vibrationdata_fluids: Equations

vibrationdata_fluids.mlapp by Tom Irvine

Twenty-six tabs cover forty equations of practical fluid mechanics, from hydrostatics, pipes, pumps and valves to vibration in fluids and convective heat transfer. Each tab has English or metric units, its equations in a strip at the top of the tab, and a results panel. This document gives the equations, assumptions and references for each tab.

The forty equations

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Units

Quantity	*English*	*metric*
Length, head, depth	ft	m
Diameter, roughness	in	mm
Velocity	ft/s	m/s
Pressure	psi	kPa
Flow (pipes), (channels)	gpm, ft^3/min, ft^3/s	L/s, m^3/h, m^3/s
Density	lbm/ft^3	kg/m^3
Kinematic viscosity	ft^2/s	m^2/s
Force, power	lbf, hp	N, kW
Modulus	psi	GPa
Volume, mass	ft^3, US gal, lbm	m^3, L, kg
Thermal conductivity	BTU/hr-ft-F	W/m-K
Convection coefficient	BTU/hr-ft^2-F	W/m^2-K

Changing the units converts the entered values. Calculations are done in SI with $g = 9.80665$ m/s^2. Fluid presets: water and seawater at 20 C, air at 15 C and 1 atm; heat transfer tab: water at 20 and 60 C, air at 20 and 100 C (Ref. 22).

1. Hydrostatic Pressure (Refs. 1, 2)

Eq. 1: $p = p_0 + \rho g h$ $\qquad p = p_0 + \sum_i \rho_i \, g \, h_i$ $\qquad \rho_i = SG_i \, \rho_w$ $\qquad p_{abs} = p_{gauge} + p_{atm}$

$h_w = \dfrac{p - p_0}{\rho_w \, g}$ $\qquad$ Eq. 29: $\dfrac{F_2}{A_2} = \dfrac{F_1}{A_1}, \quad s_2 = s_1 \, \dfrac{A_1}{A_2}$

p_0 = pressure at the free surface (gauge), h_i = thickness of layer i, SG_i = specific gravity, g = 9.80665 m/s^2.

Static fluid, layers listed from the top. h_w = equivalent head of water at 4 C, ρ_w = 1000 kg/m^3 = 62.43 lbm/ft^3.

Hydraulic press (Pascal's principle, Eq. 29): pressure applied to a confined liquid is transmitted undiminished. Pistons at the same level, no friction or leakage; $F_1 s_1 = F_2 s_2$ (force is gained, work is not).

Inputs: p0 surface, gauge, p atmospheric, Number of layers, F1 input force, D1 input piston, D2 output piston, s1 input stroke.

2. Submerged Surface (Refs. 1, 2)

Eq. 2: $F = \rho g h_c A = p_c A$ $\qquad y_{cp} = y_c + \dfrac{I_{xc}}{y_c A}$ $\qquad h_c = y_c \sin\theta$

rectangle $A = bh, \; I_{xc} = \dfrac{bh^3}{12}$ $\qquad$ circle $A = \dfrac{\pi d^2}{4}, \; I_{xc} = \dfrac{\pi d^4}{64}$ $\qquad M_{top} = F \, (y_{cp} - y_{top})$

y = distance along the plate from the free-surface line, θ = plate angle from horizontal, h_c = centroid depth,

I_{xc} = second moment of area about the centroidal axis parallel to the surface. Gauge pressure (free surface at atmosphere).

Inputs: Density, b width (rect.), h height or dia., Depth of top edge, Angle from horiz. (deg).

3. Buoyancy & Stability (Refs. 1, 14)

Eq. 3: $F_B = \rho g V$ Eq. 4: $GM = KB + BM - KG$ $BM = \dfrac{I_{wp}}{V} = \dfrac{B^2}{12\,T}$ $T = \dfrac{W}{\rho g L B}$

$KB = \dfrac{T}{2}$ $T_{roll} \approx \dfrac{2\pi k}{\sqrt{g\,GM}}$

Archimedes: buoyancy = weight of the displaced fluid. Barge: rectangular box, upright, small heel angles (metacentric height).

T = draft, K = keel, B = center of buoyancy, G = center of gravity, k = roll radius of gyration, including added inertia (blank: $k = 0.4\,B$; typical 0.35 to 0.45 B).

