Time Synchronous Averaging: Gear and Shaft Diagnostics

time_synchronous_average_a ver 1.0 by Tom Irvine

Notation

$y(t)$ vibration, T_k start time of revolution k (every P-th tach crossing, or $T_k = t_0 + 60k/n$ at a constant speed n rpm), N points per revolution, M revolutions averaged, N_t teeth on the gear on the tach shaft, N_m teeth on the mating gear, K shaft orders and H mesh harmonics removed, $\bar{x}$ mean of x over one revolution of N points.

Equations

1. Tach crossing (linear interp.) $$t_c = t_i + \frac{V_{th} - v_i}{v_{i+1} - v_i}\,(t_{i+1} - t_i)$$

2. Angle-to-time map (spline) $$t_k(\theta_j) = S\left(k + \frac{j}{N}\right), \quad S(k) = T_k, \quad \theta_j = \frac{360\,j}{N}\ \text{deg},\ j = 0 \ldots N-1$$

3. Time synchronous average $$x(\theta_j) = \frac{1}{M}\sum_{k=0}^{M-1} y\left(t_k(\theta_j)\right)$$

4. Noise reduction $$\sigma_{TSA} = \sigma/\sqrt{M}$$ (random and non-synchronous components)

5. Order spectrum $$X_o = \frac{1}{N}\sum_j x(\theta_j)\, e^{-i2\pi oj/N}, \qquad A_o = 2\,|X_o|, \quad o = 1 \ldots N/2 - 1$$

6. Mesh orders, mating gear $$o_{mesh} = h\,N_t, \quad f_{mesh} = N_t\,n/60, \quad o_{mate} = N_t/N_m$$

7. Residual signal $$r = x - \bar{x} - \{\text{orders } 1 \ldots K\} - \{\text{orders } hN_t,\ h = 1 \ldots H\}$$

8. Difference signal $$d = r - \{\text{orders } hN_t \pm 1\}$$ (first-order sidebands)

9. RMS, crest factor $$\text{RMS} = \sqrt{\frac{1}{N}\sum x_j^2}, \qquad CF = \frac{\max|x_j|}{\text{RMS}}$$

10. FM0 $$FM0 = \frac{PP_x}{\sum_{h=1}^{H} A_{hN_t}}, \qquad PP_x = \max x - \min x$$

11. FM4 $$FM4 = \frac{N\sum_j (d_j - \bar{d})^4}{\left[\sum_j (d_j - \bar{d})^2\right]^2}$$ (≈ 3 for a Gaussian d)

12. NA4 (one data set) $$NA4 = \frac{N\sum_j (r_j - \bar{r})^4}{\left[\sum_j (r_j - \bar{r})^2\right]^2}$$

- Orders are removed by zeroing the FFT bins o and $N - o$ of the TSA, which is exact because the TSA holds whole revolutions.
- With $K = 0$ the residual is the TSA minus the mesh harmonics only. $K = 1$ (default) also removes the 1x shaft order.
- NA4 normally divides by the variance averaged over all earlier data sets; with one record this reduces to the kurtosis of r.
- Hunting tooth: the same pair of teeth meets again after $N_m/\gcd(N_t, N_m)$ revolutions of the gear on the tach shaft.

Units

Time sec, speed rpm, angle deg, frequency Hz, orders per revolution: the same in English and metric. The amplitude unit (G, m/sec^2, in/sec, mm/sec, mil, micron, V) only labels the results: FM0, FM4, NA4 and crest factor are dimensionless.

Assumptions

- The shaft speed varies smoothly, so a spline through the revolution start times gives the time of each shaft angle.
- One tach crossing per revolution (or P evenly spaced ones) at a fixed shaft angle, which is 0 deg in the plots.
- Use N at least the original samples per revolution, or low-pass filter first, so orders above $N/2$ do not fold into the TSA.

Checks (test script, synthetic 24-tooth gear, ±3 % speed, defect at 120 deg)

Clean signal: TSA vs exact angle-domain waveform, rms error 5.8×10^{-4} of 0.775. Mesh order 24 largest, $A_{24} = 0.999$ (1.0).
Noise 0.5 rms: TSA noise 0.242, 0.118, 0.057 for $M = 4$, 16, 64, i.e. $\sigma\sqrt{M} = 0.484$, 0.471, 0.459.
FM4 = 3.05 without the defect, 27.5 with it; difference-signal peak at 120.9 deg. Constant-speed and tach modes agree to 0.2 %.

References

1. P.D. McFadden, A revised model for the extraction of periodic waveforms by time domain averaging, *Mech. Syst. Signal Process.* 1(1), 1987, 83–95. (TSA and the $1/\sqrt{M}$ noise reduction.)
2. R.M. Stewart, *Some Useful Data Analysis Techniques for Gearbox Diagnostics*, Univ. of Southampton, ISVR MHM/R/10/77, 1977 (FM0, FM4)
3. J.J. Zakrajsek, D.P. Townsend, H.J. Decker, *An Analysis of Gear Fault Detection Methods as Applied to Pitting Fatigue Failure Data*, NASA TM-105950, 1993 (NA4). 4. J.J. Zakrajsek, MS thesis, Cleveland State University, 1989 (FM4).
5. MathWorks, Predictive Maintenance Toolbox documentation: *gearConditionMetrics, tsaresidual, tsadifference* (formulas checked here).

Note: the FM0, FM4 and NA4 formulas and the residual / difference definitions were confirmed from the MathWorks pages (ref. 5), which cite refs. 3 and 4. Refs. 1–4 were not consulted directly.