vibrationdata_honeycomb: Equations

vibrationdata_honeycomb.mlapp by Tom Irvine

Ten tabs cover honeycomb sandwich panels: properties and Nastran PSHELL values, Hexcel beam and end-load checks, bending wave speed with core shear, critical and transition frequencies, modal density, plate natural frequencies, the core bubbling mode, sound transmission through flat panels and cylindrical shells, and damping. Each tab has English or metric units, its equations in a strip at the top of the tab, and a results panel. This document gives the equations, assumptions, sources, worked-example checks and references.

The thirty-seven equations

1. Bending stiffness, unequal faces
2. Bending stiffness, thick core
3. Mass per area
4. Shear stiffness
5. Point impedance
6. Nastran PSHELL (NASA method)
7. Hexcel beam stiffness
8. Beam deflection, bending + shear
9. Face and core stresses
10. Exact section inertia
11. End load face stress
12. Panel buckling
13. Shear crimping
14. Skin wrinkling
15. Intracell buckling
16. Wave speed (Kurtze-Watters)
17. Bending, shear, face speeds
18. Overall bending stiffness B
19. Transition frequencies
20. Critical frequency (Wijker)
21. Modal density with core shear
22. Thin plate modal density, impedance
23. Circular plate, simply supported
24. Rectangular plate, classical
25. Rectangular plate, core shear
26. Bubbling mode equation (Mead)
27. Bubbling frequency
28. Bonding mass correction
29. Panel impedance
30. Transmission coefficient
31. Field incidence average
32. Cylinder TL (Tang)
33. Shell modal impedance
34. Fluid modal impedances
35. Ring frequency
36. Kelvin-Voigt loss factor
37. Modal strain energy

Units

Quantity	*English*	*metric*
Length, thickness	in	m, mm
Face modulus	psi	GPa
Core modulus, stress	psi	MPa
Density	lbm/in^3	kg/m^3
Mass per area	lbm/in^2	kg/m^2
Force, line load	lbf, lbf/in	N, N/m
Bending stiffness per width	lbf in	N m
Wave speed, speed of sound	in/sec	m/sec
Gas density	lbm/ft^3	kg/m^3
Point impedance	lbf sec/in	N sec/m

Changing the units converts the entered values. The tabs calculate in SI; English mass is in lbm (1 lbm/in^3 = 27679.9 kg/m^3).

1. Properties & PSHELL (Refs. 1, 6, 7)

Eq. 1: $D = \dfrac{E_f}{1-\nu^2}\,\dfrac{t_1 t_2\, h^2}{t_1+t_2}, \quad h = h_c + \dfrac{t_1+t_2}{2}$ Eq. 2: $D \approx \dfrac{E_f\, h_c^2\, h_f}{4}$

Eq. 3: $m = \rho_f(t_1+t_2) + \rho_c h_c + NSM$ Eq. 4: $S = G_c h_c \left(1 + \dfrac{t_f}{h_c}\right)^2$ Eq. 5: $Z_p = 8\sqrt{D\,m}$

Eq. 6: $I = \dfrac{2}{3}\left(\dfrac{h_c}{2} + \dfrac{T}{2}\right)^3 - \dfrac{h_c^3}{12}, \quad \dfrac{12I}{T^3}, \quad \dfrac{T_S}{T} = \dfrac{h_c}{T}$

D = bending stiffness per width (faces carry the bending, core stiffness neglected), $h_f = t_1 + t_2$, m = mass per area, S = shear stiffness per width, $B_f = E_f t_f^3/12(1-\nu^2)$ = one face.

Eqs. 1 and 2: draft 7 Eqs. 8-1 and 8-2 (from MIL-HDBK-23 and Baker et al.). Eq. 4: draft 7 Eq. 9-10 (Beranek and Ver). Eq. 5: infinite thin panel, bending waves, draft 7 Eq. 9-11 (Wijker).

The app also gives $D = E_f I/(1-\nu^2)$ with the exact I of the two faces about their neutral axis (equal to Eq. 1 plus the faces' own bending).

Eq. 6: Nastran PSHELL card by the NASA method (J. Loughlin, GSFC, draft 7 section 11.3): MID1 and MID2 = face material, T = total face thickness, MID3 = core (shear modulus), $T_S = h_c$, NSM = core and bond mass per area. Not for stress recovery.

