A frequency domain approach for fatigue life estimation from Finite Element Analysis

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Abstract

Fatigue damage is traditionally determined from time signals of loading, usually in the form of stress or strain. This approach is satisfactory for periodic loading but requires very large time records to accurately describe random loading processes. These may prove prohibitive for many finite element analyses especially when modelling dynamic resonance. Alternatively, a compact frequency domain fatigue calculation can be utilised where the random loading and response are categorised using Power spectral density (PSD) functions and the dynamic structure is modelled as a linear transfer function. This paper will review the available methods for performing fatigue analysis from PSDs and shows that the Dirlik method gives the best comparable results with the traditional time domain approaches. It also demonstrates how these techniques can be implemented in the Finite Element environment to rapidly identify critical areas in the structure.

1. Introduction

It is often easier to obtain a PSD of stress rather than a time history. Take, for instance, the dynamic analysis of complicated finite element models. Here it is often beneficial to carry out a rapid frequency response (transfer function) analysis instead of a computationally intensive transient dynamic analysis in the time domain. The offshore oil industry faced this problem in the early 1980's. An offshore oil platform is a hugely complex structure that is subjected to random wind and wave loading. A typical design analysis may have to consider over 70 load combinations on the structure. The analysis is further complicated because the imposed loads are random and dynamically excite the structure. A transient dynamic analysis in the time domain proves impossible to carry out in this case.

A Finite Element analysis based in the frequency domain can simplify the problem considerably. The designer can now carry out a frequency response analysis on the FE model to determine the transfer function between wave height and stress in the structure. Using this, he simply multiplies the PSD of wave height by the transfer function to arrive at the PSD of stress. A review of this method is presented.

To take advantage of the fast frequency domain techniques for fatigue analysis, we require a method of deducing damage from the PSD of stress.

In this paper we will briefly review the technology behind time domain S-N analysis and then draw a parallel approach in the frequency domain. A comparison study between the different fatigue analysis techniques is presented. The paper concludes by describing the Finite Element approach for computing fatigue damage on vibrating components.

2. Review of S-N analysis in the time domain

The starting point for any fatigue analysis is the response of the structure or component. In the time domain this is usually expressed as a stress or strain time history. Fatigue occurs as a result of stress or strain reversals in the time history. These are known as cycles. The significant aspects of these are the

ranges of stress in the cycle and also there mean stress. Today the range and mean information is usually extracted from the time history using a procedure known as 'Rainflow Cycle Counting'. Matsuishi and Endo first introduced the concept of rainflow ranges to the scientific community over twenty years ago. An example of the way rainflow ranges are extracted from a time signal is given by Downing [1].

The output from a rainflow cycle counting exercise is usually expressed as a range mean histogram such as that shown in Figure 1[†]. The stress range of each cycle is given along the x axis, its mean stress is shown on the y axis and the z axis gives the number of cycles contained in the time history for each particular range and mean. This data was taken from the Howden HWP330 wind turbine; Bishop [2] discusses the analysis.
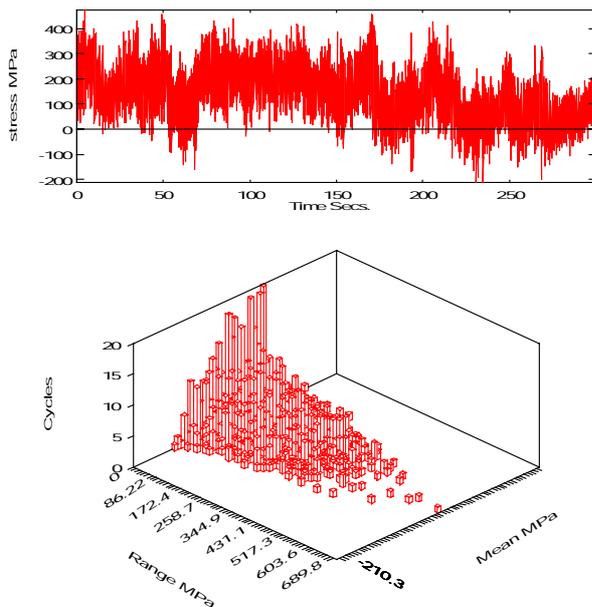

Figure 1 Range-mean histogram derived from time history by rainflow cycle counting

Each cycle will induce a certain amount of fatigue damage on the component. The total damage caused by the time history can therefore be obtained by summing the damage caused by each cycle shown in the stress range histogram. This approach is known as the Palmgren-Miner accumulated damage

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rule after the two independent people who proposed it.

