Soil Springs for Block Foundations: Rigid Footing on an Elastic Half-Space

foundation_soil_springs_a ver 1.0 by Tom Irvine

Model

Lysmer analog (Richart, Hall & Woods 1970): a rigid footing on a uniform elastic half-space is replaced by a spring k and a frequency-independent radiation dashpot for each mode. G shear modulus, ν Poisson's ratio, ρ soil mass density, m total mass, I_ψ mass moment of inertia about the rocking axis at the base, I_θ about the vertical axis, r_0 equivalent radius.

Equivalent radius (rectangle $B \times L$, L in the plane of rocking; circle: r_0 for all modes)

$r_0 = \sqrt{\dfrac{BL}{\pi}}$ (vertical, horizontal) $r_{0\psi} = \left(\dfrac{BL^3}{3\pi}\right)^{1/4}$ (rocking) $r_{0\theta} = \left(\dfrac{BL(B^2+L^2)}{6\pi}\right)^{1/4}$ (torsion)

Springs, mass ratios and damping ratios

Mode	*Stiffness k*	*Mass ratio B*	*Damping ratio D*
Vertical	$\dfrac{4Gr_0}{1-\nu}$	$\dfrac{1-\nu}{4}\,\dfrac{m}{\rho r_0^3}$ †	$\dfrac{0.425}{\sqrt{B_z}}$
Horizontal	$\dfrac{32(1-\nu)Gr_0}{7-8\nu}$	$\dfrac{7-8\nu}{32(1-\nu)}\,\dfrac{m}{\rho r_0^3}$ †	$\dfrac{0.288}{\sqrt{B_x}}$ †
Rocking	$\dfrac{8Gr_{0\psi}^3}{3(1-\nu)}$	$\dfrac{3(1-\nu)}{8}\,\dfrac{I_\psi}{\rho r_{0\psi}^5}$ †	$\dfrac{0.15}{(1+B_\psi)\sqrt{B_\psi}}$
Torsion	$\dfrac{16}{3}\,Gr_{0\theta}^3$	$\dfrac{I_\theta}{\rho r_{0\theta}^5}$ †	$\dfrac{0.50}{1+2B_\theta}$ †

† Not confirmed from a public source (Table 10-1 of Richart, Hall & Woods as commonly reproduced); check against the book. Confirmed: all four k (Richart & Whitman 1967 table), equivalent radii and embedment (TM 5-818-1), vertical 0.425 (ACI 351.3R-18), rocking D (TM 5-818-1 example: $B_\psi = 3$ gives $D_\psi \approx 0.02$; formula gives 0.0217).

Embedment (optional, vertical only, $d/r_0 \le 1$, TM 5-818-1 approximation)

$k_{zd} = k_z\,(1 + 0.6\,d/r_0)$ $D_{zd} = D_z\,(1 + 0.6\,d/r_0)$ other modes keep the surface values (conservative).

Frequencies and response (each mode, $m_i = m, m, I_\psi, I_\theta$)

$f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_i}}$ $f_d = f_n\sqrt{1-D^2}$ $c = 2D\sqrt{k\,m_i}$ $f_{peak} = f_n\sqrt{1-2D^2}$ (constant force), $\dfrac{f_n}{\sqrt{1-2D^2}}$ (unbalance)

$M = \dfrac{1}{\sqrt{(1-r^2)^2 + (2Dr)^2}}$, $r = \dfrac{N/60}{f_n}$ $A = \dfrac{F_i}{k}\,M$, $F_i = F_0,\ F_0,\ F_0 h,\ F_0 e$ (vertical, horizontal, rocking, torsion)

F_0 = unbalance force amplitude at the operating speed N; h = height of the force above the base; e = offset from the vertical axis. Speed plot: force = $F_0(n/N)^2$. Rocking and torsional displacement at the force: ψh, θe. Flag: $|r-1| < 0.2$ (rule of thumb).

Units

English: L, B, r_0, d, h, e in; m lbm and I lbm in^2 (divided by 386.088); G psi or V_s ft/s; ρ lbm/ft^3 ($\div$ 1728 $\div$ 386.088); F_0 lbf; k lbf/in and lbf-in/rad; amplitude in or rad. $G = \rho V_s^2$.

metric: m, kg, kg m^2, G Pa or V_s m/s, ρ kg/m^3, F_0 N; k N/m and N-m/rad; amplitude m or rad.

Assumptions

Rigid footing on the surface of a uniform, isotropic elastic half-space; no soil layer over rock, no material (hysteretic) damping.

Uncoupled modes: in reality horizontal sliding and rocking are coupled. For a coupled analysis enter the k values in the "Block & Foundation, Three dof" app. Constant k, D are best for $a_0 = \omega r_0/V_s$ below about 1.5 (guide value).

Lysmer-analog radiation damping is often overestimated in practice; rocking damping is low, so rocking resonance can be severe.

References

F. E. Richart, J. R. Hall and R. D. Woods, *Vibrations of Soils and Foundations*, Prentice-Hall, 1970, Ch. 10 (Table 10-1).

R. V. Whitman and F. E. Richart, Design procedures for dynamically loaded foundations, *J. SMFD ASCE*, 93(SM6), 1967.

TM 5-818-1 / AFM 88-3 Ch. 7 (UFC 3-220-03FA), Ch. 17 Machine foundations: eqs. 17-6 to 17-12, Table 17-1, embedment.

ACI 351.3R-18, *Report on Foundations for Dynamic Equipment*, ACI Committee 351 (Lysmer constant 0.425).