Craig-Bampton Reduction

English units

Mass entered in lbm, converted to consistent units: $m = W/386$ [lbf sec^2/in]

Stiffness k [lbf/in], length [in], force [lbf], acceleration [in/sec^2]

$1\,G = 386$ in/sec^2

Metric units

Mass m [kg], stiffness k [N/m], length [m], force [N]

$1\,G = 9.81$ m/sec^2

Craig-Bampton transformation (Craig and Bampton, 1968)

$\{x\} = [T]\{x_b;\, q\}$, $[T] = \begin{bmatrix} I & 0 \\ \Phi_c & \Phi_n \end{bmatrix}$, $\Phi_c = -K_{ii}^{-1} K_{ib}$

$[M_{cb}] = [T]^T [M][T] = \begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & I \end{bmatrix}$, $[K_{cb}] = \begin{bmatrix} K_{bb} & 0 \\ 0 & \Lambda \end{bmatrix}$

Fixed-base effective mass: $L = M_{qb}\{r_b\}$, $m_{eff,j} = L_j^2$, residual $= r_b^T M_{bb} r_b - \sum L_j^2$

Output transformation matrices

Displacement / acceleration: $\{x\}_{out} = [T]_{out}\{u\}$, $\{\ddot{x}\}_{out} = [T]_{out}\{\ddot{u}\}$

Interface force: $\{F_b\} = [M_{cb}]_b\{\ddot{u}\} + [K_{cb}]_b\{u\}$; spring force $f = k\,(x_q - x_p)$