machinery_statistics_a: Equations & Use

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The input is a time history with n samples x_i at times t_i, i = 1 ... n. The amplitude unit is whatever the input array uses (for example G, in/sec^2 or m/sec^2), so the same equations apply in English and metric units. Results in amplitude units are noted; all other metrics are dimensionless.

1. Time Step & Sample Rate

$$\Delta t = \frac{t_n - t_1}{n-1} \qquad sr = \frac{1}{\Delta t} \qquad sr_{min} = \frac{1}{\max(t_{i+1} - t_i)} \qquad sr_{max} = \frac{1}{\min(t_{i+1} - t_i)}$$

The minimum and maximum confirm uniform sampling. They should equal sr for data from a single digital recorder.

2. Amplitude Statistics

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2} \qquad x_{rms} = \sqrt{\sigma^2 + \mu^2}$$

Mean, standard deviation and RMS are in amplitude units. RMS is the overall vibration level and the first quantity to trend. A non-zero mean in accelerometer data usually indicates a sensor offset or a DC shift rather than vibration.

$$\text{skewness} = \frac{1}{n\,\sigma^3}\sum_{i=1}^{n}(x_i - \mu)^3 \qquad \text{kurtosis} = \frac{1}{n\,\sigma^4}\sum_{i=1}^{n}(x_i - \mu)^4$$

Skewness is zero for a symmetric signal. Kurtosis is 3 for Gaussian random vibration and 1.5 for a sine. Kurtosis above 3 indicates impulsive content such as bearing impacts, gear tooth damage, rubs or looseness.

3. Peak & Waveform Shape Metrics

$|x_i|$ is the absolute value of sample x_i: its magnitude with the sign removed, so positive and negative excursions count equally. An overbar denotes the mean over all n samples:

$$\overline{|x|} = \frac{1}{n}\sum_{i=1}^{n}|x_i| \qquad \overline{\sqrt{|x|}} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{|x_i|} \qquad x_{pk} = \max|x_i|$$

$$\text{Peak}-\text{to}-\text{Peak} = \max x_i - \min x_i \qquad \text{Crest Factor} = \frac{x_{pk}}{x_{rms}}$$

$$\text{Shape Factor} = \frac{x_{rms}}{\overline{|x|}} \qquad \text{Impulse Factor} = \frac{x_{pk}}{\overline{|x|}}$$

$$\text{Clearance Factor} = \frac{x_{pk}}{\left(\overline{\sqrt{|x|}}\right)^2} \qquad \text{K}-\text{factor} = x_{pk}\,x_{rms}$$

3. Peak & Waveform Shape Metrics (continued)

x_{pk}, peak-to-peak and the mean absolute value are in amplitude units, and K-factor is in amplitude units squared. Crest, impulse and clearance factors are dimensionless and increase with impulsiveness. Clearance factor is the most sensitive to early bearing spalls, followed by impulse factor and crest factor. Shape factor changes slowly and indicates the overall waveform type.

Reference values:

	Sine	**Gaussian random**
Kurtosis	1.5	3
Crest factor	1.414 ($\sqrt{2}$)	about 3 to 5, grows with record length
Shape factor	1.111 ($\pi/(2\sqrt{2})$)	1.253 ($\sqrt{(\pi/2)}$)
Impulse factor	1.571 ($\pi/2$)	grows with record length
Clearance factor	1.719	grows with record length

Use these metrics as trends against a baseline from the same machine, sensor and operating condition. Crest factor and kurtosis usually rise when a localized defect first appears, then fall as the damage spreads and the RMS grows, so read them together with RMS. Clearance factor needs a high sample rate: sqrt|x| changes sharply at each zero crossing, and at 200 samples per cycle a sine reads about 0.2% high.

