vibrationdata_hvac: Equations

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Twelve tabs cover everyday HVAC calculations: airflow and duct velocity, air density, psychrometrics, sensible and latent loads, cooling coils, air mixing, heat exchangers, wall U-values, duct friction, fans, capacity and units. Each tab has English or metric units, its equations in a strip at the top of the tab, and a results panel. This document gives the equations, assumptions, corrections to a common HVAC formula sheet, and references.

The twenty-seven equations

1. Airflow, area and velocity
2. Velocity pressure
3. Equivalent round duct (Huebscher)
4. Standard atmosphere pressure
5. Dry air density (ideal gas)
6. Saturation vapor pressure
7. Humidity ratio
8. Moist air enthalpy
9. Moist air specific volume
10. Humidity ratio from wet bulb
11. Sensible load
12. Latent load
13. Coil mass flow and total load
14. Bypass factor, apparatus dew point
15. Adiabatic mixing
16. Heat exchanger duty
17. Log mean temperature difference
18. LMTD correction F, 1-2 shell and tube
19. Wall resistance and U-value
20. Conduction and convection
21. Radiation
22. Darcy-Weisbach duct friction
23. Colebrook friction factor
24. Fan air and shaft power
25. Fan laws
26. Tons of refrigeration
27. COP, EER and kW per ton

Units

Quantity	*English*	*metric*
Airflow	cfm	L/s
Duct velocity	ft/min	m/s
Duct pressure, friction rate	in. w.g., in. w.g./100 ft	Pa, Pa/m
Barometric pressure	psia	kPa
Temperature	deg F	deg C
Heat rate, load	BTU/hr	kW
Power	hp	kW
Enthalpy, specific volume	BTU/lb, ft^3/lb	kJ/kg, m^3/kg
Humidity ratio	lb/lb (grains/lb)	kg/kg (g/kg)
U-value, R-value	BTU/hr-ft^2-F, hr-ft^2-F/BTU	W/m^2-K, m^2-K/W
Duct size, roughness	in	mm

Changing the units converts the entered values. Psychrometric properties use the English or SI equations of Ref. 1 directly (the enthalpy datums differ); the other tabs calculate in SI. 1 in. w.g. = 249.0889 Pa, 1 BTU/hr = 0.29307107 W (Ref. 10).

1. Airflow & Duct Velocity (Refs. 7, 10)

Eq. 1: $Q = A\,V$ $\qquad A = WH$ or $\dfrac{\pi D^2}{4}$ $\qquad$ Eq. 2: $P_v = \dfrac{\rho V^2}{2}$ $\qquad D_h = \dfrac{4A}{P}$

Eq. 3: $D_e = \dfrac{1.30\,(WH)^{0.625}}{(W+H)^{0.25}}$ $\qquad$ standard air $P_v = \left(\dfrac{V}{4005}\right)^2$ $\qquad CFM = A\,[\text{ft}^2]\;V\,[\text{ft/min}]$

Q = airflow, V = mean velocity, A = inside cross-section area, P = perimeter, D_h = hydraulic diameter, P_v = velocity pressure.

D_e = diameter of the round duct with the same friction loss and airflow (Huebscher, Ref. 7). The 4005 rule: P_v in in. w.g., V in ft/min, air at 0.075 lbm/ft^3.

Correction to the formula sheet: CFM = $A \times V \times 60$ holds only for V in ft/s. With V in ft/min, CFM = $A \times V$.

Inputs: W width (rect.), H height (rect.), D diameter (round), Q airflow, V velocity, Air density.

2. Air Density & Altitude (Refs. 1, 2)

Eq. 4: $p = 14.696\,(1 - 6.8754 \times 10^{-6}\,Z)^{5.2559}$ $\qquad p = 101325\,(1 - 2.25577 \times 10^{-5}\,Z)^{5.2559}$

Eq. 5: $\rho_{da} = \dfrac{144\,p}{R_{da}\,T}$ $\qquad \rho = \dfrac{1+W}{v}$ $\qquad R_{da} = 53.350\ \dfrac{\text{ft lbf}}{\text{lbm R}} = 287.042\ \dfrac{\text{J}}{\text{kg K}}$

Standard atmosphere pressure (psia with Z in ft, Pa with Z in m), Ref. 1 (ASHRAE Fundamentals 2017 ch. 1 Eq. 3). T = absolute temperature (R or K).

