Journal Bearing & Oil Whirl: Plain Cylindrical Fluid-Film Bearing

journal_bearing_a ver 1.0 by Tom Irvine

Notation (all values per bearing)

$D = 2R$ journal diameter, L bearing length, C radial clearance, μ dynamic viscosity, W static load, M rotor mass carried by the bearing, N speed (rpm), $\omega = 2\pi N/60$ (rad/s), e eccentricity, $\varepsilon = e/C$, ϕ attitude angle (from the load line, in the direction of rotation), $h = C(1 + \varepsilon\cos\theta)$ film thickness. Coordinates: x = direction of W, $y = 90^\circ$ ahead in the direction of rotation.

Operating point

1. Sommerfeld number
$$S = \frac{\mu N_s L D}{W}\left(\frac{R}{C}\right)^2, \qquad N_s = N/60 \text{ in rev/s}$$

2. Short bearing load
$$W = \frac{\mu\,\omega R L^3}{4\,C^2}\,\frac{\varepsilon\sqrt{16\varepsilon^2 + \pi^2(1-\varepsilon^2)}}{(1-\varepsilon^2)^2} \quad\Leftrightarrow\quad S\left(\frac{L}{D}\right)^2 = \frac{(1-\varepsilon^2)^2}{\pi\varepsilon\sqrt{16\varepsilon^2 + \pi^2(1-\varepsilon^2)}}$$

3. Short-bearing attitude angle
$$\tan\phi = \frac{\pi\sqrt{1-\varepsilon^2}}{4\,\varepsilon}$$

4. Long bearing load
$$W = \frac{\mu\,\omega R^3 L}{C^2}\,\frac{6\varepsilon\sqrt{4\varepsilon^2 + \pi^2(1-\varepsilon^2)}}{(2+\varepsilon^2)(1-\varepsilon^2)}, \qquad \tan\phi = \frac{\pi\sqrt{1-\varepsilon^2}}{2\,\varepsilon}$$

5. Film thickness
$$h_{min} = C(1-\varepsilon), \qquad e = \varepsilon C$$

6. Short-bearing pressure
$$p = \frac{3\mu\omega}{C^2}\left(\frac{L^2}{4} - z^2\right)\frac{\varepsilon\sin\theta}{(1+\varepsilon\cos\theta)^3}, \quad p_{max} \text{ at } z = 0,\ \cos\theta_m = \frac{1-\sqrt{1+24\varepsilon^2}}{4\varepsilon}$$

7. Long-bearing pressure
$$p = \frac{6\mu\omega R^2}{C^2}\,\frac{\varepsilon\sin\theta\,(2+\varepsilon\cos\theta)}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2}, \qquad p \geq 0\ (0 \leq \theta \leq \pi)$$

8. Petroff torque and power
$$T_P = \frac{2\pi\mu\omega R^3 L}{C}, \qquad P_P = T_P\,\omega$$

9. Bearing friction torque
$$T = \frac{T_P}{\sqrt{1-\varepsilon^2}} + \frac{We}{2}\sin\phi, \qquad P = T\omega, \qquad f = \frac{T}{WR}$$

Dynamic coefficients, short bearing (closed form)

$F_x = -K_{xx}x - K_{xy}y - C_{xx}\dot{x} - C_{xy}\dot{y}$ (likewise F_y); $\quad k_{ij} = K_{ij}\,C/W, \quad c_{ij} = C_{ij}\,\omega C/W$

$Q = \left[\pi^2(1-\varepsilon^2) + 16\varepsilon^2\right]^{3/2}, \quad s = \sqrt{1-\varepsilon^2}$

$$k_{xx} = \frac{4}{Q}\left[\pi^2(1+2\varepsilon^2) + \frac{32\varepsilon^2(1+\varepsilon^2)}{1-\varepsilon^2}\right] \qquad k_{xy} = \frac{\pi\left[\pi^2(1-\varepsilon^2)(1+2\varepsilon^2) + 32\varepsilon^2(1+\varepsilon^2)\right]}{\varepsilon\,s\,Q}$$

$$k_{yx} = -\frac{\pi\left[\pi^2(1-\varepsilon^2)^2 - 16\varepsilon^4\right]}{\varepsilon\,s\,Q} \qquad k_{yy} = \frac{4}{Q}\left[\pi^2(2-\varepsilon^2) + 16\varepsilon^2\right]$$

$$c_{xx} = \frac{2\pi\left[\pi^2(1-\varepsilon^2)^2 + 48\varepsilon^2\right]}{\varepsilon\,s\,Q} \qquad c_{xy} = c_{yx} = \frac{8}{Q}\left[\pi^2(1+2\varepsilon^2) - 16\varepsilon^2\right]$$

$$c_{yy} = \frac{2\pi s\left[\pi^2(1+2\varepsilon^2) - 16\varepsilon^2\right]}{\varepsilon\,Q}$$

Same set as Rao (via the COMSOL verification model), written in the x (load), y frame; checked here by finite-difference perturbation of the numerically integrated π-film force (agreement 10^{-8}). Note the squared $(1-\varepsilon^2)^2$ in k_{yx}.