Inputs: Density, W weight, V body volume, L barge length, B barge beam, Hull depth, KG, CG above keel, k roll gyration.

4. Continuity & Bernoulli (Refs. 1, 2)

Eq. 5: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2, \quad Q = AV$ Eq. 6: $p_1 + \dfrac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \dfrac{1}{2}\rho V_2^2 + \rho g z_2 + \rho g h_L$

$H = \dfrac{p}{\rho g} + \dfrac{V^2}{2g} + z$ $A = \dfrac{\pi D^2}{4}$ $\dot{m} = \rho Q$

Steady, incompressible flow along a streamline. $h_L = 0$ gives the ideal Bernoulli equation.

H = total head. A computed p_2 below vacuum means the flow is not possible (the liquid cavitates first).

Inputs: Density, Q flow, D1 inlet, D2 outlet, p1 inlet, gauge, z1 elevation, z2 elevation, hL head loss.

5. Pitot Tube (Refs. 2, 5)

Eq. 7: $V = \sqrt{\dfrac{2\,(p_0 - p)}{\rho}}$ $\qquad M = \sqrt{\dfrac{2}{\gamma - 1}\left[\left(\dfrac{p_0}{p}\right)^{(\gamma-1)/\gamma} - 1\right]}$ $\qquad \rho = \dfrac{p}{RT}$

$M > 1$: $\dfrac{p_{02}}{p_1} = \left(\dfrac{\gamma+1}{2}M^2\right)^{\gamma/(\gamma-1)} \left(\dfrac{\gamma+1}{2\gamma M^2 - \gamma + 1}\right)^{1/(\gamma-1)}$ $\qquad a = \sqrt{\gamma RT},\ V = Ma$

$$T = \frac{T_0}{1 + \frac{\gamma-1}{2}M^2}$$

$p_0 - p$ = Pitot-static differential, p = static pressure (absolute), γ = ratio of specific heats, R = gas constant.

Subsonic: isentropic. $p_0/p \geq ((\gamma+1)/2)^{\gamma/(\gamma-1)}$ (1.893 for air): Rayleigh Pitot formula, normal shock ahead of the probe.

Inputs: p0 - p, Pitot dp, p static, abs, Temperature, gamma, R, rho liquid.

6. Orifice & Venturi (Refs. 1, 4)

Eq. 8: $Q = C_d\,A_2\sqrt{\dfrac{2\,\Delta p}{\rho\,(1-\beta^4)}}$ $\qquad \beta = \dfrac{d}{D}$ $\qquad A_2 = \dfrac{\pi d^2}{4}$ $\qquad \Delta p = \dfrac{\rho\,(1-\beta^4)}{2}\left(\dfrac{Q}{C_d\,A_2}\right)^2$

orifice loss $\approx (1 - \beta^{1.9})\,\Delta p$ $\qquad Re_D = \dfrac{V_1 D}{\nu}$

Typical C_d: sharp-edged orifice 0.61, classical venturi 0.985 (0.984 to 0.995), flow nozzle 0.97 (0.95 to 0.99). For metering, use the C_d correlations of ISO 5167

(functions of Re_D and β) and the pressure tap locations of the standard.

Inputs: Density, Kin. visc., D pipe, d throat or bore, Cd discharge coef., dp differential, Q flow.

7. Reynolds Number (Refs. 1, 2)

Eq. 9: $Re = \dfrac{\rho V D}{\mu} = \dfrac{V D}{\nu}$ pipe: laminar $Re < 2300$, turbulent $Re > 4000$

$f = \dfrac{64}{Re}$ (laminar) $\dfrac{L_e}{D} \approx 0.06\, Re$ (laminar) $\dfrac{L_e}{D} \approx 4.4\, Re^{1/6}$ (turbulent)

plate $\delta = \dfrac{5.0\, x}{\sqrt{Re_x}}, \dfrac{0.37\, x}{Re_x^{0.2}}$

$\nu = \mu/\rho$ = kinematic viscosity, L_e = entrance length, δ = boundary layer thickness (laminar, turbulent; transition near $Re_x = 5 \times 10^5$).

Cylinder regimes (approximate): vortex street from $Re \approx 40$, subcritical ($St \approx 0.2$) to 2×10^5, drag crisis (critical) 2 to 5×10^5, supercritical to 3.5×10^6.

Inputs: Kin. visc., V velocity, Q flow (V blank), D length.

