′/√3, γf′ = √3 εf′). kfs is about 0.5; Sy is the cyclic yield stress from K′ and n′ at 0.2 % strain. It suits shear cracking. Both use the Poisson's ratio on the tab for the elastic strains.
- **Critical plane: normal stress.** The normal stress on every plane is rainflow counted with the S-N tab mean stress correction. It suits brittle materials and tensile cracking.
- **Crossland and Sines (endurance).** One repeating cycle is checked against the fully reversed axial and torsion limits f-1 and t-1; Sines also uses the pulsating limit f0 (or Goodman from Su when f0 = 0). The result is an index (above 1 predicts finite life) and a safety factor.

Plane step sets the search grid (10° by default; the best plane is then refined at a quarter step). The results give the damage per pass, the life in passes, the critical-plane normal and its angles (θ from z, φ from x) and, for plane stress, the biaxiality. Plot damage on planes maps the damage over every orientation.

Stress PSD. Load stress transfer functions reads the frequency, then the real and imaginary parts of each stress component per unit input: 7 columns (sx sy txy) or 13 (sx sy sz txy tyz txz), for example from a SOL 111 base-drive run. The input is the base PSD on the Vibration Fatigue tab. The criteria are:

- **Von Mises (Preumont).** The equivalent von Mises stress PSD. It is positive only, so it suits zero-mean loading.
- **Critical plane: normal stress, and shear (Tresca).** The PSD of the stress on every plane; the most damaging plane is critical.

The spectral method and exposure time are set on the tab.

Samples. samples/multiaxial_history.txt (out-of-phase bending and torsion) and samples/stress_frf_3d.txt (two modes; load base_psd_flight_a.txt as the base PSD).

Checks and limits. The tests confirm that a uniaxial history gives its uniaxial damage under every criterion, that a rotated stress state gives the same damage and finds the load axis, that pure torsion gives 2τ with k = 0, and that Crossland and Sines return an

index of 1 at their defining limits. The stress-PSD method takes one input (base drive); there is no Dang Van criterion; and SWT and Fatemi–Socie use elastic strains from the stresses (no local plasticity).

Contact fatigue

The **Contact** tab checks rolling contact fatigue (gear pitting, bearing and roller spalling): the Hertz contact is rolled over a material point, and the stress history at each depth is assessed. Body 1 is analyzed; x is the rolling direction.

Contact	Inputs	Hertz solution
Line: parallel cylinders	E, ν of both bodies, R1, R2 (0 = flat, negative = concave), load per length, traction μ	Half-width a, peak pressure p0; McEwen field with traction q = μ p
Line: spur gear at the pitch point	Pitch diameters, pressure angle, Wt, face width	R = (d/2) sin φ, w = Wt / (F cos φ), then as above
Point: spheres	R1, R2, force	Circular radius a; Huber closed form
Ellipse: general	Radii in the rolling plane (R1, R2) and across it (R1 across, R2 across), force	Semi-axes a (along x) and b (across) from the elliptic-integral equations; stresses by Boussinesq integration
Ellipse: ball bearing	Ball diameter, pitch diameter, groove conformity, contact angle, inner or outer race, ball load	Race radii (Harris), then as the general ellipse

Per depth the tab reports the largest Tresca shear, the orthogonal shear amplitude (half range of τxz) and the largest von Mises stress, with the worst value and its depth. Residual stresses sx and sy (constant with depth; negative = compressive) are added to the history.

Dang Van. With the fully reversed bending and torsion fatigue limits f-1 and t-1, the mesoscopic shear (Tresca of the deviatoric stress less the centre of its smallest enclosing sphere) and the hydrostatic stress σh give

$$\max_t\left[\hat\tau(t) + \alpha_{DV}\,\sigma_h(t)\right] \le \beta_{DV}, \qquad \alpha_{DV} = \frac{3t_{-1}}{f_{-1}} - \frac{3}{2}, \qquad \beta_{DV} = t_{-1}$$

An index above 1 predicts finite life (pitting or spalling); the Dang Van diagram plots the path at the critical depth against the limit line. **Findley life** (optional, slower) applies the Findley critical plane to the history at each depth with the S-N tab curve and the Multiaxial tab k.