4. Characteristic Frequencies

$$f_{Rice} = \frac{1}{2\pi}\,\frac{\sigma_{\dot{x}}}{\sigma_x} \qquad f_{zc} = \frac{N_{zc}}{T} \qquad f_{pk} = \frac{(N_{max} + N_{min})/2}{T}$$

T is the record duration, x-dot is the time derivative (calculated numerically), N_{zc} is the number of positive-slope zero crossings, and N_{max} and N_{min} are the numbers of local maxima and minima.
For a narrowband signal all three equal the dominant frequency. For broadband vibration the peak rate exceeds the zero-crossing rate; the ratio f_{zc} / f_{pk} (irregularity factor) approaches 1 for narrowband and 0 for very broadband data.
The Rice frequency is the expected zero-crossing rate of a Gaussian process. If the measured zero-crossing rate differs strongly from it, the signal is not Gaussian or is dominated by a few tones.

5. Histograms & Envelope

The histogram shows the amplitude distribution; a Gaussian bell shape indicates random vibration, while heavy tails indicate impulses. The absolute peak histogram uses |x| at each local maximum and minimum. The cumulative histogram is the fraction of samples at or below each amplitude. The envelope plot outlines the time history so that bursts and amplitude modulation are easy to see.

6. Gear Metrics (optional)

Inputs: shaft speed (rpm), number of gear teeth N_t, mesh harmonics removed and shaft orders removed. The shaft speed is assumed constant. The record must contain at least one revolution, and (N_t + 1) times the shaft frequency must be below the Nyquist frequency.

$$f_r = \frac{rpm}{60} \qquad f_{mesh} = N_t\, f_r \qquad \bar{y}(\theta) = \frac{1}{R}\sum_{r=1}^{R} x\left(t_0 + \frac{r-1}{f_r} + \frac{\theta}{2\pi f_r}\right)$$

Time synchronous average (TSA): the record is resampled to N points per revolution and the R complete revolutions are averaged. Components that are not synchronous with the shaft (other shafts, bearings, random noise) average toward zero.

$$d = \bar{y} - [\ \text{shaft orders } 0 \ldots k \ \ + \ \ \text{mesh harmonics } mN_t \ \ + \ \ \text{sidebands } mN_t \pm 1\ ], \quad m = 1 \ldots M$$

The difference signal d is the TSA with the regular meshing components removed, done in the FFT of one revolution where each bin is one shaft order. The regular signal r is the mesh harmonics m N_t only.

$$\mathrm{FM4} = \frac{N\sum_{i=1}^{N} d_i^4}{\left(\sum_{i=1}^{N} d_i^2\right)^2} \qquad \mathrm{M6A} = \frac{N^2\sum_{i=1}^{N} d_i^6}{\left(\sum_{i=1}^{N} d_i^2\right)^3}$$

$$\mathrm{M8A} = \frac{N^3\sum_{i=1}^{N} d_i^8}{\left(\sum_{i=1}^{N} d_i^2\right)^4} \qquad \mathrm{ER} = \frac{\mathrm{RMS}(d)}{\mathrm{RMS}(r)}$$

For a healthy gear d is close to Gaussian: FM4 is about 3, M6A about 15 and M8A about 105. Localized damage on one or two teeth (a crack, pit or chip) produces a short disturbance once per revolution and raises FM4, M6A and M8A; M6A and M8A react more strongly and earlier. These metrics fall back toward the Gaussian values when damage spreads to many teeth. ER (energy ratio) rises with heavy, distributed wear, which FM4 does not detect well.
Use more revolutions for better noise suppression; fewer than 10 is noted in the output. As with the other metrics, trends against a baseline are more reliable than single values.

References

J. J. Zakrajsek, D. P. Townsend and H. J. Decker, "An Analysis of Gear Fault Detection Methods as Applied to Pitting Fatigue Failure Data", NASA TM-105950, 1993.
R. M. Stewart, "Some Useful Data Analysis Techniques for Gearbox Diagnostics", Report MHM/R/10/77, Machine Health Monitoring Group, Institute of Sound and Vibration Research, University of Southampton, 1977.