The damage caused by each cycle is calculated by reference to the material life curve, in this case the SN curve. The SN curve shows the number of cycles to failure, N_f, for a given stress range, S. The total damage caused by N number of cycles is therefore obtained as the ratio of cycles to the number of cycles to failure. The Palmgren-Minor rule can therefore be expressed as Equation (1).

$$Accumulated\ Damage = \sum_i \frac{N_i}{N_f} \qquad (1)$$

Where, N_i is the number of cycles with a particular stress range and mean; i is a ranging variable covering all the possible range and mean combinations; and N_f is the number of cycles to failure for a particular stress range and mean.

The accumulated damage is expressed as a proportion of the damage required to fail the material. Therefore the fatigue life for the component can be determined from Equation (2).

$$Fatigue\ Life = \frac{Length\ of\ time\ history}{Accumulated\ Damage} \qquad (2)$$

The fatigue life for the data shown in figure 1 was found to be 0.2 years based on an aluminium alloy 2024_HV_T4.

3. Simple fatigue analysis using time history recreation

The most obvious method for calculating fatigue from a PSD is to regenerate a characteristic time history and proceed as described above. This section describes the theory of time history regeneration from PSDs and highlights some of the fundamental assumptions involved.

The frequency domain is simply another domain in which to view a time signal; the x-axis now represents frequency instead of time. To convert a time signal into the frequency domain we effectively split it up into a number of discrete sinusoidal waves of varying amplitude, frequency and phase. When these are added together they form the original time signal. The algorithm used to split

the time history into its constituent sinusoidal components is the 'Fourier Transform'. This returns a complex vector of values where each value represents the amplitude and phase of the particular sinusoidal wave at a particular frequency. The amplitude can be obtained from the modulus of the complex number while the phase is determined from the argument.

In practice we usually represent the frequency domain as a 'Power Spectral Density (PSD)' plot. This is a normalised density plot describing the mean square amplitude of each sinusoidal wave with respect to its frequency. A typical PSD plot is show in Figure 2. The mean square amplitude of a constituent sinusoidal wave can be determined by measuring the area under the PSD over the desired frequency range. The amplitude can then be estimated using Equation 3. The PSD does not contain any phase data.
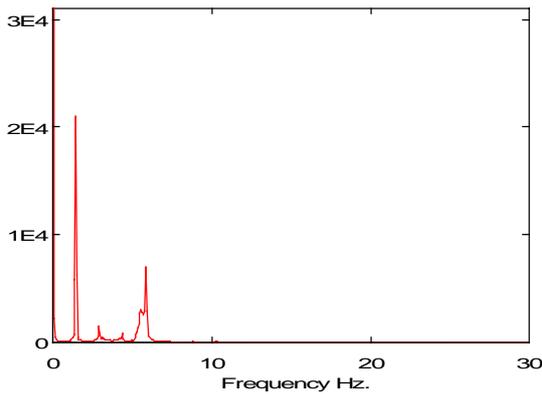

Figure 2 Typical PSD of a random time history

$$Amplitude = \sqrt{2 \cdot Mean\,Square} \qquad (3)$$

To regenerate a time signal from the frequency domain we would usually perform an 'Inverse Fourier Transformation' on the complex vector of frequency domain results. This would yield exactly the same time history as we started with. When starting with a PSD this method is inappropriate because the PSD does not contain any of the original phase information. For certain time histories, however, we are able to make assumptions about the original phase content and therefore regenerate a statistically equivalent time history. If the time history is taken from an 'ergodic stationary Gaussian random process', it is found that the phase is purely random between $-\pi$ and $+\pi$ radians.