Inputs: Ef face modulus, Face Poisson ratio, Face density, t1 face 1, t2 face 2, hc core thickness, Gc core shear mod., Core density, NSM mass/area.

2. Sandwich Beam (Hexcel) (Refs. 1, 2)

Eq. 7: $D = \dfrac{E_f t_f h^2 b}{2}, \quad S = b\,h\,G_c, \quad h = t_f + t_c$ Eq. 8: $\delta = \dfrac{k_b P l^3}{D} + \dfrac{k_s P l}{S}$

Eq. 9: $\sigma_f = \dfrac{M}{h\,t_f\,b}, \quad \tau_c = \dfrac{F}{h\,b}$ Eq. 10: $I = 2b\left(\dfrac{t_f^3}{12} + t_f y_c^2\right), \quad \sigma = \dfrac{My}{I}$

Hexcel beam formulas (Ref. 2, pages 11-13 and Appendix III): P = total load, l = span, b = width, k_b, k_s, M and F from the Hexcel table of beam coefficients for the seven support and load cases.

Eq. 10: draft 7 Eqs. 7-1 to 7-8, $y_o = h_c/2 + t_f$ (outer fiber) and $y_c = h_c/2 + t_f/2$ (face centroid). Global bending stress only: no local effects at loads, supports or inserts.

Hexcel sample (Ref. 2): b = 0.5 m, l = 2 m, P = 1500 N central, 0.5 mm aluminum faces, 25.4 mm core, G_W = 220 MPa: δ = 43 mm, σ_f = 115.8 MPa, τ_c = 0.06 MPa.

Inputs: b beam width, l span, P total load, Ef face modulus, tf each face, tc core thickness, Gc core shear mod., Face allowable, Core shear allow..

3. End Loading & Instability (Ref. 2)

Eq. 11: $\sigma_f = \dfrac{P}{2 t_f b}$ Eq. 12: $P_b = \dfrac{\pi^2 D}{l^2 + \frac{\pi^2 D}{G_c h b}}$ Eq. 13: $P_b = t_c\, G_c\, b$

Eq. 14: $\sigma_{cr} = 0.5\,(G_c E_c E_f)^{1/3}$ Eq. 15: $\sigma_{cr} = 2E_f \left(\dfrac{t_f}{s}\right)^2$

Hexcel end loading (Ref. 2, pages 18-19 and Appendix III): $P = q\,b$, Eq. 11 face stress, Eq. 12 panel buckling (D of Eq. 7), Eq. 13 shear crimping, Eq. 14 skin wrinkling, Eq. 15 intracell buckling (s = cell size).

Hexcel sample: q = 20 kN/m, b = 0.5 m, l = 2 m: σ_f = 20 MPa, P_b = 14,413 N, crimping 2.79 MN, wrinkling 1244 MPa, intracell 854 MPa.

Inputs: q end load/width, b panel width, l panel length, Ef face modulus, tf each face, tc core thickness, Gc core shear mod., Ec core compress., s cell size, Face allowable.

4. Wave Speed & Critical Freq. (Refs. 1, 8, 9, 10)

Eq. 16: $\dfrac{c_s^2}{c_b^4}\, c^6 + c^4 - c_s^2 c^2 - c_{bf}^4 = 0$

Eq. 17: $c_b = \sqrt{\omega}\left(\dfrac{B}{m}\right)^{1/4}$, $c_s = \sqrt{\dfrac{G_c h_c}{m}}$, $c_{bf} = \sqrt{\omega}\left(\dfrac{2B_f}{m}\right)^{1/4}$ Eq. 18: $B = \dfrac{E_f t_f (h_c + t_f)^2}{2(1-\nu^2)}$

Eq. 19: $\omega_1 = \dfrac{G_c h_c}{\sqrt{Bm}}$, $\omega_2 = \dfrac{G_c h_c}{\sqrt{2B_f m}}$ Eq. 20: $f_{cr} = \dfrac{1}{2\pi}\sqrt{\dfrac{c_0^4 m/B}{1 - c_0^2 m/S}}$

Eq. 16: phase speed c of bending waves with core shear (Kurtze and Watters; draft 7 Eqs. 9-1 to 9-5). It is a cubic in c^2 with one positive root. Low frequency $c \to c_b$, mid $c \to c_s$, high $c \to c_{bf}$.