8. Pipe Friction (Refs. 2, 6, 7)

Eq. 10: $h_f = f\, \dfrac{L}{D}\, \dfrac{V^2}{2g}$ Eq. 11: $\dfrac{1}{\sqrt{f}} = -2 \log_{10} \left(\dfrac{\varepsilon/D}{3.7} + \dfrac{2.51}{Re\sqrt{f}} \right)$

Haaland $\dfrac{1}{\sqrt{f}} \approx -1.8 \log_{10} \left[\left(\dfrac{\varepsilon/D}{3.7} \right)^{1.11} + \dfrac{6.9}{Re} \right]$ $\Delta p = \rho g h_f$ $f = \dfrac{64}{Re}$

Darcy-Weisbach with the Darcy friction factor f (four times the Fanning factor). Colebrook is solved by iteration from the Haaland value.

Roughness ε (typical): drawn tubing 0.0015 mm, commercial steel 0.045 mm, galvanized 0.15 mm, cast iron 0.26 mm, concrete 0.3 to 3 mm.

Inputs: Density, Kin. visc., Q flow, D inside dia., L length, Roughness.

9. Pipe System & Pump (Refs. 1, 3)

Eq. 12: $h_m = \sum K \, \frac{V^2}{2g}$ Eq. 13: $h_p = (z_2 - z_1) + \frac{p_2 - p_1}{\rho g} + h_f + h_m$ $P_{fluid} = \rho g Q h_p$

$P_{shaft} = \frac{\rho g Q h_p}{\eta}$ $h_f = f \, \frac{L}{D} \, \frac{V^2}{2g}$

Energy equation between the source and discharge free surfaces ($V \approx 0$ there); the exit loss $K = 1$ is a fitting.

K values in the table are typical textbook values and vary with size and design: use manufacturer or Crane TP-410 data.

Inputs: Density, Kin. visc., Q flow, D inside dia., L length, Roughness, z1 source, z2 discharge, p1 source, gauge, p2 disch., gauge, Pump efficiency (%).

10. Pump Affinity Laws (Ref. 13)

Eq. 14: $\frac{Q_2}{Q_1} = \frac{N_2}{N_1}\frac{D_2}{D_1}$ $\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$ $\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^3$

similar $\frac{Q_2}{Q_1} = \frac{N_2}{N_1}\left(\frac{D_2}{D_1}\right)^3$ $\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5$ $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$

Speed change or impeller trim (same pump), and geometrically similar pumps. Efficiency is assumed unchanged.

N_s in US units (rpm, gpm, ft); the dimensionless form is $\omega\sqrt{Q}/(gH)^{3/4}$. Trim beyond about 20 percent is less accurate.

Inputs: Q1 flow, H1 head, P1 shaft power, N1 speed (rpm), N2 speed (rpm), D1 impeller, D2 impeller, SG2 / SG1.

11. Momentum & Jet Force (Refs. 1, 2)

Eq. 15: $\sum \vec{F} = \dot{m}\,(\vec{V}_2 - \vec{V}_1)$ bend $R_x = \dot{m}\,(V_2\cos\theta - V_1) - p_1A_1 + p_2A_2\cos\theta$

$R_y = (\dot{m}V_2 + p_2A_2)\sin\theta$ vane $F_x = \rho A(V-u)^2(1-\cos\theta)$ $F_y = \rho A(V-u)^2\sin\theta$

Steady control volume. Bend: R = force of the bend on the fluid, the fluid load on the bend is $-R$; gauge pressures, horizontal,

p_2 from Bernoulli without loss. Vane: moving at u along the jet, free jet at atmospheric pressure, no friction.

Inputs: Density, Q flow, D1 inlet or jet, D2 outlet (bend), Turn angle (deg), p1 gauge (bend), u vane speed.

12. Drag & Lift (Refs. 2, 12)

Eq. 16: $F_D = \frac{1}{2}\rho V^2 C_D A$ $F_L = \frac{1}{2}\rho V^2 C_L A$ $q = \frac{1}{2}\rho V^2$

terminal $V_t = \sqrt{\dfrac{2\,(W-\rho g V_b)}{\rho\, C_D\, A}}$ $Re = \dfrac{V\,L}{\nu}$ $P = F_D\,V$

A = reference (frontal) area for C_D. Typical C_D at $Re = 10^4$ to 10^5: sphere 0.47, long cylinder 1.2, square plate 1.17, cube 1.05, streamlined body 0.04. C_D depends on Re, roughness and shape: check the value for the case.