A process is said to be stationary if its statistics are not affected by a shift in the time origin. (I.e. the statistics of a time history X(t) are the same as a time history X(t + τ) for all values of τ.) To test for stationarity we take a number of recordings of the random process at different times. The process is stationary if the probability distributions of the ensemble are the same for all points in time. If the ensemble probability density function is Gaussian then the process is known as a Gaussian random process. A stationary process is called an ergodic process if statistics taken from one sample are the same as those obtained for the ensemble. With an ergodic stationary random process, therefore, we can effectively take a single sampled time history from the process and safely assume that this contains all the required statistical properties of the parent process. For nonstationary processes the statistics obtained from a sampled time history would not be representative of those of the whole random process as these would be continuously changing. In practice we see that many time signals obey this criterion. Naturally occurring phenomena like wind speed, wave height, etc. usually obey this along with vibrations from generators and engines. Strictly speaking, the later examples are not random processes, however, the vibrations occur as a result of many deterministic sources and the central limit theorem tends to make them behave in the desired fashion.

To regenerate a time signal from a PSD we therefore assume that the original process was 'ergodic stationary Gaussian and random'. We can then generate random phase angles and add these to the amplitude data given in the PSD. At this point we use the Inverse Fourier Transformation to determine a statistically equivalent time history.

4. Fast fatigue analysis methods in the frequency domain

This section describes a variety of approaches for computing fatigue life, or damage, directly from the PSD of stress as opposed to a time history. The techniques fall into two broad categories, those that estimate fatigue life directly and those that compute range mean histograms as an intermediate stage. For more background information the reader is referred to Bishop [3] and Halfpenny [4]. As before, these methods are only applicable for PSDs describing an ergodic stationary Gaussian and random process.

In 1964 Bendat [5] proposed the first significant step towards a method of determining fatigue life from PSDs. Bendat showed that the probability density function (pdf) of peaks for a narrow band signal tended towards a Rayleigh distribution as the bandwidth reduced. Furthermore, for a narrow banded time history Bendat assumed that all positive peaks in the time history would be followed by corresponding troughs of similar magnitude regardless of whether they actually formed stress cycles. Using this assumption the pdf of stress range would also tend to a Rayleigh distribution. To complete his solution method, Bendat used a series of equations derived by Rice [7] to estimate the expected number of peaks using moments of area under the PSD. Bendat's narrow band solution for the range mean histogram is therefore expressed in Equation (4).

$$N(S) = E[P] \cdot T \cdot \left\{ \frac{S}{4 \cdot m_0} \cdot e^{-\frac{S^2}{8 \cdot m_0}} \right\} \quad (4)$$

Where N is the number of cycles of stress range S occurring in T seconds. m_0 is the zeroth moment of area under the PSD (i.e. the area beneath the curve), and $E[P]$ is the expected number of peaks obtained by Equation (5).

$$E[P] = \sqrt{\frac{m_4}{m_2}} \quad (5)$$

m_4 and m_2 are the 4th and 2nd moments of area of the PSD respectively where the nth moment of area is obtained as $m_n = \int f^n \cdot G(f) df$, and $G(f)$ is the value of the single sided PSD at frequency f Hz.

The term in brackets in Equation (4) is the Raleigh distribution. Figure 3 shows the range-mean histogram obtained from the PSD of the time history given in Figure 1. The range mean histogram given by Bendat contains no cycle mean data.
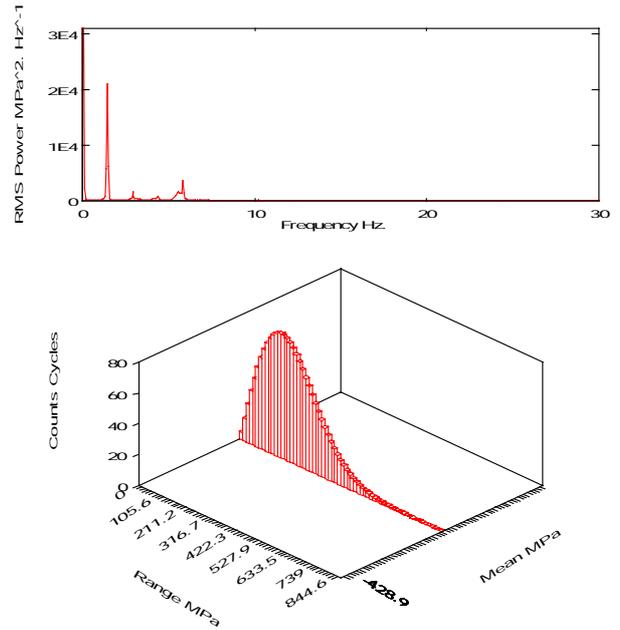

