Bode & Polar Plots: 1X / nX Tracking and Critical-Speed Assessment

bode_polar_plot_a ver 1.0 by Tom Irvine

Shaft angle and speed

1. Tach / keyphasor — $\theta(t_k) = 2\pi k/p$ at rising crossings t_k of level L (p pulses/rev), cubic between

2. Speed vs time table — $\theta(t) = 2\pi \int_{t_1}^{t} \dfrac{N(\tau)}{60}\, d\tau$

3. Constant ramp rate — $N(t) = N_0 + \dot{N}(t - t_1), \qquad \theta(t) = \dfrac{2\pi}{60}\left[N_0(t - t_1) + \dfrac{1}{2}\dot{N}(t - t_1)^2\right]$

Synchronous (per-revolution) DFT

4. Order resampling — $x_j = x\left(t(\theta_j)\right), \qquad \theta_j = \theta_0 + \dfrac{2\pi j}{M}, \qquad M = 64$ samples/rev

5. Order n phasor — $X_n = \dfrac{2}{MR} \displaystyle\sum_{j=0}^{MR-1} x_j\, e^{-in(\theta_j - \theta_0)}$ over each block of R revolutions

6. Block speed — $N = \dfrac{60\,R}{t_{end} - t_{start}}$ (rpm)

7. Amplitude and phase — $A = |X_n|$ (zero-to-peak), $2|X_n|$ (peak-to-peak), $\phi = -\arg X_n$ (phase lag, deg)

8. Slow-roll compensation — $X_{n,c}(N) = X_n(N) - \overline{X_n}, \qquad \overline{X_n}$ = mean of X_n for $N_{lo} \le N \le N_{hi}$

The phasors are sorted by speed and smoothed with a moving mean over about 1/400 of the points. $\theta_0 = 0$ at the first keyphasor pulse (or at t_1), so a heavy spot at the keyphasor gives $\phi = 0$ at low speed. Polar plot: ϕ clockwise from the top.

Critical speed, amplification factor and separation margin

9. Half-power method — $A(N_1) = A(N_2) = A_{peak}/\sqrt{2}, \qquad AF = \dfrac{N_{c1}}{N_2 - N_1}, \qquad \zeta \approx \dfrac{1}{2\,AF}$

10. Actual margin — $SM = 100\,\dfrac{N_{min} - N_c}{N_{min}}$ (below N_{min}), $\qquad SM = 100\,\dfrac{N_c - N_{mc}}{N_{mc}}$ (above MCOS N_{mc})

11. Required, $AF < 2.5$ — critically damped: no separation margin required

12. Required, below N_{min} — $SM = 17\left(1 - \dfrac{1}{AF - 1.5}\right)$, not more than 16 %

13. Required, above MCOS — $SM = 10 + 17\left(1 - \dfrac{1}{AF - 1.5}\right)$, not more than 26 %

14. Pumps (API 610 row) — $\zeta \ge 0.15$ ($AF \le 3.33$): margin not required; otherwise see the API 610 lateral-analysis annex

Not confirmed: rules 11-13 are from public summaries of API RP 684 and API 617 (8th ed.); check them against the standards. Rule 14 (API 610) was not confirmed from any source. A critical speed inside the operating range fails if $AF \ge 2.5$.

Example: Jeffcott rotor runup

15. Equation of motion — $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = e\left(\Omega^2\cos\theta + \dot{\Omega}\sin\theta\right)$

16. Steady-state response — $X = \dfrac{e\,r^2}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}, \qquad \phi = \tan^{-1}\dfrac{2\zeta r}{1 - r^2}, \qquad r = \dfrac{\Omega}{\omega_n}$

17. Peak and AF — $r_{peak} = \dfrac{1}{\sqrt{1 - 2\zeta^2}}, \qquad \phi = 90^\circ$ at $r = 1, \qquad AF \approx \dfrac{1}{2\zeta}$

Example: critical 3000 rpm, $\zeta = 0.05$, $e = 0.2$ mil, runup 300 to 6000 rpm at 50 rpm/s, 4096 samples/s, slow-roll runout 0.25 mil at 60°, 2X runout 0.05 mil, noise 0.02 mil rms, keyphasor. The finite sweep rate moves the peak about 0.25 % above r_{peak}.

Units

English: amplitude in mil (0.001 in); **metric**: amplitude in micron. The data are not converted; the setting only labels the results. Time in s, speed in rpm, phase in degrees, separation margin in % of operating speed.

References: API RP 684, *Tutorial on the API Standard Paragraphs Covering Rotor Dynamics* (AF, half-power method, separation margin); API 617, *Axial and Centrifugal Compressors*, 8th ed.; API 610, *Centrifugal Pumps*; D. E. Bently and C. T. Hatch, *Fundamentals of Rotating Machinery Diagnostics* (Bode and polar plots, slow-roll compensation); J. M. Vance, *Rotordynamics of Turbomachinery.*