EHL film. With viscosity (cP), pressure-viscosity coefficient α, entrainment speed and the two rms roughnesses, the tab gives the minimum and central film thickness (Dowson-Higginson for line contact, Hamrock-Dowson for point and ellipse) and λ = hmin / sqrt(Rq1² + Rq2²): below 1 boundary lubrication (surface distress, micropitting), 1 to 3 mixed, above 3 full film. The film check does not change the stresses; it shows whether the smooth-surface analysis applies.

Checks. Line contact matches the Flamant integral (1e-5 p0) and Johnson's 0.300 p0 at 0.786a; point contact matches the axis closed form and gives 0.31 p0 at 0.48a; the ellipse reduces to the circle (1e-4 p0) and tends to the line contact, and its pressure satisfies the surface gap condition; Dang Van reproduces f-1 and t-1 exactly. Example: a 0.5 in ball on an inner race (dm = 2.5 in, conformity 0.52, 300 lbf) gives p0 = 299 ksi, b/a = 9.3, Tresca 0.315 p0 at 0.79a.

Limits. Elastic, smooth half-spaces in steady rolling with Hertz pressure; point and elliptical contacts are frictionless and evaluated on the track centreline; ellipse depths start at 0.05 of the smaller semi-axis; gears are checked at the pitch point only (apply load-sharing and dynamic factors to Wt); enter the ball load yourself here (the Bearing Life tab splits a bearing load among the balls).

Fatigue map of a finite-element model

The **FE Map** tab assesses every element of a finite-element model and draws the damage on the mesh. Shells are checked at both fibres (Z1, Z2) and each element keeps its worse surface. The S-N curve and mean stress correction come from the S-N tab.

1. **Load mesh (Nastran BDF)** reads GRID (with CORD2R/C/S systems), CQUAD4, CTRIA3, CQUAD8, CTRIA6, CHEXA, CPENTA, CTETRA, CBAR, CBEAM and CROD in small, large or free field, with INCLUDE files. The mesh is only needed for the contour plot and the VTK export.

2. **Stresses: Nastran punch** reads SORT1 element stresses of CQUAD4 and CTRIA3 at the centroid: SOL 101 output (one subcase per load channel, in subcase order) or SOL 111 output (stress per unit base input, real/imaginary or magnitude/phase). Set **Scale to ksi** (0.001 for a model in psi). **Stresses: table** reads the same data as text, and solids with all six components.
3. Choose the analysis and run:
   - **Static:** stress history = sum of unit-load stresses x load histories (the Superposition method); measure = critical plane (normal or shear), max abs principal or signed von Mises; gate and passes.
   - **Random:** stress cross-PSD from the transfer functions and the base PSD (the Multiaxial method); criterion = von Mises stress PSD, or critical plane normal or shear; spectral method and exposure time. For shells the normal-stress critical plane is searched in the plane of the shell, about 140 times faster than the full search and within 0.4% of it.
4. **Plot** draws log damage or log life on the mesh (the worst element starred) or a bar chart of the highest-damage elements. **Export CSV** writes every element; **Export VTK** writes the mesh with damage, life and surface as cell data for ParaView.

Input	Columns or cards
Static table	ID, channel, sx, sy, txy (+ surface 1 or 2), stress per unit load
Random table	ID, surface, f, then Re and Im of sx, sy, txy (9 columns) or of six components (15 columns)
SOL 101 deck	STRESS(PUNCH,SORT1,CENTER) = ALL, one subcase per load channel
SOL 111 deck	STRESS(PUNCH,SORT1,REAL,CENTER) = ALL, unit base input

Samples. samples/fe_plate.bdf (a cantilever plate, 76 CQUAD4 + 8 CTRIA3) with fe_unit_stresses.txt and load_histories.txt (static), or fe_stress_frf.txt and base_psd_flight_a.txt (random). fe_plate_sol101.dat, fe_plate_sol111.dat and fe_plate_sol111_small.dat are Nastran decks for the same plate.

Checks. Each element reproduces the single-location result (Superposition, multiaxial PSD). The punch reader is checked on Simcenter Nastran runs of the sample decks: SOL 101 CQUAD4 stresses (mid-span sx = 1425 psi, as beam theory gives) and SOL 111

CQUAD4 and CTRIA3 stresses, including the WITH VON MISES layout. Magnitude/phase output and MSC Nastran files follow the same rules but have not been run.

Limits. Element stresses are used as given (centroid; no averaging or extrapolation). Static analysis is plane stress. Corner (BILIN) punch output is not read; use centroid output. There is no Femap or Patran results file: import the CSV.