Figure 3 Range-mean histogram derived from PSD using Bendat's method

The problem with Bendat's narrow band solution is that it is extremely conservative when wider band time histories are used. (We now observe a fatigue life of only 280.1 hours.) The reason for this lies in the assumption that peaks are matched with corresponding troughs of similar magnitude. This effect is illustrated in Figure 4. A narrow band time history is characterised by each peak having a corresponding valley of similar magnitude. In comparison, a wide band time history is characterised by smaller waves riding on a low frequency carrier. As Bendat assumes that all positive peaks are matched with corresponding valleys of similar magnitude, the damage is grossly exaggerated for wide band histories as shown.
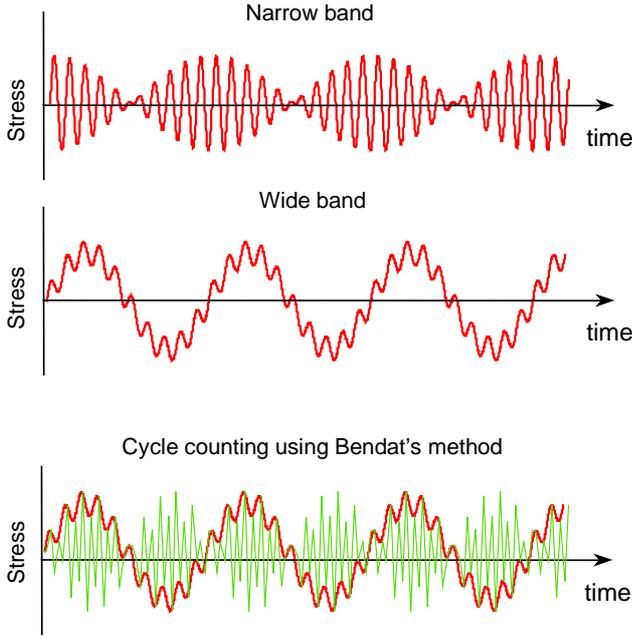
Figure 4 Why Bendat's method is conservative

During the 1980's the need for a rapid fatigue analysis method based in the frequency domain became apparent to the offshore oil industry. Large jacket platforms were being designed and fatigue failures had to be avoided. The transient dynamic analysis proved too intensive for the time domain software because of the large structural models and high number of possible load combinations. The dynamic wave and wind load data was already provided in the frequency domain and it therefore seemed sensible to make use of the speed advantages inherent in a frequency domain analysis. The problem was how to calculate a reasonably accurate fatigue life using the resultant PSDs from the frequency domain analyses.

Sea state spectra are relatively wide banded and this effectively rules out the use of Bendat's narrow band fatigue analysis because the results prove too conservative. Several methods were developed to address this problem, the notable ones being Wirsching , Kam & Dover and Hancock. These are semi-empirical approaches based on the narrow band solution. The latter two methods are both in the form of an equivalent stress parameter and neither tends to work particularly well when used outside the offshore platform industry. Wirsching's approach was developed for the offshore industry but has been found to be applicable to a wider class of industrial problems.

In other industries advances were also being made.

Steinberg and Tunna both worked on the problem for the electronics and rail industries respectively. Again neither of these methods tend to work well outside their respective industries. For more information on these methods see Bishop [3].

In 1985 Dirlik proposed an empirical closed form solution to the problem following extensive computer simulations using the Monte Carlo technique. Although apparently more complicated than some alternative methods it is still only a function of four moments of area of the PSD, these being m_0, m_1, m_2 and m_4. This method has been found to be widely applicable and constantly outperforms all of the other available methods. The Dirlik formulation is given in Equation (6).

$$N(S) = E[P] \cdot T \cdot p(S) \qquad (6)$$

Where, $N(S)$ is the number of stress cycles of range S N/mm^2 expected in time T sec. $E[P]$ is the expected number of peaks obtained by Equation (5).

