vibrationdata_rocket: Equations

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Twenty-four tabs cover thirty-three equations of rocket propulsion and launch vehicle dynamics, from thrust and specific impulse to cold gas blowdown, staging, orbit transfer, escape velocity, propellant slosh, POGO and combustion instability. Each tab has English or metric units, its equations in a strip at the top of the tab, and a results panel. This document gives the equations, assumptions and references for each tab. The ideal-rocket relations assume a perfect gas of fixed composition and isentropic flow.

The thirty-three equations

1. Thrust
2. Specific impulse, exhaust velocity
3. Ideal exit velocity
4. Total impulse
5. Specific impulse from total impulse
6. Area-Mach relation
7. Characteristic velocity c*
8. Thrust coefficient
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32. Cold gas tank blowdown
33. Cold gas thrust and impulse

Units

Quantity	*English*	*metric*
Force, total impulse	lbf, lbf-s	N, N-s
Pressure	psi	kPa
Velocity	ft/s	m/s
Mass, mass flow	lbm, lbm/s	kg, kg/s
Temperature (absolute)	R	K
Area (nozzle), diameter	in^2, in	cm^2, mm
Length, altitude	ft	m
Orbit altitude, radius	mi	km
Density, volume	lbm/ft^3, ft^3	kg/m^3, m^3
Burn rate	in/s	mm/s
Gas viscosity, stiffness	lbf-s/ft^2, lbf/in	Pa-s, N/mm

Changing the units converts the entered values. Calculations are done in SI with g_0 = 9.80665 m/s^2 and R_u = 8314.46 J/kmol-K. Specific impulse is in seconds in both systems.

1. Thrust (Refs. 1, 3)

Eq. 1: $F = \dot{m}\, V_e + (p_e - p_a)\, A_e \qquad c = \dfrac{F}{\dot{m}} = V_e + \dfrac{(p_e - p_a)\, A_e}{\dot{m}}$

Eq. 2: $I_{sp} = \dfrac{F}{\dot{m}\, g_0} = \dfrac{c}{g_0} \qquad F_{vac} = \dot{m}\, V_e + p_e A_e \qquad g_0 = 9.80665 \text{ m/s}^2$

$\dot{m}$ = propellant mass flow, V_e = exit velocity, p_e, p_a = exit and ambient pressure, A_e = exit area, c = effective exhaust velocity.

Momentum thrust plus pressure thrust. $p_e < p_a$: overexpanded; below about $0.4\, p_a$ the flow may separate (Summerfield criterion, approximate).

Inputs: mdot propellant, Ve exit velocity, pe exit, abs, pa ambient, abs, Ae exit area.

2. Exit Velocity (Refs. 1, 2, 4)

Eq. 3: $V_e = \sqrt{\dfrac{2\gamma}{\gamma - 1} \dfrac{R_u T_c}{\mathcal{M}} \left[1 - \left(\dfrac{p_e}{p_c}\right)^{(\gamma-1)/\gamma}\right]} \qquad R = \dfrac{R_u}{\mathcal{M}} \qquad V_{e,max} = \sqrt{\dfrac{2\gamma}{\gamma - 1} R T_c}$

$M_e = \sqrt{\dfrac{2}{\gamma - 1} \left[\left(\dfrac{p_c}{p_e}\right)^{(\gamma-1)/\gamma} - 1\right]} \qquad T_e = T_c \left(\dfrac{p_e}{p_c}\right)^{(\gamma-1)/\gamma}$

Ideal rocket: isentropic expansion of a perfect gas of fixed composition from the chamber ($V \approx 0$) to the exit. R_u = 8314.46 J/kmol-K,

$\mathcal{M}$ = molar mass of the combustion gas, T_c = chamber temperature, γ = ratio of specific heats. Take γ, $\mathcal{M}$ and T_c from NASA CEA (Ref. 4) or tables (Ref. 1).