Eq. 5 is in English units (p in psia; the factor 144 converts to lbf/ft^2); in SI, $\rho_{da} = p/(R_{da}T)$ with p in Pa. Correction to the sheet: with p in psia the factor 144 is needed.

Load factors from the density: $60\,\rho\,c_p$ (sensible) and $60\,\rho\,(1061 + 0.444\,t)$ (latent), with ρ = mass of dry air per volume $= 1/v$.

Inputs: Altitude, Barometric pressure, Dry bulb, Relative humidity (%).

3. Psychrometrics (Refs. 1, 2)

Eq. 6: $\ln p_{ws} = \dfrac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13}\ln T$ Eq. 7: $W = 0.621945\,\dfrac{p_w}{p - p_w}$

$\phi = \dfrac{p_w}{p_{ws}}$ Eq. 8: $h = 0.240\,t + W\,(1061 + 0.444\,t)$ Eq. 9: $v = \dfrac{R_{da}T\,(1 + 1.607858\,W)}{p}$

Eq. 10: $W = \dfrac{(1093 - 0.556\,t^*)\,W_s^* - 0.240\,(t - t^*)}{1093 + 0.444\,t - t^*}$

Saturation pressure over liquid water (Eq. 6, coefficients C_8 to C_{13} of Ref. 1 and 2); over ice below the triple point a seven-term form. t^* = wet bulb, W_s^* = saturation W at t^*.

SI forms: $h = 1.006\,t + W\,(2501 + 1.86\,t)$ kJ/kg, and Eq. 10 with 2501, 2.326, 1.006, 1.86 and 4.186. Each unit system uses its own equations (enthalpy datum 0 F or 0 C dry air).

Dew point: Eq. 6 solved for T at p_w. Wet bulb: Eq. 10 solved for t^*. Equations and coefficients as implemented in PsychroLib 2.5.0 (Ref. 1).

Corrections to the formula sheet: 0.621945 (sheet 0.62198, an older value) and 1.607858 (sheet 1.607).

Inputs: Altitude, Barometric pressure, Dry bulb, Relative humidity (%), Wet bulb, Dew point, Humidity ratio.

4. Sensible, Latent & Total Load (Refs. 1, 2)

Eq. 11: $Q_s = 60\,\rho\,(0.240 + 0.444\,W_2)\,Q\,(t_1 - t_2)$

Eq. 12: $Q_L = 60\,\rho\,(1061 + 0.444\,t_1)\,Q\,(W_1 - W_2)$ $Q_t = Q_s + Q_L = 60\,\rho\,Q\,(h_1 - h_2)$

sheet $Q_s = 1.08\,Q\,\Delta t, \quad Q_L = 4840\,Q\,\Delta W$ $SHR = \dfrac{Q_s}{Q_t}$

English units: Q in cfm, ρ = dry air per volume in lbm/ft^3, loads in BTU/hr. SI: $Q_s = \rho\,(1.006 + 1.86\,W_2)\,Q\,\Delta t$ and $Q_L = \rho\,(2501 + 1.86\,t_1)\,Q\,\Delta W$ in kW with Q in m^3/s.

Eqs. 11 and 12 split Eq. 8 exactly: $h_1 - h_2 = (0.240 + 0.444\,W_2)(t_1 - t_2) + (W_1 - W_2)(1061 + 0.444\,t_1)$.

The sheet factors: $1.08 = 60 \times 0.075 \times 0.240$ (dry air at 0.075 lbm/ft^3) and $4840 = 60 \times 0.075 \times 1076$. With moist air, $c_p = 0.240 + 0.444\,W$ gives about 1.10. W in lb water per lb dry air (not grains).

Inputs: Q airflow, t1 room / entering, t2 supply / leaving, W1 humidity ratio, W2 humidity ratio, Dry air density, Qs sensible load.

5. Coil Process (Refs. 1, 2)

Eq. 13: $\dot{m} = \dfrac{Q}{v_1}$ $\quad Q_t = \dot{m}\,(h_1 - h_2)$ $\quad \dot{m}_w = \dot{m}\,(W_1 - W_2)$ $\quad Q_L = Q_t - Q_s$

Eq. 14: $BF = \dfrac{t_2 - t_{adp}}{t_1 - t_{adp}}$ $\quad CF = 1 - BF$ $\quad W_2 + \dfrac{W_1 - W_2}{t_1 - t_2}(t_{adp} - t_2) = W_s(t_{adp})$

States from dry and wet bulb (Eq. 10), standard atmosphere at the altitude (Eq. 4). $\dot{m}$ = dry air mass flow from the entering airflow, $\dot{m}_w$ = condensate.

t_{adp} = apparatus dew point: the coil process line extended to the saturation curve (last equation), or entered. BF = bypass factor, CF = contact factor.