Eq. 19: transition frequencies where $c_b = c_s$ (ω_1, draft 7 Eq. 9-6) and $c_s = c_{bf}$ (ω_2, Eq. 9-8, with c_{bf} of Eq. 17).

Eq. 20: critical (coincidence) frequency, valid for $c_0^2 m/S < 1$ (Beranek and Ver, Wijker; draft 7 Eq. 9-9). The app also gives the frequency at which Eq. 16 gives $c = c_0$.

Draft 7 example (Table 9-2, Figure 9-2): $f_{cr} \approx$ 290 Hz and bending-to-shear transition $\approx$ 4400 Hz.

Inputs: Ef face modulus, Face Poisson ratio, Face density, tf each face, hc core thickness, Gc core shear mod., Core density, NSM mass/area, c0 sound speed, Plot fmin (Hz), Plot fmax (Hz).

5. Modal Density & Impedance (Refs. 1, 10)

Eq. 21: $n(f) = \dfrac{\pi A m f}{S}\left[1 + \dfrac{m\omega^2 + 2S^2/D}{\sqrt{m^2\omega^4 + 4m\omega^2 S^2/D}}\right]$

Eq. 22: $n = \dfrac{A}{2}\sqrt{\dfrac{m}{D}}$ (bending only), $Z_{p,\infty} = 8\sqrt{D\,m}$ $\qquad N(f) = \displaystyle\int_0^f n\,df$

Eq. 21: modal density (modes per Hz) of a flat sandwich panel of area A with core shear (Wijker, draft 7 Eq. 9-12); $D = B$ of Eq. 18. At low frequency it tends to the thin plate value of Eq. 22, at high frequency to $2\pi A m f/S$ (shear waves).

Eq. 22: point impedance of an infinite plate in bending (draft 7 Eq. 9-11). Core shear lowers the impedance at higher frequencies.

Inputs: Ef face modulus, Face Poisson ratio, Face density, tf each face, hc core thickness, Gc core shear mod., Core density, NSM mass/area, a panel length, b panel width, Plot fmin (Hz), Plot fmax (Hz).

6. Plate Natural Frequencies (Refs. 1, 11)

Eq. 23: $f_n = \dfrac{\lambda^2}{2\pi a^2}\sqrt{\dfrac{D}{m}}$, $\dfrac{J_1(\lambda)}{J_0(\lambda)} + \dfrac{I_1(\lambda)}{I_0(\lambda)} = \dfrac{2\lambda}{1-\nu}$ $\qquad$ Eq. 24: $f_{ij} = \dfrac{k^2}{2\pi}\sqrt{\dfrac{D}{m}}$, $k^2 = \left(\dfrac{i\pi}{a}\right)^2 + \left(\dfrac{j\pi}{b}\right)^2$

Eq. 25: $k = \dfrac{\omega}{c(\omega)}$

Eq. 23: fundamental frequency of a simply supported circular plate of radius a (Leissa, NASA SP-160). λ^2 from the frequency equation: 4.935 for $\nu = 0.3$, 4.979 for $\nu = 0.33$. Draft 7 Eq. 8-14 uses 4.9744.

Draft 7 example: 40 in. bulkhead, 0.063 in. aluminum faces, 1 in. core: D = 391,143 lbf in., f_n = 178 Hz (77.8 Hz with 100 lbm added). m = total mass / area (added mass has no stiffness).

Eq. 24: simply supported rectangular plate, bending only. Eq. 25: the same mode wavenumber with the core shear dispersion of Eq. 16, which lowers the higher modes.

Inputs: Ef face modulus, Face Poisson ratio, Face density, t1 face 1, t2 face 2, hc core thickness, Core density, Gc core shear mod., Added mass, Diameter, circular, a length, rect., b width, rect..