Inputs: gamma (cp/cv), Molar mass (kg/kmol), Tc chamber temp., pc chamber, abs, pe exit, abs.

3. Speed of Sound in Gas (Refs. 4, 14)

Eq. 21: $a = \sqrt{\gamma\, R\, T}$ $\qquad R = \dfrac{R_u}{\mathcal{M}}$ $\qquad R_u = 8314.46$ J/kmol-K $\qquad \gamma = \dfrac{c_p}{c_v}$ $\qquad M = \dfrac{V}{a}$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

Ideal gas, T = absolute temperature, $\mathcal{M}$ = molar mass. Typical γ (Ref. 14): dry air 1.4, liquid fuel/oxidizer combustion gas 1.22 to 1.33,

solid propellant combustion gas 1.17 to 1.26. Preset γ values are near 300 K; take combustion-gas values from NASA CEA (Ref. 4).

Inputs: gamma (cp/cv), Molar mass (kg/kmol), T temperature, abs., V velocity, opt.

4. Specific & Total Impulse (Refs. 1, 3, 9)

Eq. 4: $I_t = \displaystyle\int_0^{t_b} F\, dt = \bar{F}\, t_b$ $\qquad$ Eq. 5: $I_{sp} = \dfrac{I_t}{m_p\, g_0}$ $\qquad c = \dfrac{I_t}{m_p} = I_{sp}\, g_0$ $\qquad \dot{\bar{m}} = \dfrac{m_p}{t_b}$

class: A = 1.26 to 2.5 N-s, each letter doubles

I_t = total impulse, t_b = burn time, m_p = propellant mass, $\bar{F}$ = average thrust. Thrust-time table: trapezoidal rule.

Impulse classes of NFPA 1125: 1/2A to 1.25 N-s, A 1.26 to 2.50, B to 5.00, C to 10, D to 20, E to 40, F to 80, G to 160, H to 320, and so on.

Inputs: F average thrust, tb burn time (s), mp propellant mass.

5. Nozzle Expansion (Refs. 1, 3, 5)

Eq. 6: $\dfrac{A}{A_t} = \dfrac{1}{M}\left[\dfrac{2}{\gamma+1}\left(1+\dfrac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$ $\qquad \dfrac{p}{p_c} = \left(1+\dfrac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}}$

$\dfrac{T}{T_c} = \left(1+\dfrac{\gamma-1}{2}M^2\right)^{-1}$ $\qquad \dfrac{p_t}{p_c} = \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ $\qquad \dfrac{V_e}{V_{e,max}} = \sqrt{1-\left(\dfrac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}}$

Isentropic flow of a perfect gas, sonic at the throat ($M = 1$). The area ratio has a subsonic and a supersonic solution; the nozzle exit is supersonic.

The Mach number for a given area ratio is found by root finding. p_t = throat pressure.

Inputs: gamma (cp/cv), Ae/At area ratio, pc/pe pressure ratio.

6. c* & Thrust Coefficient (Refs. 1, 2, 3, 7)

Eq. 7: $c^* = \dfrac{p_c A_t}{\dot{m}} = \dfrac{\sqrt{R\,T_c}}{\Gamma}$ $\qquad \Gamma = \sqrt{\gamma}\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$ $\qquad$ Eq. 8: $F = C_F\, p_c A_t$

$C_F = \lambda\sqrt{\dfrac{2\gamma^2}{\gamma-1}\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\dfrac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \dfrac{p_e - p_a}{p_c}\,\dfrac{A_e}{A_t}$ $\qquad \lambda = \dfrac{1+\cos\alpha}{2}$

$I_{sp} = \dfrac{c^* C_F}{g_0}$

c^* = characteristic velocity (chamber performance), C_F = thrust coefficient (nozzle performance), α = cone half-angle ($\lambda = 1$ for $\alpha = 0$).

