Torsional Critical Speeds: Lumped Machine Train with Gear Stages

torsional_critical_speeds_a ver 1.0 by Tom Irvine

Model

Rigid inertias J_i (stations) joined by torsional springs $k_{t,e}$ (shafts, couplings, gear meshes). Shaft e joins station e to $e+1$. Undamped. Each station turns at n_i times the reference shaft speed (speed ratio).

1. Speed ratio $n_i = N_i / N_{ref}$ (reference shaft: $n = 1$)
2. Referred inertia $J_{eq,i} = J_i \, n_i^2$
3. Referred stiffness $k_{eq,e} = k_{t,e} \, n_e^2$ (n_e = ratio of the shaft $k_{t,e}$ is measured on)
4. Shaft stiffness $k_t = \dfrac{G J_p}{L}$ $J_p = \dfrac{\pi}{32}\left(D_o^4 - D_i^4\right)$
5. Mass matrix $\mathbf{M} = \text{diag}\,(J_{eq,1}, \ldots, J_{eq,N})$
6. Stiffness matrix $\mathbf{K} = \sum_e k_{eq,e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ at stations $e,\ e+1$
7. Eigenvalue problem $\left[\mathbf{K} - \omega^2 \mathbf{M}\right]\{\phi\} = \mathbf{0}$ $f_n = \omega / 2\pi$
8. End conditions free-free: drop the rigid-body mode $\omega = 0$; grounded: $\phi_1 = 0$

Mode shapes

Twist ϕ is the referred angle, normalized to a peak of 1. A node (zero twist) lies where ϕ changes sign.

Modal strain energy in shaft e: $SE_e = 100 \dfrac{k_{eq,e}(\phi_{e+1} - \phi_e)^2}{\sum_e k_{eq,e}(\phi_{e+1} - \phi_e)^2}$ % (the shaft to stiffen or soften)

Interference speeds

Excitation order h on a shaft with speed ratio s has frequency $f = h \, s \, N_{ref}/60$. It meets mode f_n at

$$N_{cr} = \frac{60 \, f_n}{h \, s} \quad \text{rpm of the reference shaft}$$

With *Apply orders to every shaft speed* checked, the orders are applied to each distinct speed ratio, otherwise to $s = 1$ only.

Separation margin (operating range N_{min} to N_{max}, reference shaft)

$SM = 100 \dfrac{N_{min} - N_{cr}}{N_{min}}$ % below the range, $SM = 100 \dfrac{N_{cr} - N_{max}}{N_{max}}$ % above the range

An interference speed with SM less than the required margin is flagged.

Set the required margin from the applicable standard.

Units

English: J lbm-in^2, k_t lbf-in/rad, L, D in, G psi. Inertias are divided by 386.088 in/s^2 to give lbf-in-s^2.

metric: J kg-m^2, k_t N-m/rad, L, D m, G Pa.

Speeds in rpm and frequencies in Hz in both systems. Steel: $G \approx$ 11.5e6 psi (79.3 GPa).

Reference: W. Ker Wilson, *Practical Solution of Torsional Vibration Problems*, Chapman & Hall.

The Holzer method on the Rod tab gives the same frequencies for a chain with all speed ratios = 1.