Seam weld fatigue

The **Seam Weld** tab assesses fatigue at each point along a weld toe under load histories. You give the stress per unit load of each load channel at each weld point, from your model; the stress history at a point is the sum of unit stress x load history (the Superposition method), counted by rainflow, with a Miner sum per pass.

Method	Table input (per point and channel)	S-N curve
Structural stress (Dong, master S-N)	[point channel sm sb] membrane and bending stress at the toe, or [point channel x y z Fn Mb] nodal force and moment from a shell model	Master S-N curve, -3 SD design (or mean, +/-2 SD, +/-3 SD)
Hot-spot stress (IIW)	[point channel s1 s2 (s3)] surface stresses at the read-out points of the chosen rule	FAT 90 or 100, thickness correction (25 mm/t)^n
Effective notch stress (IIW)	[point channel sn] stress at a 1 mm notch radius	FAT 225

Structural stress. Nodal forces F and moments along the weld are converted to line force f and line moment m by work equivalence (a linear distribution between nodes), so the result does not depend on the mesh: sm = f/t, sb = 6m/t^2. For each rainflow cycle the membrane and bending ranges give the bending ratio Rb and the equivalent structural stress range:

```
\Delta S_s = \frac{\Delta\sigma_s}{t^{*(2-m)/(2m)}\,I(R_b)^{1/m}}, \qquad
\Delta S_s = C\,N^{-0.3195}, \qquad m = 3.6, \; t^* = t/1\,\mathrm{mm}
```

C = 19930.2 MPa (mean), 13875.8 (-2 SD) and 11577.9 (-3 SD, the design curve). The constants and the I(Rb) polynomial are those tabulated from ASME BPVC VIII-2 in Metals 11 (2021) 1952; the code text itself was not checked, and the ASME thickness limits are not applied.

Hot-spot and notch. The hot-spot stress is extrapolated to the toe from read-out points on the plate surface: type a fine (0.4t, 1.0t), type a fine quadratic (0.4t, 0.9t, 1.4t), type a coarse (0.5t, 1.5t), type b fine (4, 8, 12 mm) and type b coarse (5, 15 mm), per Hobbacher's IIW recommendations. The IIW curve has slope 3 to a knee at 1e7 cycles, then m2 = 22 (variable amplitude), Haibach or a fatigue limit, with the IIW residual stress f(R) factor.

Outputs. A table of damage per block, life in passes, the largest stress range and Rb at each point; plots of damage and stress range along the weld and the S-N curve; CSV and workspace export.

Samples and checks. samples/weld_structural_stress.txt, weld_nodal_forces.txt (the same weld as nodal forces) and weld_hotspot_readout.txt, with load_histories.txt. The tests check that the extrapolation rules are exact for linear or quadratic fields (within the published rounding of 1.67 and 0.67), that work equivalence recovers a linear line force exactly, the master curve constants and thickness and bending effects, and hand calculations of the damage. The nodal-force sample reproduces the stress-table damage to 1e-6.

Limits. The stresses come from your model as tables; the app does not find weld lines or read grid point forces from Nastran files. No spot welds, weld root checks or multiaxial (Mode III) weld criteria.

Bearing life (ISO 281)

The **Bearing Life** tab gives the ISO 281 basic and modified rating life of a radial or thrust ball bearing over a duty cycle of load cases. Enter the catalogue ratings (C, C0 and the fatigue load limit Cu), the equivalent-load factors X, Y and e, the geometry (pitch diameter, ball diameter, number of balls, contact angle and groove conformities) and the contamination factor ec. Then fill the duty-cycle table (time fraction, speed in rpm, Fr, Fa and the lubricant viscosity at the operating temperature) and press **Calculate**. The defaults are a 6205-size deep groove ball bearing with representative values. Use the catalogue values for your bearing.

```
L_{10} = \left(\frac{C}{P}\right)^3 10^6\,\mathrm{rev}, \qquad L_{nm} =
a_1\,a_{ISO}\,L_{10}, \qquad \frac{1}{L_h} = \sum_i \frac{t_i}{L_{h,i}}
```

$$a_{ISO} = 0.1\left[1 - \left(2.5671 - \frac{A}{\kappa^{B}}\right)^{0.83}\left(\frac{e_C C_u}{P}\right)^{1/3}\right]^{-9.3} \le 50, \qquad \kappa = \frac{\nu}{\nu_1}$$

Inputs. The **Reliability** list sets a1 from the ISO 281 table: 1 at 90 % down to 0.077 at 99.95 %. The **Cleanliness** list puts the middle of each ISO guide range into ec, which depends on whether the pitch diameter is below 100 mm. Typing ec switches the list to User ec. The reference viscosity ν1 comes from the speed and pitch diameter, and κ above 4 is taken as 4. Thrust ball bearings use P = Fa and ec Cu / 3P, and the tab disables X, Y and e for them.