Choked mass flow $\dot{m} = \Gamma\, p_c A_t/\sqrt{R\,T_c}$ (Ref. 3). The c^* efficiency scales the ideal c^* (typical 95 to 99%).

Inputs: gamma (cp/cv), Molar mass (kg/kmol), Tc chamber temp., pc chamber, abs, Dt throat diameter, Ae/At area ratio, pa ambient, abs, c* efficiency (%), Cone half-angle (deg).

7. Nozzle Sizing (Refs. 1, 2, 7)

Eq. 9: $A_t = \dfrac{F}{C_F\, p_c}$ $\qquad D_t = \sqrt{\dfrac{4A_t}{\pi}}$ $\qquad A_e = \dfrac{A_e}{A_t}\, A_t$ $\qquad \dot{m} = \dfrac{p_c A_t}{c^*}$

$\dfrac{A_e}{A_t}$ from $M_e(p_e/p_c)$ $\qquad$ Eq. 10: $V_c = L^* A_t$ $\qquad L_n = \dfrac{D_e - D_t}{2\tan\alpha}$

Design point: the exit pressure fixes the exit Mach number and area ratio; the required thrust then fixes the throat. $p_e = p_a$ gives optimum expansion.

L^* = characteristic chamber length (Ref. 2), V_c = chamber volume to the throat, L_n = conical divergent length (no throat radius of curvature).

Inputs: gamma (cp/cv), Molar mass (kg/kmol), Tc chamber temp., F required thrust, pc chamber, abs, pe design exit, abs, pa ambient, abs, L* char. length, Cone half-angle (deg).

8. Altitude Performance (Refs. 1, 8)

Eq. 11: $F(h) = F_{vac} - p_a(h)\, A_e$ $\qquad I_{sp}(h) = \dfrac{F(h)}{\dot{m}\, g_0}$ $\qquad \dot{m} = \dfrac{F_{vac}}{I_{sp,vac}\, g_0}$

$p_a(h)$: U.S. Standard Atmosphere 1976, 0 to 86 km $\qquad p_a = p_e \;\Rightarrow$ optimum expansion

Fixed nozzle, constant chamber pressure and flow: the momentum thrust and $p_e A_e$ do not change with altitude, so thrust rises by $p_a A_e$ from sea level to vacuum.

Above 86 km the ambient pressure is taken as zero.

Inputs: F vacuum thrust, Isp vacuum (s), Ae exit area, pe exit, abs, Altitude for results.

9. Rocket Equation (Refs. 1, 3)

Eq. 12: $\Delta v = I_{sp}\, g_0\, \ln \dfrac{m_0}{m_f}$ $\qquad MR = \dfrac{m_0}{m_f} = e^{\Delta v/(I_{sp} g_0)}$ $\qquad m_p = m_0 - m_f$

$\Delta v_{vertical} = I_{sp}\, g_0\, \ln \dfrac{m_0}{m_f} - g_0\, t_b$ $\qquad \zeta = \dfrac{m_p}{m_0}$

Ideal (Tsiolkovsky) rocket equation: no gravity or drag, constant effective exhaust velocity $c = I_{sp}\, g_0$ (Ref. 3). m_0 = initial mass, m_f = burnout mass.

With a burn time, the vertical gravity loss $g_0\, t_b$ is subtracted (Ref. 3). ζ = propellant mass fraction.

Inputs: Isp (s), m0 initial mass, mp propellant mass, mf burnout mass, Delta-v required, tb (s), optional.

10. Multistage (Ref. 1)

Eq. 13: $\Delta v = \sum_{k=1}^{N} I_{sp,k}\, g_0\, \ln \dfrac{m_{0,k}}{m_{f,k}}$ $\qquad m_{0,k} = m_{pl} + \sum_{j \geq k} (m_{p,j} + m_{d,j})$

$m_{f,k} = m_{0,k} - m_{p,k}$ $\qquad$ payload fraction $= \dfrac{m_{pl}}{m_{0,1}}$

Serial staging: each stage burns all its propellant and drops its dry mass before the next stage ignites. m_{pl} = payload, m_d = stage dry mass.