Inputs: Density, Kin. visc., V velocity, A frontal area, CD drag coef., CL lift coef., L length for Re, W weight, Body volume.

13. Vortex Shedding (Refs. 10, 11)

Eq. 17: $f_s = \dfrac{St\,V}{D}$ $\qquad V_r = \dfrac{V}{f_n D}$ $\qquad V_{cr} = \dfrac{f_n D}{St}$ $\qquad F_L' = C_L\,\dfrac{1}{2}\rho V^2 D$ $\qquad Sc = \dfrac{4\pi\, m\,\zeta}{\rho D^2}$

lock-in $f_s \approx f_n$, in-line $2f_s \approx f_n$

$St \approx 0.2$ for a circular cylinder, about $300 < Re < 2 \times 10^5$. m = mass per unit length, ζ = damping ratio.

The app flags f_s (or $2f_s$) within 20 percent of f_n. Sc as in EN 1991-1-4 ($2\delta_s m/\rho b^2$, $\delta_s = 2\pi\zeta$).

Inputs: Density, Kin. visc., V velocity, D diameter, Strouhal number, fn natural freq. (Hz), m mass/length, Damping ratio, CL lift coef..

14. Compressible Flow (Ref. 5)

Eq. 18: $a = \sqrt{\gamma R T},\ M = \dfrac{V}{a}$ $\qquad \dfrac{T_0}{T} = 1 + \dfrac{\gamma - 1}{2} M^2$ $\qquad \dfrac{p_0}{p} = \left(\dfrac{T_0}{T}\right)^{\gamma/(\gamma-1)}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(2\gamma-2)} \qquad \text{shock } M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

Perfect gas, isentropic flow. $\rho_0/\rho = (T_0/T)^{1/(\gamma-1)}$. Air: $\gamma = 1.4$, $R = 287.05$ J/kg-K = 1716 ft-lbf/slug-R. CO_2: γ at 25 C.

For $M > 1$ the app gives the Mach angle $\sin^{-1}(1/M)$ and the normal shock M_2, p_2/p_1, T_2/T_1 and p_{02}/p_{01} (NACA 1135).

Inputs: M Mach number, V (if M blank), T static, p static, abs, gamma, R.

15. Water Hammer (Ref. 8)

Eq. 19: $\Delta p = \rho\, a\, \Delta V$ $\qquad a = \sqrt{\dfrac{K/\rho}{1 + c_1 \frac{KD}{Ee}}}$ $\qquad T_r = \dfrac{2L}{a}$ $\qquad \Delta h = \dfrac{a\,\Delta V}{g}$

Michaud, $t_c > \dfrac{2L}{a}$: $\Delta p \approx \dfrac{2\rho L\, \Delta V}{t_c}$ $\qquad \sigma_h = \dfrac{\Delta p\, D}{2e}$

Joukowsky surge for a closure faster than the round trip time $2L/a$. K = fluid bulk modulus, E = pipe modulus, e = wall. $c_1 = 1$ with expansion joints; anchored pipe: $1 - \nu^2$ (throughout) or $1 - \nu/2$ (upstream end only), Wylie and Streeter.

Pipe E presets: steel 207, ductile iron 165, copper 117, PVC 2.76, HDPE 0.8 GPa (HDPE ranges 0.7 to 1.1 GPa by grade).

Inputs: Density, K bulk modulus, D inside dia., e wall thickness, E pipe modulus, c1 restraint, L pipe length, Velocity change, Closure time (sec).

16. Open Channel (Refs. 9, 23, 26)

Eq. 20: $V = \dfrac{k}{n} R_h^{2/3} S^{1/2}$ $\qquad k = 1.486$ (ft), $k = 1$ (m) $\qquad Q = V A,\ R_h = \dfrac{A}{P}$ $\qquad Fr = \dfrac{V}{\sqrt{gA/T}}$

Eq. 39: $A = \dfrac{D^2}{8}(\theta - \sin\theta),\ P = \dfrac{D\theta}{2}$ $\qquad \theta = 2\cos^{-1}\left(1 - \dfrac{2y}{D}\right)$ $\qquad$ Eq. 40: $S = \left(\dfrac{n\,Q}{k\,A R_h^{2/3}}\right)^2$

Manning, uniform flow. P = wetted perimeter, T = top width, S = bed slope (S in % = 100 S; 1 : N with $N = 1/S$). $Fr < 1$ subcritical, $Fr > 1$ supercritical; critical flow $Q^2T/(gA^3) = 1$; bed shear $\tau_0 = \rho g R_h S$.