$$p(S) = \frac{\frac{D_1}{Q} \cdot e^{\frac{-Z}{Q}} + \frac{D_2 \cdot Z}{R^2} \cdot e^{\frac{-Z^2}{2 \cdot R^2}} + D_3 \cdot Z \cdot e^{\frac{-Z^2}{2}}}{2 \cdot \sqrt{m_0}}$$

$$D_1 = \frac{2 \cdot (x_m - \gamma^2)}{1 + \gamma^2} \qquad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R}$$

$$D_3 = 1 - D_1 - D_2 \qquad Z = \frac{S}{2 \cdot \sqrt{m_0}}$$

$$Q = \frac{1.25 \cdot (\gamma - D_3 - D_2 \cdot R)}{D_1}$$

$$R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2} \qquad \gamma = \frac{m_2}{\sqrt{m_0 \cdot m_4}}$$

$$x_m = \frac{m_1}{m_0} \cdot \sqrt{\frac{m_2}{m_4}}$$

Figure 5 shows the range mean histogram obtained using Dirlik's method for the wind turbine data used in Figure 1. Again the method ignores the cycle mean stresses but now gives an improved range mean histogram that is comparable with that

obtained using a time domain approach. The fatigue life is correctly calculated as 0.2 years.
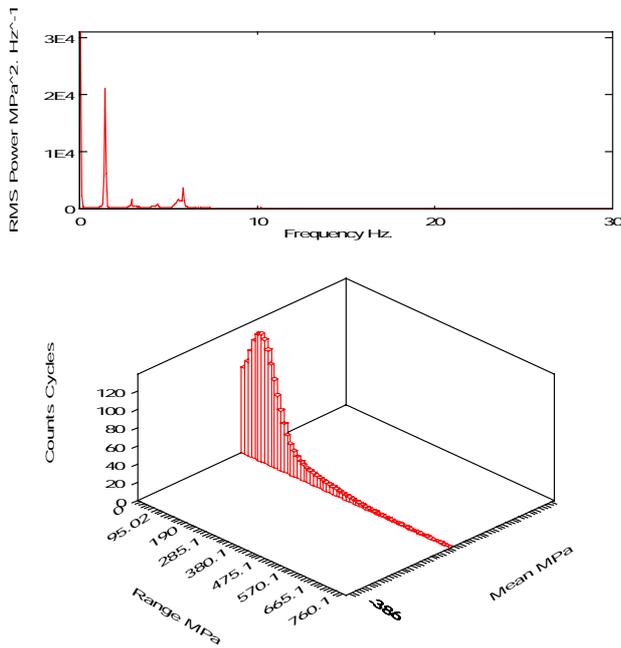

Figure 5 Range-mean histogram derived from PSD by Dirlik's method

In terms of accuracy, Dirlik's empirical formula for rainflow ranges has been shown to be far superior to the previously available correction factors. However, the need for certification of the technique before its use meant that theoretical verification was required. Bishop [6] achieved this when a theoretical solution for predicting rainflow ranges from the moments of area of the PSD was produced. The theoretical approach devised by Bishop is computationally intensive and shows little improvement on accuracy over Dirlik's empirical approach. For this reason, Bishop's method gives credence to the Dirlik method but is seldom used for analysis purposes.

5. Comparison between fatigue analysis techniques

Many comparative studies have been carried out to ascertain the accuracy of all the various methods. The studies made by Bishop [2] are of notable significance. These studies took data from the Howden HWP330 wind turbine, computed fatigue lives in the frequency domain and then compared these with the life obtained from the time domain

approach. A summary of the results is given in Table 1.

Load Case	Narrow Band	Dirlik	Wirsching	Bishop	Chaudhury	Hancock
y12a	5.14	1.03	3.91	1.52	2.13	2.75
y19a	5.15	1.00	3.92	1.54	2.14	2.77
y27a	14.34	1.59	10.91	1.74	5.12	5.83
y35a	81.87	2.34	62.23	1.95	30.08	25.08
y12b	1.91	0.77	1.46	1.13	0.98	1.25
y19b	1.98	0.81	1.50	1.22	1.04	1.31
y27b	3.67	1.07	2.79	1.29	1.47	1.92
y35b	18.34	1.48	13.95	1.84	5.68	6.10
y12c	1.98	0.76	1.51	0.86	0.95	1.25
y19c	1.87	0.73	1.43	0.86	0.92	1.20
y27c	2.03	0.74	1.54	0.72	0.87	1.14
y35c	3.22	0.76	2.45	0.66	1.15	1.42
y12d	2.09	0.84	1.59	1.15	1.03	1.33
y19d	2.03	0.83	1.54	1.17	1.02	1.31
y27d	2.92	1.01	2.22	1.15	1.23	1.62
y35d	7.50	1.12	5.70	1.23	2.75	3.29
y12e	2.80	0.99	2.13	1.27	1.50	1.95
y19e	3.06	1.01	2.33	1.44	1.64	2.12
y27e	3.50	1.03	2.67	1.53	1.65	2.16
y35e	8.81	1.11	6.71	1.99	3.31	4.15
y12f	3.86	0.98	2.93	1.43	1.66	2.18
y19f	3.97	1.00	3.02	1.61	1.78	2.33
y27f	3.96	1.01	3.01	1.57	1.76	2.31
y35f	5.59	0.98	4.25	1.65	2.17	2.80
avrg	7.98	1.04	6.08	1.36	3.08	3.32