Outputs. For each case the results table lists P, L10h, ν1, κ, aISO, Lnm in hours, the heaviest ball load Qmax (Stribeck, rigid rings, zero clearance) and the Hertz peak pressure on the inner and outer race. The text area gives the duty-cycle L10h and Lnm in hours and in millions of revolutions, plus the static safety factor s0 = C0/P0. It also warns when P > 0.5 C, s0 < 1, the contact stress exceeds 4200 MPa or aISO reaches 50. The plots are life per case, the aISO chart with each case's operating point, and the ball loads around the bearing.

Checks. The tests reproduce the a1 table, check aISO for continuity at κ = 0.4 and 1 and against its limits, check ball-load equilibrium and Stribeck's 4.37 factor, and check that the duty-cycle life equals the cubic-mean-load result. With the 6205 catalogue values, 5 C0/Z and 5 Cu/Z give 4192 and 1468 MPa through the Hertz solution, close to the 4200 MPa (ISO 76) and 1500 MPa bases of those ratings.

Limits. Ball bearings only. aISO uses the compact form of the ISO 281:2007 equations from Machines 12 (2024) 444, and a1 is the table published from ISO 281 / JIS B 1518. The standard itself was not checked. ec is entered rather than computed from filter ratings. The life does not use the contact stresses. The method does not cover water in the lubricant, wear, misalignment or speeds above the limiting speed.

Mission synthesis (FDS and ERS)

The **Mission Synthesis** tab turns a life-cycle profile into one random vibration test, following Lalanne (Mechanical Vibration and Shock, Vol. 5, ch. 11). Add each event of the life cycle with its type and number of repeats:

- **Random (base PSD):** a PSD file [f G²/Hz] or the Vibration Fatigue tab base PSD, and its duration.
- **Sine dwell:** frequency, amplitude (G, zero to peak) and duration.

- **Sine sweep:** segments [f1 f2 A; …], log (octaves/min) or linear (Hz/min) rate, and the number of passes.

Compute reference gives each event's fatigue damage spectrum (FDS) and extreme response spectrum (ERS) on the Vibration Fatigue tab basis: SDOF absolute acceleration, pseudo-damage in G^b. The reference FDS is the **sum** of the event FDSs (events in series accumulate damage) or, if you choose it, their envelope; the reference ERS is the envelope of the event ERSs.

```
D_{ref} = \sum_e r_e\,D_e, \qquad D_{sine} =
f\,T\left(\left|H\right|A\right)^b, \qquad D_{sweep} = n_{pass}\int
f\left(\left|H\right|A\right)^b\,\frac{dt}{df}\,df
```

Synthesize test PSD finds, for each b, the random PSD whose FDS over the test time equals the reference (Lalanne eq. [11.11]), and takes their envelope as the specification. It then compares the test ERS with the reference ERS. Below the reference, the test does not reach the largest field response (shorten it); above, it over-tests the peak response (lengthen it, or accept it). Plots show the FDS of each event with the reference and the test, both ERSs, the test PSD and the FDS ratio. **Export test PSD** writes [f G²/Hz].

Checks. Lalanne Vol. 5 eq. [4.9] (white noise, narrowband) is reproduced to 0.07 %; Example 4.3 gives a damage ratio of 15.33 against the book's asymptotic 16 and a narrowband share of 6.5 %; the sweep FDS agrees with a time-domain swept sine through the SDOF filter and rainflow counting within 0.5 %.

Limits. Sweeps are quasi-steady (slow next to the resonance build-up). Random events are Gaussian and stationary. The test PSD is defined on the fn grid, so the match is looser (about 5 %) at the two highest natural frequencies. No test or uncertainty factors, and no shock (SRS) check yet. Units are G and Hz.

