Aero Loads: Static-Elastic, Gust and Buffet

Units (English and metric)

English: mass [lbm] /386, x, R, L_g [in], velocity [in/sec], q [psi], S [in^2], EI [lbf in^2],

forces T, V, P [lbf], moment M [lbf in], $1\,G = 386$ in/sec^2

Metric: [kg], [m], [m/sec], [Pa], [m^2], [N m^2], [N], [N m], $1\,G = 9.81$ m/sec^2

Static-elastic airload and trim (pitch plane, x positive aft)

Local airload: $N_i = q\,S\,C_{N\alpha,i}\,(\alpha - \theta_i)$, $\theta = dw/dx$ (Euler-Bernoulli beam elements)

Mean axis: $\sum m_i w_i = 0$, $\sum m_i w_i (x_i - x_{cg}) = 0$

$([K] + q[A])\{u\} + [B]\{p\} = q\,S\,\alpha\,\{C_{N\alpha}\}$, $A_{w_i\theta_i} = S\,C_{N\alpha,i}$

Trim, $p = \{a_z;\ \delta\}$: $\sum N_i + T\delta = m_{tot}\,a_z$, $\sum N_i (x_i - x_{cg}) + T\delta\,(x_g - x_{cg}) = 0$

No gimbal, $p = \{a_z;\ \ddot{\theta}\}$: $a_i = a_z - \ddot{\theta}\,(x_i - x_{cg})$, rigid: $\ddot{\theta} = -\sum N_i (x_i - x_{cg})/I_{cg}$

Divergence: smallest $q > 0$ with $\det \begin{bmatrix} K + qA & B \\ C & 0 \end{bmatrix} = 0$; clamped check: $q_D = 2EI/(S\,C_{N\alpha} L^2)$

Running loads

Net nodal load $p_i = N_i - m_i a_i + T\delta\,[i = g]$ (g = gimbal station)

Shear $V_j = \sum_{i \le j} p_i$; moment $M_j = \sum_{i<j} p_i (x_j - x_i) = EI\,w''$

Axial: $a_x = (T - qS\sum C_{A,i})/m_{tot}$, $P_j = \sum_{i \le j} (m_i a_x + qSC_{A,i}) - T\,[j \ge g]$ (compression +)

Equivalent axial load (thin cylinder, peak line load $\times\, 2\pi R$): $P_{eq} = P + 2|M|/R$

Follower: lateral thrust $T(\delta - \theta_g)$; P-delta: $[K - K_G(P)]$, $M_j \mathrel{+}= \sum_{i<j} X_i (w_i - w_j)$, $X_i = m_i a_x + qSC_{A,i} - T[i = g]$

Gust (criteria: NASA/TM-2008-215633, sec. 2.3.7)

1997 model (Eq. 2.77): $v = \frac{V_m}{2}(1 - \cos\frac{\pi d}{d_m})$, $0 \le d \le 2d_m$; V_m from Table 2-63, or $V_m/\sigma = 2.8\sqrt{1 - \frac{2}{\pi}\tan^{-1}\frac{1}{\pi d_m/L}}$ (2.82)

Classical (2.73-2.74): 30 m 1-cos edges, λ = 60–300 m; $A = 6$ ($H_b < 300$ m), $6 + \frac{3}{700}(H_b - 300)$, 9 m/s ($> 1000$ m)

$\Delta\alpha_i = v(Vt - (x_i - x_1))/V - \dot{w}_i/V - \theta_i$ (aero on), $\theta = T_\theta w$; $F_i = q\,S\,C_{N\alpha,i}\,\Delta\alpha_i + T\delta\,[i = g]$

Control: $\delta = -(K_p\theta_s + K_d\dot{\theta}_s)$, $K_p = I_{cg}\omega_c^2/(T\,l_g)$, $K_d = 2\zeta_c\omega_c I_{cg}/(T\,l_g)$, $l_g = x_g - x_{cg}$

$[M]\{\ddot{w}\} + [C_s + C_a + C_c]\{\dot{w}\} + [K_c + K_a + K_k]\{w\} = \{F_g\}$, $C_a = \frac{qS}{V}[C_{N\alpha}]$, $K_a = qS[C_{N\alpha}]T_\theta$

$M_j(t) = \sum_{i<j}(F_i - m_i\ddot{w}_i)(x_j - x_i)$; quasi-static: rigid, no control; DAF $= \max|M_{dyn}|/\max|M_{qs}|$

Buffet

$n = fD/V$, $\Phi_{C_N}(f) = nF(n)/f$ or $\Phi_{C_N,z}(f)$ per zone [1/Hz]; $S_{F,ij} = q^2 A_i A_j \Phi_{C_N}$ (same zone; 0 between zones)

$\ddot{q}_k = H_k\,\phi_k^T F$, $H_k = -\omega^2/(\omega_k^2 - \omega^2 + 2i\zeta\omega_k\omega)$; $\sigma_M^2 = \int S_M\,df$, peak $= k\sigma$

Combined: STEL $\pm$ (GUST $+k\sigma$), RSS, JSC-65829 or SP-8099 Eq. 3 forms (Load Factors tab)