Long bearing coefficients

Film force from the infinitely-long Reynolds equation $\frac{d}{d\theta}\left(h^3\frac{dp}{d\theta}\right) = 6\mu R^2\left(\omega\frac{dh}{d\theta} + 2\frac{\partial h}{\partial t}\right)$, integrated numerically (16384 points), ambient pressure at the maximum film, negative pressure set to zero (π-film). The 8 coefficients are central finite differences of this force about the closed-form equilibrium. Check: the stiffness equals the derivative of the closed-form static force, and the whirl-velocity damping obeys $c\,\hat{t} = -2/\varepsilon\,\hat{x}$.

Caution: long π-film damping depends on the ambient-pressure convention; its light-load whirl ratio is 0.59.

Oil-whirl stability of a rigid, symmetric rotor (Lund's threshold)

10. Equivalent stiffness $\quad k_{eq} = \dfrac{k_{xx}c_{yy} + k_{yy}c_{xx} - k_{xy}c_{yx} - k_{yx}c_{xy}}{c_{xx} + c_{yy}}, \qquad K_{eq} = k_{eq}\,W/C$

11. Whirl frequency ratio $\quad \gamma^2 = \dfrac{(k_{xx} - k_{eq})(k_{yy} - k_{eq}) - k_{xy}k_{yx}}{c_{xx}c_{yy} - c_{xy}c_{yx}}$

12. Threshold speed $\quad \dfrac{M\,C\,\omega_{th}^2}{W} = \dfrac{k_{eq}}{\gamma^2} \qquad \Rightarrow \qquad \omega_{th} = \dfrac{1}{\gamma}\sqrt{\dfrac{k_{eq}\,W}{M\,C}}$

13. Whirl frequency $\quad \nu = \gamma\,\omega_{th}$ (short bearing: 0.5 X at light load, up to 0.52 X near $\varepsilon = 0.3$, less at high ε)

Derived from $\det\left[K - M\nu^2 I + i\nu C\right] = 0$ (real and imaginary parts). If $\gamma^2 \le 0$ or $k_{eq} \le 0$ the rotor is stable at any speed: for the short bearing this is $\varepsilon > 0.756$. Because ε falls as speed rises, the **onset speed** is found where $MC\omega^2/W = k_{eq}/\gamma^2$ along the operating curve $\varepsilon(\omega)$ (bisection in ε). Above it the rotor whirls at $\gamma\,\omega$ (oil whirl). Oil *whip* of a flexible shaft, which locks to its first critical speed, is not modelled.

Units

English: D, L, C in, μ reyn (lbf·s/in^2) or cP, W lbf, M lbm (divided by 386 to give lbf·s^2/in), K lbf/in, C_{ij} lbf·s/in, h mil, p psi, T lbf·in, P hp.

metric: D, L, C mm, μ Pa·s or cP, W N, M kg, K N/m, C_{ij} N·s/m, h μm, p MPa, T N·m, P W.

1 reyn = 6894.76 Pa·s, 1 cP $= 10^{-3}$ Pa·s $= 1.4504 \times 10^{-7}$ reyn. Speeds in rpm; S uses rev/s.

Assumptions

- Plain 360° cylindrical bearing, aligned, rigid; laminar, isoviscous, incompressible film; no inertia, no thermal effects.
- Half-Sommerfeld (π-film) cavitation. Short bearing best for $L/D \le 0.5$ (fair to 1); long bearing for $L/D \ge 2$.
- Friction torque uses full-film Couette shear (no reduction in the cavitated region). Rigid rotor, equal bearings.

Worked example (Load Example; checked in the test script)

$D = 4$ in, $L = 2$ in, $C = 0.004$ in, $\mu = 2 \times 10^{-6}$ reyn, $W = 1000$ lbf, $M = 1000$ lbm, 3600 rpm, short bearing: $S = 0.24$, $S(L/D)^2 = 0.06$, $\varepsilon = 0.6072$, $\phi = 45.79°$, $h_{min} = 1.571$ mil, $T = 24.72$ lbf·in, $P = 1.412$ hp, onset 7511 rpm with whirl 0.520 X (eigenvalues of the rigid rotor confirm), stable at 3600 rpm.

Load at a known eccentricity: $\varepsilon = 0.5 \Rightarrow S(L/D)^2 = 0.10605$, $\phi = 53.68°$.

References

1. G. B. DuBois, F. W. Ocvirk, *Analytical derivation and experimental evaluation of short-bearing approximation for full journal bearings*, NACA Report 1157, 1953 (short-bearing theory).
2. J. S. Rao, *Rotor Dynamics*, New Age International, 2014, Sec. 7.6; as listed in the COMSOL Application Gallery model *Journal Bearing Dynamic Coefficients* (short-bearing k_{ij}, c_{ij}, attitude angle).
3. M. I. Friswell, J. E. T. Penny, S. D. Garvey, A. W. Lees, *Dynamics of Rotating Machines*, Cambridge, 2010, Ch. 5.
4. D. Childs, *Turbomachinery Rotordynamics*, Wiley, 1993, Ch. 3; J. W. Lund, rigid-rotor threshold.
5. L. San Andrés, *Hydrodynamic Fluid Film Bearings and Their Effect on the Stability of Rotating Machinery*, NATO RTO-EN-AVT-143, paper 10, 2006.

Note: only ref. 2 (via COMSOL) was read online. Refs. 3 to 5 were not consulted directly. The k_{yx} as read from that page lacks the square on $(1 - \varepsilon^2)$; the squared form above matches the numerical check. The stability formulas were derived here and checked against rigid-rotor eigenvalues. Long-bearing coefficients are this app's own numerical result.