Fretting fatigue

The **Fretting** tab analyzes a cylindrical pad pressed on a flat specimen (Hertz line contact, same material) under an oscillating tangential load Q in phase with a bulk stress in the specimen. The contact is in partial slip (Mindlin–Cattaneo, with the bulk-stress offset of Hills and Nowell): a stick zone of half-width c, shifted by e, that reverses as Q goes from Qmax to Qmin.

```
\frac{c}{a} = \sqrt{1 - \frac{\left|Q\right|}{\mu P}}, \qquad \frac{e}{a} =
\frac{\sigma_B}{4\mu p_0}
```

Outputs. The stick and slip zones at Qmax and Qmin, the slip amplitude across the contact, the peak surface stress, and the Ruiz parameter F2 = max σx · max(|q| δ), whose peak marks the likely crack site. With **SWT and Fatemi–Socie life maps** ticked, the stress history at every point of a map under the contact is assessed on the critical plane with the Initiation tab strain-life constants, and the shortest life, its location and its plane angle are reported. The plots are the shear traction with the stick zones, the surface stress and slip, the Ruiz parameter, the two life maps and the stress history at the hot spot.

Checks. The tab reproduces the closed-form stick zone c/a and offset e/a; the engine tests (FrettingTest, 23 tests) cover traction equilibrium, the stick condition, the trailing-edge stress and the Hertz and full-sliding limits.

Limits. Elastic, plane strain, pad and specimen of one material, constant normal load, Q and the bulk stress in phase, Coulomb friction with one coefficient. No gross slip, wear or plasticity. The life maps take one cycle per point and search planes containing the out-of-plane axis. There is no critical-distance (gradient) correction, so surface hot-spot lives are conservative for small pads, and crack growth from the hot spot is not included. Not yet checked against published fretting test data.

Fitting FNK constants

On the Material tab, **Fit FNK to da/dN data** reads three columns (R, ΔK, da/dN) and fits C, n, p, q and ΔK0 by least squares on log da/dN. The closure constants (α, Smax/σ0) and the toughness fit are kept, and the plot shows data and fit at each R. Type a name and press **Save as library material** to append it to materials.csv with status "fitted - check" and the data file as its source.

Saving, loading and reports

Save model and **Load model** on the top bar store every tab's inputs and loaded data, in English units whatever units are on screen. **Report** writes an HTML page with the material, geometry and loading, every results panel that has output, the superposition ranking, and the plots, and opens it in the browser; print it to PDF from there.

Units

The **Units** selector at the top right switches between English (ksi, in) and Metric (MPa, mm). Fields, results text, plots and the .spc and CSV exports follow it. The engine always runs in English units, so a case gives the same life in either system (Example 1: 16,347 cycles).

Quantity	English	Metric
Stress	ksi	MPa
Length, crack size	in	mm
K, ΔK0	ksi√in	MPa√m
da/dN	in/cycle	mm/cycle
C (FNK, Paris)	in/cycle, ksi√in	mm/cycle, MPa√m

- C converts with the exponent, C_SI = 25.4 C / 1.0988^n, so set n before C when typing metric constants.
- File value × scale factor is in the selected units. Switching units converts the scale factor only when data is already loaded.
- Model files and workspace exports are always in English units. The weld S-N inputs stay in MPa and mm.

Scripting and tests

The calculations live in the `+vdfat` package, so they run without the GUI. `vdfat.crack.model` builds a run and `vdfat.crack.run` returns the result.

```
G   = struct('type','CC','W',4,'t',0.25);
M   = vdfat.crack.model('7075-T651 plate (L-T)', G, [20 2 1e8], 'ai', 0.05);
res = vdfat.crack.run(M);   % res.Nf = 16,347 cycles (Example 1)
```

Package	Contents
`vdfat.crack`	Growth laws, K solutions, integrators, Willenborg, residual strength, Monte Carlo
`vdfat.cycle`	Turning points, gate, rainflow, half cycles

Package	Contents
`vdfat.psd`	PSD statistics, Dirlik and Rayleigh, synthesis, sine sweep
`vdfat.fatigue`	Initiation and weld S-N
`vdfat.io`	Nastran punch files
`vdfat.mat`	Material library from `materials.csv`

The Verification tables above are automated tests (`tests/VerificationTest.m`), with a GUI smoke test that presses the main buttons. Run them headless in R2026b with `run_tests` .