No gravity or drag losses.

Inputs: Number of stages, Payload mass.

11. Thrust & Burn Time (Refs. 1, 3)

Eq. 14: $\dot{m} = \dfrac{F}{I_{sp}\, g_0}$ $\quad t_b = \dfrac{m_p}{\dot{m}}$ $\quad \dfrac{T}{W} = \dfrac{F}{m\, g_0}$ $\quad v_{bo} = c \ln \dfrac{m_0}{m_f} - g_0 t_b$

$h_{bo} = c\, t_b \left[1 - \dfrac{m_f}{m_p} \ln \dfrac{m_0}{m_f}\right] - \dfrac{g_0 t_b^2}{2}$ $\quad h_{coast} = \dfrac{v_{bo}^2}{2 g_0}$

Constant thrust and mass flow, vertical flight, constant gravity, no drag: the burnout altitude and coast height are upper bounds.

h_{bo} is the exact integral of $v(t) = c \ln[m_0/(m_0 - \dot{m}t)] - g_0 t$. The vehicle lifts off only if $T/W > 1$.

Inputs: F thrust, Isp (s), m0 liftoff mass, mp propellant mass.

12. Solid Motor (Refs. 1, 6)

Eq. 15: $r = a\, p_c^n$ $\quad$ Eq. 16: $p_c = (a\, \rho_p\, c^*\, K_n)^{1/(1-n)}$ $\quad K_n = \dfrac{A_b}{A_t}$

$\dot{m} = \rho_p A_b\, r = \dfrac{p_c A_t}{c^*}$ $\quad F = C_F\, p_c A_t$ $\quad t_b = \dfrac{w}{r}$ $\quad \dfrac{dp_c}{p_c} = \dfrac{1}{1-n} \dfrac{dK_n}{K_n}$

Steady state: the gas generated by the burning surface equals the nozzle flow (Refs. 1, 6). a, n = burn-rate coefficient and exponent (Saint-Robert law),

ρ_p = propellant density, A_b = burning area, w = web thickness. Neutral burning assumed for t_b; $n < 1$ for stable burning.

Inputs: a burn-rate coef., n pressure exponent, rho propellant, c* char. velocity, Ab burning area, Dt throat diameter, CF thrust coefficient, Web thickness.

13. Hybrid Motor (Refs. 1, 11)

$$\text{Eq. 17: } \dot{r} = a\, G_{ox}^{n} \qquad G_{ox} = \frac{4\,\dot{m}_{ox}}{\pi D^2} \qquad \dot{m}_f = \rho_f\, \pi D L\, \dot{r} \qquad O/F = \frac{\dot{m}_{ox}}{\dot{m}_f}$$

$$D^{2n+1} = D_0^{2n+1} + (2n+1)\, 2a \left(\frac{4\dot{m}_{ox}}{\pi}\right)^{n} t$$

Single circular port, constant oxidizer flow, regression rate set by the oxidizer mass flux (Refs. 1, 11). As the port grows, G_{ox} falls and the mixture ratio shifts.

Burn time from the closed-form port growth, $dD/dt = 2\dot{r}$.

Inputs: mdot oxidizer, D port, initial, D port, final, L grain length, a regression coef., n flux exponent, rho fuel density.

