Campbell Diagram: Finite Element Rotor with Disks & Bearings

campbell_diagram_a ver 1.0 by Tom Irvine

Rotor model

Isotropic rotor, complex coordinates $w = x + i\,y$, slope θ. Euler-Bernoulli shaft elements with rotary inertia and gyroscopics, rigid disks, undamped isotropic bearings.

1. Section properties $\quad A = \frac{\pi}{4}(D_o^2 - D_i^2) \qquad I = \frac{\pi}{64}(D_o^4 - D_i^4)$

2. Element stiffness $\quad \mathbf{K}_e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$

3. Translational mass $\quad \mathbf{M}_T = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$

4. Rotary inertia $\quad \mathbf{M}_R = \frac{\rho I}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}$

5. Shaft gyroscopic matrix $\quad \mathbf{G}_e = 2\,\mathbf{M}_R \qquad$ (polar = 2 × diametral)

6. Disk at node j $\quad M_{w_j} \mathrel{+}= m \qquad M_{\theta_j} \mathrel{+}= I_d \qquad G_{\theta_j} \mathrel{+}= I_p$

7. Bearing at node j $\quad K_{w_j} \mathrel{+}= k \qquad K_{\theta_j} \mathrel{+}= k_r$

Whirl and critical speeds

8. Equation of motion, spin speed Ω $\quad \mathbf{M}\ddot{\mathbf{q}} - i\,\Omega\,\mathbf{G}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}$

9. Whirl, $\mathbf{q} = \mathbf{Q}e^{i\nu t}$ $\quad \left[\mathbf{K} + \Omega\,\nu\,\mathbf{G} - \nu^2\mathbf{M}\right]\mathbf{Q} = \mathbf{0}$

10. Whirl direction $\quad \nu > 0$ forward, $\quad \nu < 0$ backward, $\quad f = |\nu|/2\pi$

11. Forward critical, order n ($\nu = n\Omega$) $\quad \left[\mathbf{K} - \nu^2\,(\mathbf{M} - \mathbf{G}/n)\right]\mathbf{Q} = \mathbf{0}$

12. Backward critical ($\nu = -n\Omega$) $\quad \left[\mathbf{K} - \nu^2\,(\mathbf{M} + \mathbf{G}/n)\right]\mathbf{Q} = \mathbf{0}$

13. Critical speed $\quad N_{cr} = \frac{60}{2\pi}\,\frac{|\nu|}{n}$ rpm $\qquad f_{cr} = n\,\frac{N_{cr}}{60}$ Hz

Separation margin (operating range N_{min} to N_{max})

$SM = 100\,\frac{N_{min} - N_{cr}}{N_{min}}$ % below the range, $\qquad SM = 100\,\frac{N_{cr} - N_{max}}{N_{max}}$ % above the range

A critical with SM less than the required margin is flagged. Set the required margin from the applicable standard.

Units

English: E psi, ρ lbm/in^3, lengths in, m lbm, I_p, I_d lbm-in^2, k lbf/in, k_r lbf-in/rad.
Mass terms are divided by 386.088 in/s^2 to give lbf-s^2/in.

metric: E Pa, ρ kg/m^3, lengths m, m kg, I_p, I_d kg-m^2, k N/m, k_r N-m/rad.

Speeds in rpm and frequencies in Hz in both systems.

Thin uniform disk (radius r, thickness h): $\quad I_p = \frac{1}{2}mr^2, \quad I_d = \frac{1}{12}m(3r^2 + h^2)$

Reference: H. D. Nelson and J. M. McVaugh, *The Dynamics of Rotor-Bearing Systems Using Finite Elements*, ASME Journal of Engineering for Industry, 98(2), 1976, pp. 593-600.