7. Core Bubbling Mode (Mead) (Ref. 3)

Eq. 26: $\mu\lambda\,\phi\tan\phi = 1, \quad \mu = \dfrac{h_2}{h_1}, \quad \lambda = \dfrac{\rho_2}{\rho_1}$ Eq. 27: $\omega_0 = \dfrac{\phi}{h_1}\sqrt{\dfrac{C}{\rho_1}}$

Eq. 28: $\left(\dfrac{\omega}{\omega_0}\right)^2 = \dfrac{\rho_1 + 3\rho_2\mu(1+\mu)}{\rho_1 + 3\rho_2\mu(1+\mu) + 3\tau/h_1}$

Mead and Pretlove (Ref. 3) Eqs. 58, 52 and 59: symmetric bubbling (wrinkling) mode of a flat sandwich plate, faces moving in opposite directions and the core in tension and compression. Almost independent of wavelength.

h_1 = half the core depth, h_2 = face thickness, ρ_1 = core density, ρ_2 = face density, C = core modulus in the thickness direction, τ = bonding mass per area of each face. Eq. 28 is Mead's Eq. 59 in consistent units (his $36\tau/h_1$ has h_1 in inches and τ in lb/ft^2).

Mead Table 3 (Aeroweb 142, aluminum faces, $\lambda = 41.9$): $\phi = 0.640$ at $\mu = 0.05$, 0.470 at 0.10, 0.389 at 0.15, 0.339 at 0.20. Mead notes the mode is lightly damped and may matter for fatigue near the core.

Inputs: tf each face, hc core thickness, Face density, Core density, Ec core compress., Bond mass/area.

8. Flat Panel Transmission Loss (Refs. 1, 5, 9, 12)

Eq. 29: $Z_s = \dfrac{D(\omega)(1+i\eta)\,k_0^4\sin^4\theta - \omega^2 m}{i\omega}, \quad D(\omega) = \dfrac{m_0\,c(\omega)^4}{\omega^2}$ Eq. 30: $\tau(\theta) = \left|1 + \dfrac{Z_s\cos\theta}{2\rho_0 c_0}\right|^{-2}$

Eq. 31: $\bar{\tau} = \dfrac{\int_0^{78^\circ}\tau\sin\theta\cos\theta\,d\theta}{\int_0^{78^\circ}\sin\theta\cos\theta\,d\theta}, \quad TL = -10\log\bar{\tau}$

Infinite flat panel between two half spaces of the same gas (Cremer and Heckl; Fahy). $D(\omega)$ = bending stiffness that gives the sandwich wave speed $c(\omega)$ of Eq. 16 with the panel mass m_0; m = panel plus added limp mass; $k_0 = \omega/c_0$; η = loss factor.

Coincidence: $c(\omega) = c_0/\sin\theta$. At normal incidence Eq. 30 is the mass law $10\log[1 + (\omega m/2\rho_0 c_0)^2]$. Field incidence: Eq. 31.

Klos, Robinson and Buehrle (Ref. 5) measured sonic and supersonic waves in a curved honeycomb panel and a TL well below the mass law; lead vinyl that doubled the mass made the waves subsonic and gave 14 dB above 1500 Hz.

Inputs: Ef face modulus, Face Poisson ratio, Face density, tf each face, hc core thickness, Gc core shear mod., Core density, NSM mass/area, Limp mass/area, Loss factor, Gas density, c0 sound speed, Angle (deg), Plot fmin (Hz), Plot fmax (Hz).

9. Cylinder Transmission Loss (Ref. 4)

Eq. 32: $TL = -10 \log \sum_{n=0}^{\infty} \frac{2\epsilon_n}{k_{1r} R} \frac{\mathrm{Re}[Z_n^R]\,\mathrm{Re}[Z_n^T]}{|Z_n^R + Z_n^T + Z_n^S|^2}$ Eq. 33: $Z_n^S = \frac{1}{i\omega D_n}$

Eq. 34: $Z_n^R = -i\omega \frac{\rho_1}{k_{1r}} \frac{H_n^{(2)}(k_{1r}R)}{H_n^{(2)\prime}(k_{1r}R)}, \quad Z_n^T = i\omega \frac{\rho_2}{k_{2r}} \frac{H_n^{(1)}(k_{2r}R)}{H_n^{(1)\prime}(k_{2r}R)}$ Eq. 35: $f_R = \frac{1}{2\pi R} \sqrt{\frac{2E_f t_f}{(1-\nu^2)\, m}}$

Tang, Robinson and Silcox (Ref. 4) Eqs. 35-42 with no airflow: infinite cylinder of radius R, oblique plane wave at γ_1 to the axis ($k_{1z} = k_1 \cos\gamma_1$, $k_{1r} = k_1 \sin\gamma_1$), fluid 1 outside, absorbing fluid 2 inside, $\epsilon_0 = 1$, $\epsilon_n = 2$.

