Full Spectrum: Forward and Reverse Whirl from an X-Y Proximity-Probe Pair

full_spectrum_a ver 1.0 by Tom Irvine

Probe geometry (viewed from the driver end)

Probe angles θ_X, θ_Y from vertical, + right, − left (Bently convention: $\theta_X = +45°$, $\theta_Y = -45°$).

Each probe reads the shaft motion toward it. u = horizontal (right), v = vertical (up). Probe DC (gap) is removed first.

1. Probe signals $\quad s_X = u\sin\theta_X + v\cos\theta_X, \qquad s_Y = u\sin\theta_Y + v\cos\theta_Y$

2. Resolve to orthogonal axes $\quad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sin\theta_X & \cos\theta_X \\ \sin\theta_Y & \cos\theta_Y \end{bmatrix}^{-1} \begin{bmatrix} s_X \\ s_Y \end{bmatrix}, \quad \det = \sin(\theta_X - \theta_Y) \neq 0$

3. Complex orbit $\quad z(t) = u + j\,v$ (CCW rotation), $\qquad z(t) = u - j\,v$ (CW rotation)

Positive frequency = forward whirl (with rotation) for either direction; no X/Y probe-order rule is needed.

Full spectrum

4. Segment FFT, window w_n $\quad Z_k = \dfrac{\sum_{n=0}^{L-1} w_n\, z_n\, e^{-j2\pi kn/L}}{\sum_{n=0}^{L-1} w_n}, \qquad f_k = \dfrac{k}{L\,\Delta t}, \quad k = -\dfrac{L}{2} \ldots \dfrac{L}{2} - 1$

5. Average of N_s segments $\quad A(f_k) = \sqrt{\dfrac{1}{N_s}\sum_{i=1}^{N_s} |Z_{k,i}|^2}, \qquad L = \text{floor}(N/N_s), \quad \Delta f = \dfrac{1}{L\,\Delta t}$

6. Windows (periodic) $\quad$ Hanning: $w_n = \dfrac{1}{2} - \dfrac{1}{2}\cos\dfrac{2\pi n}{L}$; $\quad$ flat top: $\displaystyle\sum_{m=0}^{4} (-1)^m a_m \cos\frac{2\pi mn}{L}$

A at $+f$ = 0-pk radius of the forward circle, at $-f$ the reverse circle. Flat top a_m = 0.21557895, 0.41663158, 0.277263158, 0.083578947, 0.006947368 (as MATLAB `flattopwin`). Orders axis: f/f_1, f_1 = shaft speed (Hz).

Orders and orbit ellipse

7. Shaft angle $\quad \phi = 2\pi f_1 (t - t_0)$ (constant rpm); $\quad \phi(t_m) = 2\pi m$ at tach pulse m, linear between

8. Least-squares order fit $\quad u + jv = c_0 + \sum_{n=1}^{N_o} \left(P_n e^{jn\phi} + Q_n e^{-jn\phi}\right)$

9. Forward / reverse $\quad F_n = P_n,\ R_n = Q_n$ (CCW); $\qquad F_n = Q_n,\ R_n = P_n$ (CW)

10. Ellipse semi-axes $\quad a_n = |F_n| + |R_n|, \qquad b_n = \big|\,|F_n| - |R_n|\,\big|, \qquad$ pk-pk along major axis $= 2a_n$

11. Inclination of major axis $\quad \psi_n = \dfrac{1}{2}\arg(P_n Q_n)$ (CCW from horizontal)

12. Whirl direction $\quad |F_n| > |R_n| \Rightarrow$ forward, $\quad |F_n| < |R_n| \Rightarrow$ reverse, $\quad |F_n| = |R_n| \Rightarrow$ line orbit

13. Filtered orbit, order n $\quad u_n + jv_n = P_n e^{j\theta} + Q_n e^{-j\theta}, \quad \theta = 0 \ldots 2\pi, \quad$ keyphasor dot at $\theta = 0$

Check: probe sinusoids $s_X = A_X\cos(\phi + \alpha_X)$, $s_Y = A_Y\cos(\phi + \alpha_Y)$ on orthogonal probes give

$$a^2, b^2 = \tfrac{1}{2}\left[A_X^2 + A_Y^2 \pm \sqrt{(A_X^2 + A_Y^2)^2 - 4A_X^2 A_Y^2 \sin^2(\alpha_Y - \alpha_X)}\right]$$

The least-squares fit is locked to the shaft angle, so the order table has no window scalloping or leakage error.

The spectrum plot is the windowed FFT. With a tach, the record is trimmed to the first and last pulse.

Units

English: displacement in mil (0.001 in). **metric**: displacement in μm. Time s, speed rpm, frequency Hz, angles deg.

Amplitudes are reported in the unit of the input data (0-pk circle radii, ellipse semi-axes). No unit conversion is applied.

Assumptions: probes non-parallel, linear, equally scaled; uniform time step; no runout compensation.

References: D. E. Bently and C. T. Hatch, *Fundamentals of Rotating Machinery Diagnostics*, Bently Pressurized Bearing Press, 2002.

"Application of full spectrum to rotating machinery diagnostics," Bently Nevada *Orbit* (P. Goldman and A. Muszynska, c. 1999; authors and date not confirmed from a source, check before citing).