Circular pipe, Eq. 39: full flow at $y = D$ ($\theta = 2\pi$). With constant n the largest gravity flow is 1.076 Q_{full} at $y/D = 0.938$, and the largest velocity is at $y/D = 0.81$. A flow above the largest gravity flow surcharges the pipe.

Solve modes: depth for Q (normal depth, the lower root for a pipe); diameter for Q at a design y/D, from $Q \propto D^{8/3}$, rounded up to the next Schedule 40 inside diameter (Ref. 23); slope for Q (Eq. 40).

Self-cleaning check: the Ten States Standards ask for at least 2.0 ft/s (0.6 m/s) with the sewer flowing full, using $n = 0.013$ (Ref. 26; check the edition your jurisdiction adopts).

Typical n (Ref. 9, Table 5-4 normal values for closed conduits): cast iron coated 0.013, uncoated 0.014, vitrified clay sewer 0.014, concrete sewer with manholes 0.015, corrugated metal storm drain 0.024; open channels: finished concrete 0.012, unfinished 0.014, clean earth 0.022, gravel 0.025, natural stream 0.030. Smooth plastic (PVC, HDPE) 0.009; 0.010 to 0.011 is often used for design.

Inputs: b bottom width, z side slope (H:V), D diameter (circ.), y flow depth, y/D fill (%), circ., Q flow (solve), Q2 flow, S slope (ratio), S slope (%), n Manning, V min (self-clean), Design y/D (%), D mode.

17. Added Mass (Refs. 10, 15)

Eq. 21: $m_a = C_a\,\rho\,V_{ref}$ cylinder, strip $V_{ref} = \dfrac{\pi D^2}{4}$ per length sphere $V_{ref} = \dfrac{\pi D^3}{6}$

$f_{wet} = f_{dry}\sqrt{\dfrac{m}{m + m_a + m_i}}$ $m_i = \rho_i\,\dfrac{\pi D_i^2}{4}$ C_a: cylinder 1, strip 1, sphere 0.5

Potential-flow added mass of an isolated body in an unbounded, still fluid. D = cylinder diameter, strip width or sphere diameter.

m_i = contained fluid (pipe contents) moving with the wall. A nearby wall, other bodies or a free surface change C_a.

Inputs: Density, D dia. or width, Ca coefficient, m mass/length, M mass (sphere), fn in air, empty (Hz), Di contained dia., rho contained, L length (total).

18. Liquid Sloshing (Ref. 16)

Eq. 22: $\omega_n^2 = g\,k_n \tanh(k_n h)$ rectangle $k_n = \dfrac{(2n-1)\,\pi}{L}$ cylinder $k_n = \dfrac{\xi_n}{R},\ J_1'(\xi_n) = 0$

$f_n = \dfrac{\omega_n}{2\pi}$ $\dfrac{m_1}{m_L} = \dfrac{8\tanh(\pi h/L)}{\pi^3\,h/L}$ $\dfrac{m_1}{m_L} = \dfrac{2R\tanh(\xi_1 h/R)}{\xi_1(\xi_1^2-1)\,h}$

Linear, inviscid sloshing in a rigid tank. ξ_n = 1.8412, 5.3314, 8.5363, ... g = acceleration normal to the free surface.

Modes excited by lateral motion: odd (antisymmetric) modes of a rectangular tank, $m = 1$ modes of a cylinder. m_1 = first-mode slosh mass, m_L = liquid mass.

Inputs: Density, L length (rect.), W width (rect.), D diameter (cyl.), h liquid depth, Acceleration (g), Number of modes.

19. Acoustic Modes (Ref. 17)

Eq. 23: $f_n = \dfrac{n\,c}{2L_{eff}}$ open-open, closed-closed closed-open $f_n = \dfrac{(2n-1)\,c}{4L_{eff}}$

$L_{eff} = L + N_{open}\,\Delta L$ $\Delta L = 0.6133\ r$ unflanged, $0.8216\ r$ flanged cut-on $f_c = \dfrac{1.8412\,c}{\pi D}$

Plane-wave modes of a uniform pipe or duct, no mean flow. N_{open} = number of open ends, $r = D/2$.