D_n = radial displacement per unit pressure of mode n from a Donnell-type shell with membrane, bending, rotary inertia and transverse shear S of Eq. 4 (first-order shear); thin shell: $S \times 10^6$. Complex moduli $E(1+i\eta)$, $G(1+i\eta)$.

Tang used the Reissner-Naghdi-Berry thin shell and a first-order theory with layer integrals; this app uses sandwich stiffnesses (faces carry membrane and bending, core carries shear). Checked against Tang Fig. 3(a).

Eq. 35: ring frequency of the sandwich (membrane stiffness of the faces). Tang normalized by the monocoque value $\sqrt{E/\rho}/2\pi R = 445$ Hz.

Inputs: Ef face modulus, Face Poisson ratio, Face density, tf each face, hc core thickness, Gc core shear mod., Core density, R mean radius, Loss factor, gamma1 angle (deg), Plot fmin (Hz), Plot fmax (Hz), rho1 outside, c1 outside, rho2 inside, c2 inside.

10. Damping & Loss Factor (Ref. 1)

Eq. 36: $\sigma = E\varepsilon + \eta_v \dot{\varepsilon}, \quad \eta = \frac{\omega\, \eta_v}{E}$ $E^*(\omega) = E + i\omega\eta_v$ Eq. 37: $\eta_{sys} = \frac{\sum \eta_i U_i}{U_{total}}$

$\zeta = \frac{\eta}{2}, \quad Q = \frac{1}{\eta}$

Eq. 36: Kelvin-Voigt model, spring and dashpot in parallel (draft 7 Eqs. 12-1 to 12-4). The same form applies to core shear: $G^* = G_c + i\omega\eta_s$.

Eq. 37: modal strain energy method (draft 7 Eq. 12-12): U_i = strain energy of component i in the mode, for example from a Nastran modal analysis.

Loss factors of honeycomb sandwich (draft 7 Table 12-1): NASA SP-8050 0.06 to 0.12; NASA/TM-2016-219347 0.004 to 0.04; USAF graphite/epoxy 0.026 to 0.054.

Inputs: f frequency (Hz), E or G modulus, Viscosity.

References

1. T. Irvine, *Honeycomb Sandwich Panels*, draft 7, Vibrationdata, January 24, 2026. Equation numbers chapter-number, e.g. Eq. 9-1.
2. Hexcel Composites, *HexWeb Honeycomb Sandwich Design Technology*, Publication AGU 075b, 2000.
3. D.J. Mead and A.J. Pretlove, *On the Vibrations of Cylindrically Curved Elastic Sandwich Plates*, Aeronautical Research Council R. & M. No. 3363, 1964.
4. Y.Y. Tang, J.H. Robinson and R.J. Silcox, Sound transmission through a cylindrical sandwich shell with honeycomb core, NASA Langley Research Center (AIAA paper).
5. J. Klos, J.H. Robinson and R.D. Buehrle, Sound transmission through a curved honeycomb composite panel, NASA Langley Research Center (AIAA paper).
6. *MIL-HDBK-23, Composite Construction for Aircraft Structures*, U.S. Department of Defense, 1968 (as cited in Ref. 1).
7. J. Loughlin, PSHELL card for honeycomb panels, NASA Goddard Space Flight Center (as given in Ref. 1, section 11.3).
8. Kurtze and Watters, New wall design for high transmission loss or high damping, *J. Acoust. Soc. Am.*, Vol. 31, 1959 (as cited in Ref. 1).
9. L. Cremer and M. Heckl, *Structure-Borne Sound*, Springer-Verlag, 1988 (as cited in Ref. 1).
10. L. Beranek and I. Ver, *Noise and Vibration Control Engineering*, Wiley, 1992; J. Wijker, *Random Vibrations in Spacecraft Structure Design*, Springer, 2009 (as cited in Ref. 1).
11. A.W. Leissa, *Vibration of Plates*, NASA SP-160, 1969 (as cited in Ref. 1).
12. F. Fahy, *Sound and Structural Vibration: Radiation, Transmission and Response*, Academic Press (as cited in Ref. 5).

Refs. 6 to 12 were not consulted directly: their equations are as given in Refs. 1, 4 and 5. Material properties are inputs; take core shear and compression moduli from the core supplier data for the ribbon (L) and transverse (W) directions.