The sheet formulas $1.08\,Q\,\Delta t$ and $4840\,Q\,\Delta W$ are also shown for comparison (standard air).

Inputs: Altitude, Q airflow, entering, t1 entering dry bulb, t1 entering wet bulb, t2 leaving dry bulb, t2 leaving wet bulb, ADP (blank = solve).

6. Air Mixing (Refs. 1, 2)

Eq. 15: $W_m = \dfrac{\dot{m}_1 W_1 + \dot{m}_2 W_2}{\dot{m}_1 + \dot{m}_2}$ $\quad h_m = \dfrac{\dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_1 + \dot{m}_2}$ $\quad \dot{m}_i = \dfrac{Q_i}{v_i}$ $\quad t_m = \dfrac{h_m - 1061\,W_m}{0.240 + 0.444\,W_m}$

sheet $t_{mix} = \dfrac{t_1 Q_1 + t_2 Q_2}{Q_1 + Q_2}$

Adiabatic mixing of two moist air streams at constant pressure: mass and energy balances. States from dry bulb and RH (Eqs. 6 and 7).

The sheet's volume-weighted t_{mix} is an approximation; the app shows both. A mixture above 100 % RH is supersaturated (fog).

Inputs: Altitude, Q1 airflow, t1 dry bulb, RH1 (%), Q2 airflow, t2 dry bulb, RH2 (%).

7. Heat Exchanger (LMTD) (Refs. 3, 4)

Eq. 16: $Q = UAF\,\Delta T_{lm}$ Eq. 17: $\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$ $R = \dfrac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}},\ P = \dfrac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$

Eq. 18: $F = \dfrac{\sqrt{R^2+1}}{R-1}\,\dfrac{\ln\left[(1-P)/(1-PR)\right]}{\ln\frac{2-P\,(R+1-\sqrt{R^2+1})}{2-P\,(R+1+\sqrt{R^2+1})}}$ counterflow $\Delta T_1 = T_{h1} - T_{c2}$

parallel $\Delta T_1 = T_{h1} - T_{c1}$

$F = 1$ for counterflow and parallel flow. Eq. 18: one shell pass and two (or an even number of) tube passes, with ΔT_{lm} for counterflow (Refs. 3, 4). For $R = 1$ the limit is used.

Effectiveness $\varepsilon = Q/(C_{min}(T_{h1} - T_{c1}))$ and $NTU = UA/C_{min}$ are also given. Correction to the sheet: U in BTU/hr-ft^2-F and A in ft^2.

Inputs: U overall coefficient, A area, Required Q (optional), Hot fluid in, Hot fluid out, Cold fluid in, Cold fluid out.

8. Wall U-Value & Heat Transfer (Ref. 3)

Eq. 19: $R = \dfrac{1}{h_i} + \sum \dfrac{L_j}{k_j} + \dfrac{1}{h_o}$ $U = \dfrac{1}{R}$ $Q = UA\,(t_i - t_o)$

Eq. 20: $q = -k\,\dfrac{dT}{dx},\ \ Q = hA\,\Delta T$ Eq. 21: $Q = \varepsilon\,\sigma A\,(T_s^4 - T_{sur}^4)$

$\sigma = 5.670374 \times 10^{-8}\ \mathrm{W/m^2K^4}$

Plane wall, steady one-dimensional conduction with films on both sides (Ref. 3). Surface temperatures step through the resistances. Radiation: gray surface in large surroundings, T absolute.

The film coefficients, layer conductivities and emissivity are inputs; the defaults are examples only.

Inputs: hi inside, ho outside, A wall area, ti inside air, to outside air, Ts radiating surface, Tsur surroundings, Emissivity.

9. Duct Friction Loss (Refs. 5, 6, 7, 9)

Eq. 22: $\Delta p = f \, \dfrac{L}{D_h} \, \dfrac{\rho V^2}{2}$ Eq. 23: $\dfrac{1}{\sqrt{f}} = -2 \log_{10} \left(\dfrac{\varepsilon / D_h}{3.7} + \dfrac{2.51}{Re\sqrt{f}} \right)$ $D_h = \dfrac{2WH}{W + H}$

$Re = \dfrac{V D_h}{\nu}$ $\Delta p_{fit} = \sum n \, C \, P_v$ $f = \dfrac{64}{Re}$ (laminar)

Darcy-Weisbach with the Colebrook friction factor, solved by iteration from Haaland (Refs. 5, 6). The friction rate is given per 100 ft (in. w.g.) or per m (Pa).

