Order Tracking: Sample Rate, Nyquist Frequency & Anti-Alias Low-Pass Filter

order_tracking_sample_rate_a ver 1.2 by Tom Irvine

Speeds in rpm, frequencies in Hz and sample rates in samples per second (sps): the same in English and metric units.

1. Shaft frequency $f_{shaft} = \dfrac{N}{60}$ N = shaft speed (rpm)

2. Maximum order (order Nyquist) $O_{max} = \dfrac{S}{2}$ S = samples per revolution

3. Clean orders (filter margin) $O_{clean} \approx \dfrac{S}{2.56}$

4. Order-domain Nyquist at top speed $f_{max} = O_{max}\, f_{shaft,max} = O_{max}\, \dfrac{N_{max}}{60}$

5. Bandwidth to capture (passes O_{hi}) $f_{bw} = O_{bw}\, f_{shaft,max}$ $O_{bw} = \max\left(O_{max},\, \dfrac{O_{hi}}{k_{LP}}\right)$

6. Nyquist minimum (brick-wall filter only) $f_s > 2\, f_{bw}$

7. Recommended sample rate (analyzer) $f_s = k_{fs}\, f_{bw}$ $k_{fs} = 2.56$

8. Nyquist frequency $f_{Nyq} = \dfrac{f_s}{2}$

9. Folding frequency and roll-off room $f_{fold} = f_s - f_{bw}$ roll-off ratio $= \dfrac{f_s - f_{bw}}{f_{bw}}$

10. Anti-alias low-pass, fixed (top speed) $f_{LP} = k_{LP}\, f_{bw} \geq O_{hi}\, f_{shaft,max}$ $k_{LP} \approx 0.9$

11. Anti-alias low-pass, tracking speed $f_{LP}(t) = k_{LP}\, O_{bw}\, f_{shaft}(t)$

12. Resampled rate and Nyquist $f_{eff} = S\, \dfrac{N}{60}$ $f_{Nyq,eff} = O_{max}\, \dfrac{N}{60}$

13. Order resolution, block size, duration $\Delta O = \dfrac{1}{R}$ $n = R\, S$ $T_B = \dfrac{60\, R}{N}$

14. Smallest power-of-2 S keeping O_{hi} clean $S = 2^{\lceil \log_2(2.56\, O_{hi}) \rceil}$

$O_{bw} > O_{max}$ only when $O_{hi} > k_{LP}\, O_{max}$: the sample rate is then raised so the low-pass does not cut off order O_{hi}. O_{bw} is capped at O_{max}/k_{LP}. If $O_{hi} > O_{max}$, increase S. Without the raise ($O_{bw} = O_{max}$), $f_{bw} = f_{max}$.

Worked example (slides 8 and 10): $N_{max} = 3000$ rpm, $N_{min} = 800$ rpm, $S = 64$, $R = 16$

$f_{shaft,max} = 3000/60 = 50$ Hz, $O_{max} = 64/2 = 32$, $f_{max} = 32 \times 50 = 1600$ Hz

$f_s > 2 \times 1600 = 3200$ sps minimum; $f_s = 2.56 \times 1600 = 4096$ sps; $f_{Nyq} = 2048$ Hz

Fixed low-pass $f_{LP} = 0.9 \times 1600 = 1440$ Hz; tracking at 800 rpm: $0.9 \times 32 \times 13.33 = 384$ Hz

$\Delta O = 1/16 = 0.0625$; $T_B = 0.32$ s at 3000 rpm, 1.2 s at 800 rpm

Raised sample rate: same, with $O_{hi} = 20$ and $k_{LP} = 0.6$ $(0.6 \times 32 = 19.2 < 20)$

$O_{bw} = \max(32,\ 20/0.6) = 33.33$, $f_{bw} = 33.33 \times 50 = 1666.7$ Hz

$f_s = 2.56 \times 1666.7 = 4266.7$ sps; $f_{Nyq} = 2133.3$ Hz; $f_{LP} = 0.6 \times 1666.7 = 1000$ Hz $= 20 \times 50$

Source: T. Irvine, Order Tracking and Phase Analysis, revC, slides 7-10.