14. Cold Gas Blowdown (Refs. 1, 3)

$$\text{Eq. 32: } \dot{m} = \frac{\Gamma\, p\, A_t}{\sqrt{R\,T}} \qquad \tau = \frac{V}{\Gamma A_t \sqrt{R\,T_0}} \qquad \text{isothermal: } \frac{p}{p_0} = e^{-t/\tau} \qquad m = \frac{p\,V}{R\,T}$$

$$\text{isentropic: } \frac{p}{p_0} = \left(1 + \frac{\gamma-1}{2}\,\frac{t}{\tau}\right)^{-\frac{2\gamma}{\gamma-1}} \qquad \frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{Eq. 33: } F = A_t \left(C_{F,vac}\, p - p_a\, \frac{A_e}{A_t}\right) \qquad I_t = \int_0^{t_b} F\, dt$$

Blowdown: the gas tank feeds the nozzle with no regulator, so the chamber pressure is the tank pressure. Choked throat, Γ as in Eq. 7. V = tank volume, p_0, T_0 = initial tank state.

Isothermal (slow, heat from the tank walls) and isentropic (fast, adiabatic) blowdowns bound the real process (Ref. 1). $C_{F,vac}$ = ideal vacuum thrust coefficient. Ideal gas: at 200 bar N_2 has $Z \approx 1.05$.

Inputs: gamma (cp/cv), Molar mass (kg/kmol), T0 initial gas temp., p0 initial tank, abs, Vt tank volume, Dt throat diameter, Ae/At area ratio, pa ambient, abs, pf end tank pressure.

15. Propellant Tanks (Refs. 1, 2)

Eq. 18: $m_{ox} = m_p \dfrac{O/F}{1+O/F}$ $\qquad m_f = \dfrac{m_p}{1+O/F}$ $\qquad V = \dfrac{m}{\rho}\left(1+\dfrac{u}{100}\right)$

$\rho_{bulk} = \dfrac{m_p}{m_{ox}/\rho_{ox} + m_f/\rho_f}$ $\qquad D_{sphere} = \left(\dfrac{6V}{\pi}\right)^{1/3}$

u = ullage (%). Preset densities (kg/m^3): LOX 1141, LH2 70.8 and LCH4 422.6 at the normal boiling point; N2O4 1443, 90% H2O2 1390, RP-1 810,

MMH 874, UDMH 791 and ethanol 789 near 20 C (Refs. 1, 2). Densities change with temperature; use the value at the tank conditions.

Inputs: mp total propellant, O/F mixture ratio, rho oxidizer, rho fuel, Ullage (%).

16. Propellant Slosh (Ref. 13)

Eq. 22: $\omega_n^2 = \dfrac{\bar{g}}{a}\,\epsilon_n \tanh\left(\epsilon_n \dfrac{h}{a}\right)$ $\qquad m_n = 2\,m_L\,\dfrac{\tanh(\epsilon_n h/a)}{\epsilon_n\,(h/a)\,(\epsilon_n^2-1)}$ $\qquad m_L = \rho\,\pi a^2 h$

$l_n = \dfrac{a}{\epsilon_n}\tanh\left(\epsilon_n \dfrac{h}{a}\right)$ $\qquad L_n = \dfrac{a}{\epsilon_n}\coth\left(\epsilon_n \dfrac{h}{a}\right)$ $\qquad l_{pn} = h - l_n + L_n$

$\epsilon_n = 1.841,\ 5.331,\ 8.536,\ 11.706$

Upright circular cylinder with a flat bottom, inviscid liquid, linear lateral slosh (Ref. 13). a = tank radius, h = liquid height, $\bar{g}$ = axial acceleration,

m_n = slosh mass, l_n = depth of m_n below the free surface, L_n = pendulum length, l_{pn} = pivot above the bottom, $k_n = m_n\omega_n^2$. Valid in the barrel section.

Inputs: D tank diameter, h liquid height, rho liquid, Axial accel. (g), Number of modes.

