Machine Fault Signal Synthesis: Acceleration Time History with Selected Faults

machine_fault_synthesis_a ver 1.0 by Tom Irvine

Shaft speed and angle

1. Speed (constant: $f_1 = f_0$) $\quad f(t) = f_0 + (f_1 - f_0)\,\dfrac{t}{T}, \qquad f_0 = \dfrac{N_0}{60}, \quad f_1 = \dfrac{N_1}{60}$

2. Shaft angle $\quad \phi(t) = 2\pi \int_0^t f\, dt = 2\pi \left[f_0 t + \dfrac{(f_1 - f_0)\, t^2}{2T} \right]$

Every component is a function of ϕ, so all orders track a run-up. φ_i are random phases from the seed.

Fault models (A = amplitude entered, 0-peak)

3. Unbalance $\quad A \sin(\phi + \varphi_1)$

4. Misalignment $\quad A\,[0.5 \sin(\phi + \varphi_2) + \sin(2\phi + \varphi_3) + 0.3 \sin(3\phi + \varphi_4)]$

5. Mechanical looseness $\quad \displaystyle\sum_{n=1}^{10} \frac{A}{n} \sin(n\phi + \varphi_{4+n}) + \sum_{k=1}^{10} \frac{A}{2k} \sin\left[(k - 1/2)\phi + \varphi_{14+k}\right]$

6. Gear mesh, Z_1 teeth $\quad A\,[1 + 0.4 \cos(\phi + \varphi_{25}) + 0.4 \cos((Z_1/Z_2)\phi + \varphi_{26})] \sin(Z_1\phi + \varphi_{27}) + 0.5A \sin(2Z_1\phi + \varphi_{28})$

$\text{GMF} = Z_1 f$, sidebands GMF $\pm f$ and GMF $\pm \dfrac{Z_1}{Z_2} f$, each $0.2A$

7. White noise $\quad \sigma\, w(t)$, $\quad w$ Gaussian, unit variance (σ = rms level)

Rolling element bearing (inner race rotating with the shaft, outer race fixed)

8. Defect orders, $c = \frac{d}{D} \cos\theta$ $\quad \text{FTF} = \dfrac{1}{2}(1 - c) \qquad \text{BPFO} = \dfrac{n}{2}(1 - c) \qquad \text{BPFI} = \dfrac{n}{2}(1 + c) \qquad \text{BSF} = \dfrac{D}{2d}(1 - c^2)$

9. Impact angles $\quad \phi_k = \phi_0 + \dfrac{2\pi}{O} \displaystyle\sum_{j<k} (1 + s\, u_j), \quad u_j \sim U(-1, 1), \quad O = \text{BPFO, BPFI, 2BSF or FTF}$

10. Impact times $\quad \phi(t_k) = \phi_k$ (inverted by interpolation)

11. Response to each impact $\quad a_k\, e^{-\zeta \omega_n \tau} \sin(\omega_d \tau), \quad \tau = t - t_k \geq 0, \quad \omega_n = 2\pi f_n, \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}$

12. Impact amplitude $\quad a_k = A$ (outer race, cage); $A[0.6 + 0.4 \cos(\phi_k + \varphi_{29})]$ (inner race, load zone at 1x);

$A[0.6 + 0.4 \cos(\text{FTF}\, \phi_k + \varphi_{30})]$ (rolling element, impacts at 2BSF, carried by the cage)

s = random slip (fraction). Real bearings slip about 1-2 %, which smears the high harmonics in the spectrum. Only the ratio d/D enters, so d and D may be in inches (English) or mm (metric).

Tach pulse and spectra

13. Tach (once per rev) $\quad$ 0 to 5 V pulse, about 10° wide, 3° ramps, rising edge (2.5 V) at $\phi = 2\pi m$

14. FFT (Hann window) $\quad A(f_k) = \dfrac{2\,|X_k|}{\sum w_i}, \quad X_k = \sum (y_i - \bar{y})\, w_i\, e^{-j2\pi k i/N}, \quad w_i = \dfrac{1}{2} - \dfrac{1}{2} \cos\dfrac{2\pi i}{N}$

15. Envelope (FFT only) $\quad e(t) = \left|\text{IFFT}\{H_k X_k\}\right|, \quad H_k = 2$ for $f_n - B \leq f_k \leq f_n + B$, else 0; then the FFT of e

16. Envelope band $\quad B = \min\left[0.5 f_n,\ \max(0.2 f_n,\ 2.5 f_{max})\right], \quad f_{max}$ = highest of BPFO, BPFI + 1x, 2BSF + FTF

17. Statistics $\quad \text{crest} = \dfrac{\max |y|}{y_{rms}}, \qquad \text{kurtosis} = \dfrac{\overline{(y - \bar{y})^4}}{\left[\overline{(y - \bar{y})^2}\right]^2}$

Units and output

Amplitude unit G or m/s^2 (1 G = 9.80665 m/s^2); switching converts the amplitudes and noise level. Speed rpm, frequency Hz. **English**: d, D in. **metric**: d, D mm. Exported arrays are $n \times 2$: [time (sec), acceleration] and [time (sec), tach (V)], the same convention the other vibrationdata apps read. Every component must be below $f_s/2$.

Assumptions

The amplitude ratios (misalignment $0.5 : 1 : 0.3$, looseness $1/n$, gear sidebands $0.2A$, load-zone depth 0.4) are teaching choices, not values from a standard. One structural resonance is excited by all bearing impacts; transfer paths and speed-dependent amplitudes are ignored. Bearing frequency formulas are kinematic (no slip in the mean, no load effect on θ).

References: R. B. Randall and J. Antoni, Rolling element bearing diagnostics – a tutorial, *Mech. Systems and Signal Processing* 25 (2011) 485-520; R. B. Randall, *Vibration-based Condition Monitoring*, Wiley, 2011; P. D. McFadden and J. D. Smith, Model for the vibration produced by a single point defect in a rolling element bearing, *J. Sound Vib.* 96 (1984) 69-82; Bruel & Kjaer, Detecting faulty rolling element bearings (BO0501). The 1-2 % Randall & Antoni from secondary summaries; check against the paper.