Size round duct: the diameter that gives the target friction rate at the airflow. Roughness presets as in vibrationdata_fluids; duct construction (joints, seams) can raise the effective roughness.

Correction to the sheet: Eq. 22 gives Δp in lbf/ft^2 (Pa); divide by 5.2023 (249.09) for in. w.g. Standard air: 0.075 lbm/ft^3, $\nu = 1.516 \times 10^{-5}$ m^2/s (Ref. 9).

Inputs: W width (rect.), H height (rect.), D diameter (round), L duct length, Q airflow, Air density, Kinematic viscosity, Roughness, Target rate.

10. Fan Power & Fan Laws (Ref. 8)

Eq. 24: $P_{air} = Q \, p_t, \quad P_{shaft} = \dfrac{Q \, p_t}{\eta_f}$ sheet $BHP = \dfrac{CFM \times TP}{6356 \, \eta_f}$ $kW = \dfrac{0.746 \, BHP}{\eta_m}$

Eq. 25: $\dfrac{Q_2}{Q_1} = \dfrac{N_2}{N_1} \left(\dfrac{D_2}{D_1} \right)^3$ $\dfrac{p_2}{p_1} = \left(\dfrac{N_2}{N_1} \right)^2 \left(\dfrac{D_2}{D_1} \right)^2 \dfrac{\rho_2}{\rho_1}$ $\dfrac{P_2}{P_1} = \left(\dfrac{N_2}{N_1} \right)^3 \left(\dfrac{D_2}{D_1} \right)^5 \dfrac{\rho_2}{\rho_1}$

p_t = fan total pressure, η_f = fan total efficiency (use static pressure only with static efficiency), η_m = motor efficiency.

The exact constant is 33000/5.2023 = 6343 with the inch of water at 39 F (249.09 Pa); the sheet's 6356 corresponds to water near 70 F. The difference is 0.2 %.

Fan laws (Eq. 25): geometrically similar fans at the same point of rating, Ref. 8. The plot shows the system curve $p_t \propto Q^2$ through condition 1.

Inputs: Q1 airflow, Fan total pressure, Fan total eff. (%), Motor efficiency (%), rho1 air density, N1 speed (rpm), N2 speed (rpm), D1 wheel diameter, D2 wheel diameter, rho2 air density.

11. Capacity & Efficiency (Ref. 10)

Eq. 26: 1 ton = 12,000 BTU/hr = 3.5169 kW $\qquad \text{tons} = \dfrac{\text{BTU/hr}}{12{,}000} \qquad$ Eq. 27: $COP = \dfrac{Q_c}{P_{in}}$

$$EER = \frac{Q_c\,[\text{BTU/hr}]}{P_{in}\,[\text{W}]} = 3.412\,COP \qquad \frac{\text{kW}}{\text{ton}} = \frac{P_{in}\,[\text{kW}]}{\text{tons}}$$

Ton of refrigeration: 12,000 BTU/hr. 1 BTU/hr = 0.29307107 W (International Table BTU, Ref. 10).

Inputs: Cooling capacity, Electrical input.

12. Unit Converter (Ref. 10)

1 cfm = 4.719474×10^{-4} m^3/s $\qquad$ 1 in. w.g. = 249.0889 Pa $\qquad$ 1 psi = 6894.757 Pa

1 BTU/hr = 0.29307107 W $\qquad$ 1 BTU/lb = 2.326 kJ/kg

$$t_F = \frac{9}{5}\,t_C + 32, \quad T_R = t_F + 459.67$$

Conversion factors from Ref. 10. The inch of water is the conventional value, $\rho = 1000$ kg/m^3 and $g = 9.80665$ m/s^2 ($0.0254 \times 1000 \times 9.80665$).

Inputs: Value.

References

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10. A. Thompson and B.N. Taylor, *Guide for the Use of the International System of Units (SI)*, NIST Special Publication 811, 2008, Appendix B.

Film coefficients, conductivities, emissivity, roughness and fitting loss coefficients are inputs. Check them against the references or manufacturer data for the case at hand.