Table 1 Comparison between different frequency domain fatigue analyses

The table clearly shows that the Dirlik approach is remarkably robust. It shows an average discrepancy of only 4% from the fatigue life calculated in the time domain.

6. FE based vibration analysis in the frequency domain

The previous section discussed methods of determining fatigue life from PSDs of stress. This section considers how these PSDs are obtained from finite element analysis.

The dynamic behaviour of components can be determined both in the time and frequency domains. In the time domain this involves a complicated and often lengthy transient analysis. In the frequency domain it uses a very fast 'transfer function' technique. Essentially the frequency domain breaks down a signal into its constituent sinusoidal waves following Fourier's theory. The transfer function relates the amplitude of the input force or moment to the amplitude of the output stress for each frequency of sinusoidal wave. An example transfer

function plot is shown in Figure 6. This relates input force (N) to output stress (MPa) for a certain node on a FE model. The peaks represent resonant modes in the component.
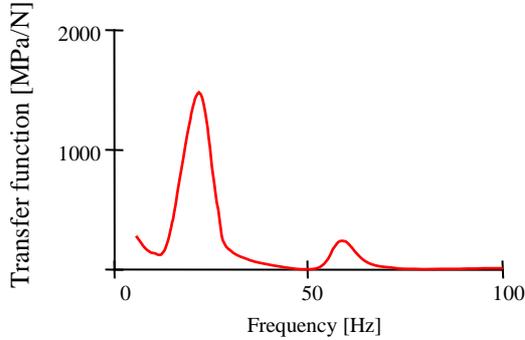

Figure 6 Transfer function relating input force to output stress at a node

6.1 Calculating the linear transfer function

Consider the simple bracket component shown in Figure 7 subjected to a single input load. PSDs of stress are required at each node on the model in order to determine the fatigue life of the component.

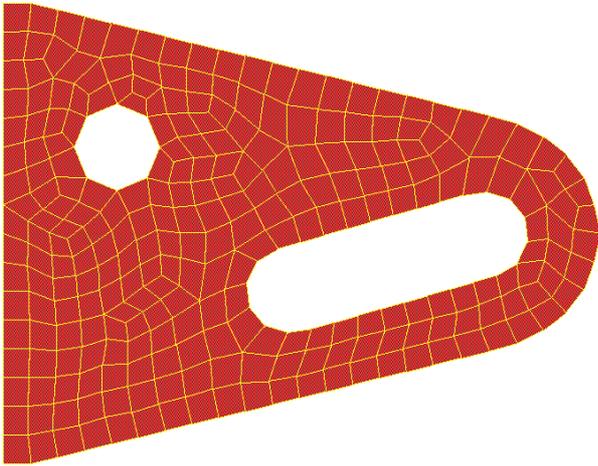The equation of motion for the component is given in Equation (7).

Figure 7 Simple FE model of a mounting bracket

$$[M] \cdot \ddot{x}(t) + [C] \cdot \dot{x}(t) + [K] \cdot x(t) = f(t) \qquad (7)$$

Where: $[M]$ is the global mass matrix
$[C]$ is the global damping matrix
$[K]$ is the global stiffness matrix
$x(t)$ is the vector of nodal displacements
$f(t)$ is the vector of nodal forces

In the case of a single input system the vector $f(t)$ has only one non-zero component.

If the input load is sinusoidally varying with respect to time then the displacement vector $v(t)$ will also be sinusoidally varying. Expressing this in exponential form yields the expressions for load and displacement shown in Equation (8).

$$f(t) = F \cdot e^{i \cdot \omega \cdot t} \qquad x(t) = X \cdot e^{i \cdot \omega \cdot t} \qquad (8)$$

Where: F is the amplitude of the sinusoidally varying force of frequency ω *rad/sec* and X is the amplitude of the induced displacement.