Above the cut-on frequency f_c the first cross mode propagates and the plane-wave model no longer applies.

Inputs: c speed of sound, L length, D diameter, Number of modes.

20. NPSH & Cavitation (Ref. 18)

Eq. 24: $NPSH_a = \dfrac{p_s - p_v}{\rho g} + z_s - h_{f,s}$ margin $NPSH_a - NPSH_r$ ratio $\dfrac{NPSH_a}{NPSH_r}$

p_s = surface pressure (abs), p_v = vapor pressure $z_s > 0$ flooded, $z_s < 0$ suction lift

cavitation if $NPSH_a < NPSH_r$

Net positive suction head available at the pump inlet, from the suction tank surface. $h_{f,s}$ = suction line loss at the flow.

Water presets: density and vapor pressure at 20 to 100 C. ANSI/HI 9.6.1 gives recommended NPSH margin ratios.

Inputs: Density, Vapor pressure, abs, p at surface, abs, Surface above pump, Suction line loss, NPSHr from pump.

21. Standard Atmosphere (Ref. 19)

Eq. 25: $T = T_b + L_b\,(H - H_b)$ $\qquad p = p_b \left(\dfrac{T}{T_b}\right)^{-g_0/(RL_b)}$ $\qquad p = p_b\, e^{-g_0(H-H_b)/(RT_b)}$ $(L_b = 0)$

$H = \dfrac{r_0\, z}{r_0 + z}$ $\qquad \rho = \dfrac{p}{RT},\ a = \sqrt{\gamma RT}$ $\qquad \mu = \dfrac{\beta\, T^{3/2}}{T + S}$

U.S. Standard Atmosphere 1976, 0 to 86 km. Layer bases H_b = 0, 11, 20, 32, 47, 51, 71 km′, lapse rates L_b = −6.5, 0, 1.0, 2.8, 0, −2.8, −2.0 K/km′.

$R = R^*/M_0$ = 287.053 J/kg-K, r_0 = 6356.766 km, β = 1.458e-6 kg/(m-s-K$^{1/2}$), S = 110.4 K, γ = 1.4.

Inputs: Altitude, geometric, ISA + dT.

22. Convective Heat Transfer (Refs. 20, 21, 22)

Eq. 26: $Nu = \dfrac{h\, L_c}{k}$ $\qquad$ tube, laminar $Nu_D = 3.66\ (T_s),\ \ 4.36\ (q_s'')$

Eq. 27: $Nu_D = 0.023\, Re_D^{0.8}\, Pr^{\,n}$ $\qquad$ Eq. 28: $\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3}$

mixed $\overline{Nu}_L = (0.037\, Re_L^{4/5} - 871)\, Pr^{1/3}$ $\qquad q = hA\,(T_s - T_\infty)$

h = convection coefficient, k = fluid thermal conductivity, $L_c = D$ (tube) or L (plate), $Pr = \nu/\alpha = \mu c_p/k$. Properties at the bulk mean temperature (tube) or the film temperature $(T_s + T_\infty)/2$ (plate).

Tube: laminar ($Re_D < 2300$), hydrodynamically and thermally fully developed, constant wall temperature T_s or uniform heat flux q_s''. Dittus-Boelter: $Re_D \geq 10^4$, $0.7 \leq Pr \leq 160$, $L/D \geq 10$, $n = 0.4$ heating ($T_s > T_m$), 0.3 cooling.

Plate: smooth, isothermal, parallel flow, average over length L. Laminar: $Re_L < 5 \times 10^5$, $Pr \geq 0.6$. Mixed: $5 \times 10^5 \leq Re_L \leq 10^7$, $0.6 \leq Pr \leq 60$, transition at $Re_{x,c} = 5 \times 10^5$.

Inputs: Kin. visc., k thermal cond., Pr Prandtl number, V mean velocity, D tube inside dia., L tube length, L plate length, b plate width, Ts - T fluid.