17. Chamber Acoustic Modes (Refs. 14, 17)

Eq. 23: $f_n = \dfrac{n\,c}{2L}$ $\qquad c = \sqrt{\gamma\,\dfrac{R_u}{\mathcal{M}}\,T}$ $\qquad \lambda_n = \dfrac{2L}{n}$ $\qquad$ Eq. 24: $f_{mn} = \dfrac{\alpha_{mn}\,c}{\pi D}$

1T 1.841, 2T 3.054, 1R 3.832, 3T 4.201 $\qquad f_{1L1T} = c\sqrt{\left(\dfrac{1}{2L}\right)^2 + \left(\dfrac{\alpha_{11}}{\pi D}\right)^2}$

Longitudinal modes of a closed-closed cavity (small throat), Ref. 14. Castor 4B example: $\gamma = 1.26$, $\mathcal{M} = 20$, $T = 1922$ K, $L = 27$ ft: $c = 3292$ ft/s, $f_1 = 61$ Hz.

Transverse modes of a rigid cylinder (Ref. 17): α_{mn} = roots of $J'_m(\alpha) = 0$. Frequency classes (Ref. 14): low 10 to 200 Hz, intermediate 200 to 1000 Hz, high above 1000 Hz.

Inputs: gamma (cp/cv), Molar mass (kg/kmol), T gas temperature, c sound speed, opt., L cavity length, D diameter, opt., Longitudinal modes.

18. Frequency Sweep (Ref. 14)

Eq. 25: $f = \dfrac{c}{\lambda}$ $\qquad \dfrac{df}{dt} = -\dfrac{c}{\lambda^2}\,\dfrac{d\lambda}{dt} = -\dfrac{f}{\lambda}\,\dfrac{d\lambda}{dt}$ $\qquad f(t) = \dfrac{c}{\lambda + \dot{\lambda}\,t}$

SR-19 example: $\dfrac{df}{dt} = -\dfrac{625\text{ Hz}}{68\text{ in}}\,(1.64\text{ in/s}) \approx -15\text{ Hz/s}$

As the propellant burns, the cavity grows and the acoustic wavelength increases, so the mode frequency sweeps down (Ref. 14). Constant sound speed assumed.

For longitudinal mode n, $\lambda = 2L/n$ and $d\lambda/dt = (2/n)\,dL/dt$.

Inputs: f frequency (Hz), lambda wavelength, d(lambda)/dt, Duration (s), opt.

19. Helmholtz Resonator (Refs. 14, 16)

Eq. 26: $f_H = \dfrac{c}{2\pi}\sqrt{\dfrac{A}{V\,L_{eff}}}$ $\qquad c = \sqrt{\gamma\,\dfrac{R_u}{\mathcal{M}}\,T}$ $\qquad A = \dfrac{\pi d^2}{4}$

tuned to the chamber mode to be damped

Helmholtz resonator, as used in acoustic liners and cavities that damp combustion instability (Refs. 14, 16). A = opening area, V = cavity volume,

L_{eff} = effective length of the opening (neck length plus end corrections), c = speed of sound of the gas in the resonator. Lumped model: resonator small against the wavelength.

Inputs: gamma (cp/cv), Molar mass (kg/kmol), T gas temperature, c sound speed, opt., A opening area, V cavity volume, Leff opening length.

20. SRM Vortex Shedding (Ref. 14)

Eq. 27: $f_v = Sr\,\dfrac{U}{L_v}$ $\qquad Sr = \dfrac{m-\alpha}{M+1/k}$ $\qquad M = \dfrac{U}{c}$ $\qquad f_n = \dfrac{n\,c}{2L}$

lock-in when f_v is near an acoustic mode

Vortex shedding in a solid rocket motor port (Ref. 14, after Dotson, Womack and Grosserode). U = mean port velocity, L_v = distance from vortex initiation to impingement,

m = number of vortices in the path, k = vortex convection parameter, α = normalized acoustic time lag (k and α are empirical). The 10% lock-in flag is a screening aid.

Inputs: U mean port velocity, Lv vortex path length, c sound speed, L cavity length, opt., Sr Strouhal, opt., m vortices in path, k convection param., alpha time lag.

