Gear Mesh Frequencies: Parallel-Axis Gear Trains & Planetary Stages

gear_mesh_frequencies_a ver 1.1 by Tom Irvine

Parallel-axis gear train

Stage i: driver gear with Z_1 teeth on shaft i, driven gear with Z_2 teeth on shaft $i+1$. Shaft 1 turns at the input speed.

1.	Shaft speed	$f_{i+1} = f_i \dfrac{Z_1}{Z_2}$ $\quad f_1 = N_{in}/60$
2.	Gear mesh frequency	$GMF = Z_1 f_i = Z_2 f_{i+1}$ $\quad$ harmonics $h\,GMF$
3.	Sidebands	$GMF \pm k\,f_i$ $\quad GMF \pm k\,f_{i+1}$ $\quad k = 1, 2, \ldots$
4.	Assembly phases	$N_a = \gcd(Z_1, Z_2)$
5.	Hunting tooth frequency	$f_{HT} = \dfrac{GMF\,N_a}{Z_1 Z_2} = \dfrac{f_i\,N_a}{Z_2}$
6.	Assembly phase frequency	$f_{AP} = GMF/N_a$ $\quad (N_a > 1)$
7.	Stage ratio	Z_2/Z_1 $\quad$ overall ratio f_1/f_{n+1}

With $N_a = 1$ every tooth of one gear meets every tooth of the other; a damaged tooth pair repeats at f_{HT}.

Planetary stage

Sun Z_s, planet Z_p, ring Z_r teeth, N_p planets. Speeds f_s, f_c, f_r (sun, carrier, ring). One member is fixed, one is the input, the third is the output.

8.	Kinematics (Willis)	$Z_s\,(f_s - f_c) + Z_r\,(f_r - f_c) = 0$
9.	Ring fixed, sun input	$f_c = f_s \dfrac{Z_s}{Z_s + Z_r}$
10.	Gear mesh frequency	$GMF = Z_s\,\lvert f_s - f_c\rvert = Z_r\,\lvert f_r - f_c\rvert$
11.	Planet spin	$f_p - f_c = -\dfrac{Z_s}{Z_p}(f_s - f_c)$ $\quad \lvert f_p - f_c\rvert = GMF/Z_p$
12.	Sun local fault	$N_p\,GMF/Z_s = N_p\,\lvert f_s - f_c\rvert$
13.	Ring local fault, planet pass	$N_p\,GMF/Z_r = N_p\,\lvert f_r - f_c\rvert$
14.	Planet local fault	GMF/Z_p (one side), $\quad 2\,GMF/Z_p$ (both sides)
15.	Sidebands	$GMF \pm k\,f_c$ (ring or sun fixed), $\quad GMF \pm k\,f_s,\ \pm k\,f_r$ (carrier fixed)
16.	Geometry checks	$Z_r = Z_s + 2Z_p$, $\quad$ equal spacing needs $(Z_s + Z_r)/N_p$ integer

Notes

All frequencies in Hz and cpm; order = frequency / input speed. Speeds are listed as magnitudes with direction.

The planet spin relative to the carrier equals the one-sided planet fault frequency GMF/Z_p.

The plotted spectrum is schematic: line heights show the type of component, not predicted amplitudes.

Sideband amplitudes in planetary gearboxes depend on planet phasing and the sensor location.

Reference for the planetary local fault frequencies: Z. Feng and M. J. Zuo, *Vibration signal models for fault diagnosis of planetary gearboxes*, Journal of Sound and Vibration, 331(22), 2012, pp. 4919-4939.

Gear Geometry and Torque: Parallel-Axis Stages

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Standard full-depth spur gears. Metric: module m (mm). English: diametral pitch P_d (1/in), $m = 25.4/P_d$.
In the English column every length is in inches.

Quantity	Metric (mm)	English (in)
17. Module, diametral pitch	$m = D/Z$	$P_d = Z/D$
18. Pitch diameter	$D = m\,Z$	$D = Z/P_d$
19. Centre distance	$a = m\,(Z_1 + Z_2)/2$	$a = (Z_1 + Z_2)/(2P_d)$
20. Circular pitch	$p_c = \pi\,m$	$p_c = \pi/P_d$
21. Addendum	$h_a = m$	$h_a = 1/P_d$
22. Dedendum (full depth)	$h_f = 1.25\,m$	$h_f = 1.25/P_d$
23. Whole depth	$h = 2.25\,m$	$h = 2.25/P_d$
24. Outside diameter	$D_a = m\,(Z + 2)$	$D_a = (Z + 2)/P_d$
25. Root diameter	$D_f = m\,(Z - 2.5)$	$D_f = (Z - 2.5)/P_d$

Speed and torque

26. Ratio and speed: $i = \dfrac{Z_2}{Z_1} = \dfrac{N_1}{N_2}$ $\qquad N_2 = N_1\,\dfrac{Z_1}{Z_2}$

27. Torque through a stage: $T_2 = T_1\,i\,\eta$ $\qquad$ ($\eta = 1$ ideal; η = mesh efficiency)

28. Torque from power: $T = P/\omega$, $\qquad \omega = 2\pi N/60$ rad/s $\quad$ (N in rpm)

Metric: T (N m) $= \dfrac{60000\,P\ \text{(kW)}}{2\pi N} = 9549.3\,\dfrac{P}{N} \approx 9550\,\dfrac{P}{N}$

English: T (in-lbf) $= \dfrac{396000\,P\ \text{(hp)}}{2\pi N} = 63025\,\dfrac{P}{N}$ $\qquad (= 5252\,P/N$ lbf-ft)

29. Output power: $P_{out} = P_{in}\,\eta^n$ $\qquad n$ = number of stages

Checks

$Z_1 = 20$, $Z_2 = 60$, $m = 3$ mm, 1500 rpm, 100 N m: $D = 60$ and 180 mm, $a = 120$ mm, 500 rpm, 300 N m ($\eta = 1$).

15 kW at 1450 rpm: $T = 9549.3 \times 15/1450 = 98.8$ N m.

Notes

The torque of each gear is the torque of its shaft; η is applied once per mesh.

A root diameter at or below zero (too few teeth) is flagged. Undercut and profile shift are not modelled.

Tooth bending and contact stresses are not calculated (see AGMA 2001 or ISO 6336).