Differentiating displacement with respect to time yields expressions for the velocity and acceleration of the component. These are given in Equation (9).

$$\dot{x}(t) = i \cdot \omega \cdot X \cdot e^{i \cdot \omega \cdot t} \qquad (9)$$
$$\ddot{x}(t) = -\omega^2 \cdot X \cdot e^{i \cdot \omega \cdot t}$$

Substituting these into the Equation (7) and simplifying yields the following formula for the amplitude of displacement with respect to frequency.

$$X(\omega) = H(\omega) \cdot F(\omega) \qquad (10)$$

Where: $H(\omega)$ is the linear transfer function given by:

$$H(\omega) = \left[-[M] \cdot \omega^2 + [C] \cdot i \cdot \omega + [K] \right]^{-1}$$

X(ω) is the vector of displacement amplitude
F(ω) is the vector of force amplitude

It is usual to compute the transfer function between input force and output stress instead of displacement shown in Equation (10). This is carried out in the FE program following the back substitution stage.

Classical FE analysis results in a tensor of 6 component stresses at each node. These consist of 3 axial and 3 shear stress components and are expressed mathematically in Equation (11).

Axial $S_{xx}(\omega),\ S_{yy}(\omega),\ S_{zz}(\omega) \qquad (11)$

Shear $\quad S_{xy}(\omega),\ S_{yz}(\omega),\ S_{xz}(\omega)$

N.B. for the two dimensional plate elements shown in Figure 7, the values of $S_{zz}(\omega)$, $S_{yz}(\omega)$ and $S_{xz}(\omega)$ are all zero.

For the purpose of fatigue analysis a single PSD is required for each node showing some stress invariant such as von-Mises, Tresca or the absolute maximum principal stress. These are obtained by taking the eigenvalues of the stress tensor matrix given in Equation (12).

$$
\begin{bmatrix} \sigma_1(\omega) \\ \sigma_2(\omega) \\ \sigma_3(\omega) \end{bmatrix} = eigenvalues \left\{ \begin{bmatrix} S_{xx}(\omega) & S_{xy}(\omega) & S_{xz}(\omega) \\ S_{xy}(\omega) & S_{yy}(\omega) & S_{yz}(\omega) \\ S_{xz}(\omega) & S_{yz}(\omega) & S_{zz}(\omega) \end{bmatrix} \right\}
$$

(12)

The desired stress invariant can be obtained from Equation (12). The modulus sign is necessary because the principal stresses returned by Equation (12) are complex.

Maximum principal: (13)

$$
\sigma_{\max}(\omega) = \max\big(|\sigma_1(\omega)|, |\sigma_2(\omega)|, |\sigma_3(\omega)|\big)
$$

von-Mises:

$$
\sigma_{von-Mises}(\omega) = \sqrt{ \tfrac{1}{2} \cdot \left(\begin{array}{c} |\sigma_1(\omega) - \sigma_2(\omega)|^2 + \\ |\sigma_2(\omega) - \sigma_3(\omega)|^2 + \\ |\sigma_3(\omega) - \sigma_1(\omega)|^2 \end{array} \right) }
$$

Tresca:

$$
\sigma_{Tresca}(\omega) = \max \left(\begin{array}{c} |\sigma_1(\omega) - \sigma_2(\omega)|, \\ |\sigma_2(\omega) - \sigma_3(\omega)|, \\ |\sigma_3(\omega) - \sigma_1(\omega)| \end{array} \right)
$$

Where $\sigma_{xxx}(\omega)$ is the transfer function with respect to frequency ω rad/sec expressed as a stress invariant. Frequency can also be expressed in terms of Hertz after using the appropriate conversion.

