Belt Drive Frequencies: Two-Sheave Open Belt Drive

belt_drive_frequencies_a ver 1.0 by Tom Irvine

Notation

D_1, D_2 driver and driven pitch diameters, N_1 driver speed (rpm), C center distance, L belt pitch length, s slip (fraction), T static tension per span, μ belt mass per unit length, n mode or harmonic number.

Geometry

1. Belt pitch length (approximate) $$L = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C}$$

2. Center distance from L $$C = \frac{1}{4}\left[b + \sqrt{b^2 - 2(D_2 - D_1)^2} \right], \qquad b = L - \frac{\pi}{2}(D_1 + D_2)$$

3. Exact open-belt length $$L_{ex} = 2C\cos\phi + \frac{\pi}{2}(D_1 + D_2) + \phi\,(D_2 - D_1), \qquad \sin\phi = \frac{D_2 - D_1}{2C}$$

4. Wrap angles $$\theta_1 = \pi - 2\phi \qquad \theta_2 = \pi + 2\phi$$

5. Free span length $$L_s = \sqrt{C^2 - \left(\frac{D_2 - D_1}{2}\right)^2}$$

6. Sheaves must not overlap $$C > \frac{1}{2}(D_1 + D_2)$$

Speeds and frequencies

7. Driven speed with slip $$N_2 = N_1 \frac{D_1}{D_2}(1 - s)$$

8. Belt speed (driver surface) $$V = \frac{\pi D_1 N_1}{60}$$

9. Shaft frequencies (Hz) $$f_1 = \frac{N_1}{60} \qquad f_2 = \frac{N_2}{60} \qquad \text{and } 2f_1,\ 2f_2$$

10. Belt pass frequency $$f_b = \frac{V}{L}, \qquad \text{harmonics } nf_b \quad (2f_b \text{ is often dominant})$$

11. Span natural frequencies $$f_n = \frac{n}{2L_s}\sqrt{\frac{T_s}{\mu}}, \qquad n = 1, 2, 3$$

12. Span tension used $$T_s = T \quad \text{or, if selected,} \quad T_s = T - \mu V^2 \quad \text{(centrifugal tension)}$$

13. Coincidence check $$100\,\frac{|f_n - f_e|}{f_e} < \text{margin} \;\Rightarrow\; \text{flagged}, \quad f_e = \text{shaft or belt frequency}$$

The span model is a taut string. Belt bending stiffness and the axial motion of the belt are neglected. Frequency in cpm = 60× Hz. Order = f/f_1.

Units

English: D, C, L, L_s in, N rpm, T lbf, μ lbm/in, V reported in ft/min.
μ is divided by 386.088 in/s^2 to give lbf-s^2/in^2, so $\sqrt{T/\mu}$ is in in/s.
metric: D, C, L, L_s mm (converted to m internally), N rpm, T N, μ kg/m, V m/s.

References: belt length and center distance, *Machinery's Handbook*, belt drive section; J. E. Shigley, *Mechanical Engineering Design*, flexible elements chapter. Taut string: R. D. Blevins, *Formulas for Natural Frequency and Mode Shape*.