23. Pipe Sizing & Velocity (Refs. 1, 23, 24)

Eq. 30: $V = \dfrac{Q}{A} = \dfrac{4\,Q}{\pi D^2}$ Eq. 31: $D = \sqrt{\dfrac{4\,Q}{\pi V}}$ $A = \dfrac{\pi D^2}{4}$ 1 m³/h = 1/3600 m³/s

next Schedule 40 size up $V_{min} \le V \le V_{max}$

Mean velocity in a full circular pipe. Eq. 31 gives the inside diameter for a design velocity; it is rounded up to the next Schedule 40 steel inside diameter (ASME B36.10M; NPS 14 from OD 14.000 in and wall 0.438 in), which lowers V.

Typical velocity ranges (m/s): domestic water 0.6–2.0, chilled water 0.6–2.5, condenser water 1.0–3.0, fire protection 1.5–4.5, drainage 0.6–3.0, steam 15–30 (low pressure) and 20–40 (high pressure). Low V: sedimentation, scaling, stagnation. High V: friction loss, noise, erosion, water hammer.

These ranges are rules of thumb from common design guides. Check them against the project specification or the pipe sizing chapter of the ASHRAE Handbook, Fundamentals (Ref. 24), and check the pressure drop on the Pipe Friction tab.

Inputs: Q flow, Q2 flow, D inside dia., V design velocity, V minimum, V maximum.

24. Control Valve Cv (Ref. 25)

Eq. 32: $C_v = Q\sqrt{\dfrac{SG}{\Delta p}}$ (gpm, psi) $K_v = Q\sqrt{\dfrac{SG}{\Delta p}}$ (m³/h, bar) $C_v = 1.156\,K_v$

$\Delta p = p_1 - p_2$ $Q = C_v\sqrt{\dfrac{\Delta p}{SG}}$ liquid, non-choked, turbulent

ISA-75.01.01 / IEC 60534-2-1 sizing equation for incompressible liquids in non-choked turbulent flow, with the piping geometry factor $F_p = 1$ (valve the same size as the pipe): $Q = N_1 F_p C\sqrt{\Delta p/(\rho_1/\rho_0)}$, $N_1 = 1$ for C_v in gpm and psi, 1 for K_v in m³/h and bar, 0.865 for C_v in m³/h and bar.

$SG = \rho_1/\rho_0$ with ρ_0 = water at 15 C. Not included: choked flow ($\Delta p > F_L^2(p_1 - F_F p_v)$), reducers ($F_p < 1$) and laminar flow ($F_R$). Use the valve maker's C_v and F_L data for the valve opening.

Inputs: Q flow, Q2 flow, SG specific gravity, p1 inlet pressure, p2 outlet pressure, Cv (Q from Cv).

25. Pump Power & Efficiency (Refs. 1, 22)

Eq. 33: $P_h = Q\left[\Delta p + \frac{\rho}{2}\left(V_2^2 - V_1^2\right) + \rho g\,\Delta z\right]$ $\quad V = \frac{4Q}{\pi D^2}$ $\quad$ Eq. 37: $P_e = \sqrt{3}\,V_L I_L \cos\phi$

Eq. 34: $\eta_{pump} = \frac{P_h}{\eta_{motor}\,P_e}$ $\quad H = \frac{P_h}{\rho g Q}$ $\quad$ Eq. 38: $Q = \frac{\eta_{motor}\,\eta_{pump}\,P_e}{\rho g H}$

Energy equation between the pump inlet (1) and outlet (2): the hydraulic power is the rise in flow energy (pressure, kinetic and potential) of the liquid. Steady, incompressible flow.

P_e = electric power drawn by the motor, $\eta_{motor}P_e$ = shaft power into the pump. η_{pump} is the pump (mechanical) efficiency in the sense of Cengel and Cimbala; $\eta_{overall} = P_h/P_e$.

Eq. 37: balanced 3-phase supply, line-to-line voltage V_L, line current I_L, power factor $\cos\phi$. Eq. 38 (Q from power): $H = \Delta p/(\rho g) + \Delta z$; with D_1 and D_2 the kinetic term of Eq. 33 is kept and the cubic in Q is solved. With 100 % efficiencies Q is an upper bound: the running flow is where the pump curve meets the system curve. 1 kgf/cm^2 = 98.0665 kPa = 10.0 m of water.

Inputs: Density, Q flow, Q2 flow, D1 inlet (suction), D2 outlet (disch.), Line voltage (V), Line current (A), p2 - p1 rise, p2 - p1 (kgf/cm^2), z2 - z1, Pe electric input, Power factor, Motor efficiency (%), Pump eff. (%), Q mode.