21. Particle Damping (Ref. 14)

Eq. 28: $\tau_p = \dfrac{\rho_p\, d_p^2}{18\,\mu_g}$ $\qquad Stk = \omega\,\tau_p$ $\qquad \omega = 2\pi f$ $\qquad d_{opt} = \sqrt{\dfrac{18\,\mu_g}{\rho_p\,\omega}}$ $\;(Stk = 1)$

Condensed particles (for example alumina) in the combustion gas, Stokes drag (Ref. 14). $Stk \ll 1$: particles follow the gas; $Stk \gg 1$: particles nearly stationary;

$Stk \approx 1$: most acoustic damping. ρ_p = particle density, d_p = particle diameter, μ_g = gas viscosity.

Inputs: rho particle, d particle (um), mu gas viscosity, f frequency (Hz).

22. POGO Frequency Check (Refs. 14, 15)

Eq. 29: $\omega_L = \sqrt{\dfrac{k_L}{m}}$ $\qquad f_s = \dfrac{\omega_L}{2\pi}$ $\qquad \dfrac{f_p}{f_s}$ $\qquad$ POGO band: 5 to 60 Hz

Structural longitudinal mode as an SDOF system (Ref. 14): k_L = effective axial stiffness, m = effective vibrating mass. POGO couples this mode with the propulsion

(feedline, pump, engine) dynamics; typical POGO frequencies are 5 to 60 Hz (Refs. 14, 15). The 10% flag is a screening aid; use NASA SP-8055 (Ref. 15) for design.

Inputs: kL axial stiffness, m effective mass, f propulsion (Hz), opt.

23. Orbit & Hohmann Transfer (Refs. 10, 12)

Eq. 19: $v_c = \sqrt{\dfrac{\mu}{r}}$ $\quad T = 2\pi\sqrt{\dfrac{r^3}{\mu}}$ $\quad v_{esc} = \sqrt{\dfrac{2\mu}{r}}$ $\quad a_t = \dfrac{r_1 + r_2}{2}$

Eq. 20: $\Delta v_1 = \sqrt{\mu\left(\dfrac{2}{r_1} - \dfrac{1}{a_t}\right)} - \sqrt{\dfrac{\mu}{r_1}}$ $\quad \Delta v_2 = \sqrt{\dfrac{\mu}{r_2}} - \sqrt{\mu\left(\dfrac{2}{r_2} - \dfrac{1}{a_t}\right)}$ $\quad t_t = \pi\sqrt{\dfrac{a_t^3}{\mu}}$

Two-body motion, circular coplanar orbits, impulsive burns (Ref. 10). $r = R + h$, $\mu = GM$ of the central body (Ref. 12).

The Hohmann transfer is the two-burn minimum-energy transfer between circular orbits (for $r_2/r_1 < 11.94$).

Inputs: mu = GM, R body radius, h1 initial altitude, h2 final altitude.

24. Escape Velocity (Refs. 10, 12)

Eq. 30: $v_{esc} = \sqrt{\dfrac{2\mu}{r}} = \sqrt{2}\, v_c$ $\quad r = R + h$ $\quad v_c = \sqrt{\dfrac{\mu}{r}}$ $\quad$ Eq. 31: $v_{bo} = \sqrt{v_\infty^2 + \dfrac{2\mu}{r}}$

$C_3 = v_\infty^2$ $\quad \Delta v = v_{bo} - v_c$

Two-body motion, no atmosphere (Ref. 10): the speed at which kinetic energy equals the gravitational potential energy, $\frac{1}{2}v^2 = \mu/r$. $\mu = GM$ and R from Ref. 12.

Hyperbolic departure: v_∞ = hyperbolic excess speed, C_3 = characteristic energy; Δv = impulsive burn from a circular orbit at radius r. Earth surface: 11.19 km/s.

Inputs: mu = GM, R body radius, h altitude, v infinity, opt.

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The example inputs (combustion properties, burn-rate and regression constants, densities) are illustrative. Use NASA CEA (Ref. 4), test data or the references for the propellant and conditions at hand.