6.2   Single force input

Having found the relationship between nodal stress invariant and applied load, the stress PSD $G(f)$ for a single input load PSD $W(f)$ can be found very

rapidly by multiplying the input PSD by the transfer function as Equation (14).

$$
G(f) = |H(f)|^2 \cdot W(f)
$$

(14)

Figure 8 shows the fatigue log damage plot arising on the bracket due to a vertical vibration at the bolt hole. The critical fatigue life is determined as 206 hours.
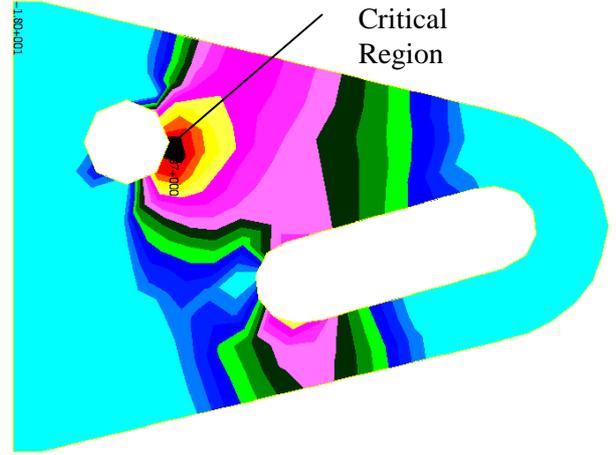

Figure 8 Fatigue contor plot of Log Damage

6.3   Multiple force input

To this point we have only considered the response on an element resulting from a single random input. This section introduces the analysis of multiple random inputs. Figure 9 shows a simple example of a flagpole with two flags flying at different heights. A typical application would be to determine the bending stress at the base of the flagpole as a result of the two random wind speeds seen by the flags.

The time history of wind speed at the location of the flags can be recorded using anemometers. PSDs of wind speed can be calculated from these; however, the PSDs alone do not provide information on the phase relationships between the two measured time histories.
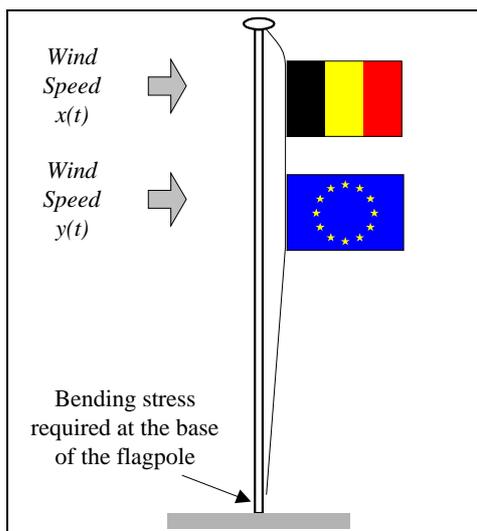
Figure 9 Example of two random process inputs

With multiple random processes we also require the sequential relationship between the two time histories. If the two flags are far enough apart then the wind speed witnessed by one will be completely independent of that on the other. As they are moved closer together then a correlation between the two time histories will be seen. The two time histories are correlated because the random wind turbulence incident on one flag has a sufficiently large range of influence to also affect the response at the other.

To calculate the bending stress at the base of the flagpole it is insufficient to simply sum the reactions from the two input PSDs, instead we must sum the reactions from the input and cross-power spectra. The cross-power spectra contain information on the joint statistics of the two processes. If the two processes are correlated then the sequencing effects may act to increase or decrease the base bending stress depending on whether the forces are in or out-of-phase. For a mathematical explanation of this see Newland [8]. The single-sided PSD function of stress at the base of the flagpole, $G_{zz}(f)$, is therefore determined by Equation (15).

$$G_{zz}(f) = \sum_{a=1}^{2} \sum_{b=1}^{2} H_a(f) \cdot H_b(f)^* \cdot W_{ab}(f) \quad (15)$$

$H_1(f)$ and $H_2(f)$ are the transfer functions relating stress at the base of the flagpole to wind load incident on flags x and y, respectively. The asterisk indicates the complex conjugate. $W_{11}(f)$ and $W_{22}(f)$ are the PSD functions of wind speed at flags x and y, respectively, and $W_{12}(f)$ and $W_{21}(f)$ are the cross-

power spectral density functions. For a general loading with n simultaneous forcing functions the PSD of stress can be obtained from Equation (16).

$$G_{zz}(f) = \sum_{a=1}^{n} \sum_{b=1}^{n} H_a(f) \cdot H_b(f)^* \cdot W_{ab}(f) \quad (16)$$

7. Conclusions

The concept of frequency domain fatigue analysis has been presented, where the random loading and response are categorised using Power spectral density (PSD) functions. All the current methods are briefly reviewed and conclusions are drawn showing that the Dirlik method is recommended for general use. The Dirlik formulation is presented along with an explanation of the FE analysis involved.

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