26. Tank Volume

Eq. 35: $V = \frac{\pi D^2}{4}H$ $\quad$ rectangular $V = L\,W\,H$ $\quad$ 1 ft^3 = 28.317 L = 7.4805 US gal

Eq. 36: $A_{seg} = R^2\cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh-h^2}$ $\quad V_{liq} = A_{seg}\,L$ $\quad m = \rho\,V_{liq}$

Inside dimensions; heads, nozzles and internals are not included. Vertical cylinder and rectangular tank: liquid volume = base area × depth h.

Horizontal cylinder of radius $R = D/2$ and length L: the liquid cross-section is a circular segment of depth h (Eq. 36).

Inputs: D inside diameter, H height or length, W width (rect.), h liquid depth, Density.

References

1. B.R. Munson, D.F. Young and T.H. Okiishi, *Fundamentals of Fluid Mechanics*, Wiley.
2. F.M. White, *Fluid Mechanics*, McGraw-Hill.
3. Crane Co., *Flow of Fluids Through Valves, Fittings, and Pipe*, Technical Paper No. 410.
4. ISO 5167, *Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full*, Parts 1 to 4.
5. Ames Research Staff, *Equations, Tables, and Charts for Compressible Flow*, NACA Report 1135, 1953.
6. C.F. Colebrook, Turbulent flow in pipes, with particular reference to the transition region between the smooth and rough pipe laws, *J. Institution of Civil Engineers*, 1939.
7. S.E. Haaland, Simple and explicit formulas for the friction factor in turbulent pipe flow, *J. Fluids Engineering*, 1983.
8. E.B. Wylie and V.L. Streeter, *Fluid Transients in Systems*, Prentice Hall.
9. V.T. Chow, *Open-Channel Hydraulics*, McGraw-Hill, 1959.
10. R.D. Blevins, *Flow-Induced Vibration*, Van Nostrand Reinhold.
11. EN 1991-1-4, *Eurocode 1: Actions on structures, Part 1-4: General actions, Wind actions.*
12. S.F. Hoerner, *Fluid-Dynamic Drag*, 1965.
13. I.J. Karassik et al., *Pump Handbook*, McGraw-Hill.
14. E.V. Lewis (editor), *Principles of Naval Architecture*, SNAME.
15. R.D. Blevins, *Formulas for Natural Frequency and Mode Shape*, Van Nostrand Reinhold.
16. H.N. Abramson (editor), *The Dynamic Behavior of Liquids in Moving Containers*, NASA SP-106, 1966.
17. L.E. Kinsler, A.R. Frey, A.B. Coppens and J.V. Sanders, *Fundamentals of Acoustics*, Wiley.
18. ANSI/HI 9.6.1, *Rotodynamic Pumps Guideline for NPSH Margin*, Hydraulic Institute.
19. *U.S. Standard Atmosphere, 1976*, NOAA-S/T 76-1562, NASA-TM-X-74335, U.S. Government Printing Office, 1976.
20. T.L. Bergman, A.S. Lavine, F.P. Incropera and D.P. DeWitt, *Fundamentals of Heat and Mass Transfer*, Wiley, Chapters 7 and 8.
21. F.W. Dittus and L.M.K. Boelter, Heat transfer in automobile radiators of the tubular type, *University of California Publications in Engineering*, Vol. 2, 1930.
22. Y.A. Cengel and J.M. Cimbala, *Fluid Mechanics: Fundamentals and Applications*, 3rd ed., McGraw-Hill, 2014, Tables A-3 and A-9.
23. ASME B36.10M, *Welded and Seamless Wrought Steel Pipe*, American Society of Mechanical Engineers.
24. *ASHRAE Handbook, Fundamentals*, Pipe Design chapter (Chapter 22 in the 2017 edition), ASHRAE.
25. ANSI/ISA-75.01.01 (IEC 60534-2-1), *Industrial-Process Control Valves, Part 2-1: Flow capacity, Sizing equations for fluid flow under installed conditions.*
26. Great Lakes - Upper Mississippi River Board, *Recommended Standards for Wastewater Facilities* (Ten States Standards), Section 33.41.

Coefficients in the app (Cd, CD, K, roughness, Manning n, pipe modulus) are typical textbook values. Check them against the references or manufacturer data for the case at hand.